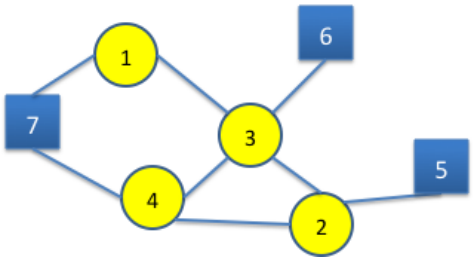
(a) (1 points) The graph on the right represents a netlist of standard cells. Circles represent N movable gates (modules) and squares represent fixed I/O pads. No weight is assumed on the edges for now. We would like to formulate the wire length minimization placement problem using quadratic optimization in the form of:



$$\text{Total cost} = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + d_x^T\mathbf{x} + \frac{1}{2}\mathbf{y}^T Q\mathbf{y} + d_y^T\mathbf{y} + \text{const}$$

where **x** and **y** are the x and y coordinate vectors of the gates, Q is an N×N matrix derived from the netlist, and the "d" vectors are derived from the fixed I/O coordinates.

Assuming that the (x,y) coordinates of I/O pads are $(x_5, y_5)=(10, 5)$, $(x_6,y_6)=(5, 8)$, $(x_7,y_7)=(0, 1)$, write the Q matrix as well as the d vectors. Show the details of your calculations.

The matrix Q is formed by setting the diagonal values to be the degree of the node, and setting the off-diagonal entries to be -1 whenever there is a connection between the two nodes and 0 otherwise.

For the netlist in this problem, matrix Q is going to be:

$$Q = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 3 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ 0 & -1 & -1 & 3 \end{bmatrix}$$

The d vectors are formed by setting the i^{th} entry to be the sum of the x (y) coordinates of all fixed I/O pads connected to the i^{th} gate:

d_x = [0 -10 -5 0]

d_y = [-1 -5 -8 -1]

(b) (0.6 points) Show the derivation of a set of linear equations from the quadratic equation in the previous part. Show the matrix form of the set of equations as well.

By taking the partial derivative of the quadratic cost function with respect to individual x_i and y_i variables, we get a set of N linear equations for the x coordinates, and N linear equations for the y coordinates (x and y coordinates can be solved independently).

For example, the terms of the cost function relevant to x1 are:

$\text{Cost}_{x1} = (x_1-x_7)^2 + (x_1-x_3)^2 = (x_1-0)^2 + (x_1-x_3)^2$

Taking the partial derivative of cost with respect to x1, we get:

$2x_1 + 2(x_1 - x_3) = 0$ (drop the coefficients "2")

The matrix form of the equations would be:

$Q\, x + d_x = 0$

$Q\, y + d_y = 0$

(c) (0.8 points) Solve the set of equations associated with the netlist of part (a). ONLY SOLVE THE X EQUATIONS. List the x coordinates of all the gates.

The set of linear equations for x coordinates is:

$2x_1 - x_3 = 0$

$3x_2 - x_3 - x_4 = 10$

$-x_1 - x_2 + 4x_3 - x_4 = 5$

$-x_2 - x_3 + 3x_4 = 0$

Solving them yields: $x_1=2$, $x_2=5.75$, $x_3=4$, $x_4=3.25$.

(d) (0.8 points) The wire length is represented as a quadratic function in the above formulation.

- Name two disadvantages of the quadratic wire length model:

It is not an accurate representation of the half-perimeter wire length, which is a much better approximation of the routing cost of a net. It also cannot directly model hyperedges.

- Suggest a heuristic that can help reduce the gap between the desired linear cost function and the quadratic function.

1- After each iteration of the quadratic optimization, we can move cells greedily to locations that improve the linear wire length (used by the FastPlace paper by Chu et. al.).

2- To keep the quadratic engine from snapping them back to the original location, we can add dummy terminals to pull them towards the location preferred by the linear model.

(e) (0.8 points) List two well-known methods for removing the overlap between gates. Comment on the (rough) time complexities of the methods.

1- Use “field” forces calculated as global forces that tend to move gates from highly congested areas to sparser areas. This is used by the Kraftwerk paper. It has a high time complexity, both in terms of calculating the field forces, and in terms of requiring many iterations to converge.

2- Use bin stretching as is done in the FastPlace paper. Time complexity is low and the x- and y-coordinates can be handled in parallel.