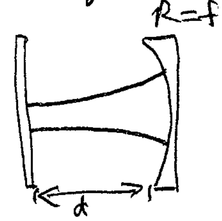
Optics Problem #7

a) This resonator is equivalent to a curved mirror with roc = f



(because a double passed lens has a focal length of f/2, but a mirror with RoC R has a focal length of R/2)

In this configuration, the beam waist is at the flat mirror on the left.

b) We want to establish a mode waist size of ω_0, corresponding to a Rayleigh range of $z_0 = \pi\omega_0^2/\lambda$. We know that the RoC of the Gaussian beam at the curved mirror has to match the RoC of the mirror (for a stable cavity). Thus, from the Gaussian beam propagation equations:

$$R = z\left[1 + \frac{z_0^2}{z^2}\right] = d\left[1 + \frac{z_0^2}{d^2}\right]$$

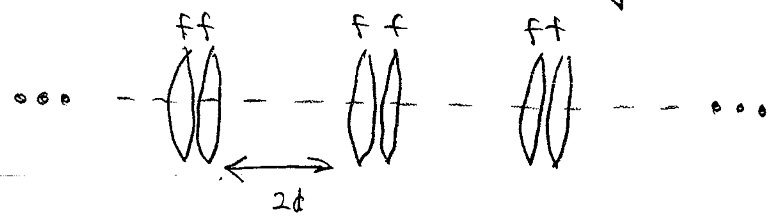
for a propagation distance of d.

$$\therefore \quad R = d + \frac{z_0^2}{d} \quad \text{or} \quad d^2 - Rd + z_0^2 = 0$$

$$\Rightarrow \quad d = \frac{R \pm \sqrt{R^2 - 4z_0^2}}{2} = \frac{R}{2} \pm \sqrt{\frac{R^2}{4} - z_0^2}, \quad (R = f)$$

where $z_0 = \dfrac{\pi\omega_0^2}{\lambda}$

c) The resonator has the following lens train analog:



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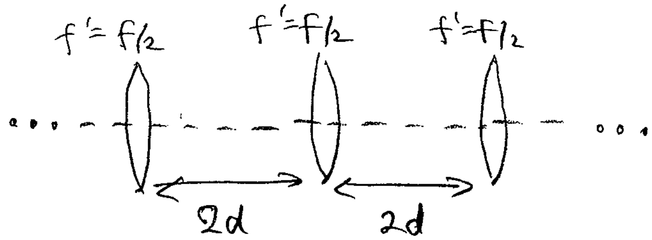


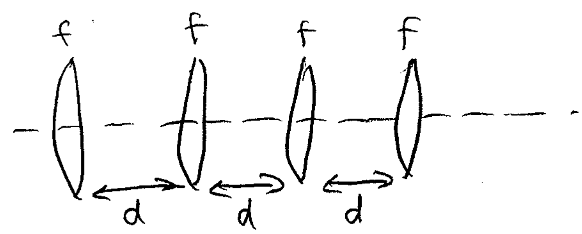
For a single-lens system, the stability is given by

$$0 \le d' \le 4f'$$

where $d' = 2d$
$f' = f/2$

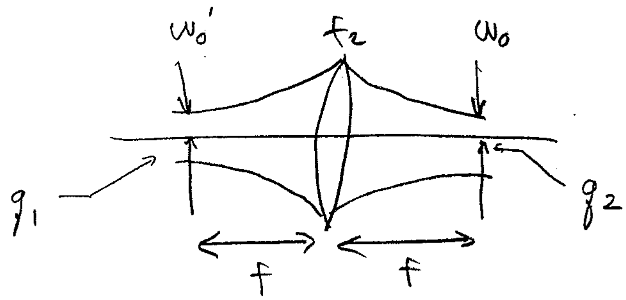
$\therefore \; 0 \le 2d \le 2f$, or $d \le f$

d) If the lens is move to the center of the resonator, then the lens train analog is given by:



and we have $0 \le d \le 4f$

e) Use ABCD matrix to relate waist in front focal plane of lens to waist in back focal plane:



$$\begin{bmatrix} 1 & f_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & f_2 \\ -\frac{1}{f_2} & 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

ABCD law states $q_2(z=f_2) = \dfrac{Aq_1 + B}{Cq_1 + D} = \dfrac{f_2}{-(q_1/f_2)} = -\dfrac{f_2^2}{q_1}$

Recall definition of q_1:

$$\frac{1}{q_1} = \underbrace{\frac{1}{R(z)} - i\frac{\lambda}{\pi n \omega^2(z)}}_{\text{In general}} = \underbrace{-\frac{i\lambda}{\pi n \omega_0'^2}}_{\text{our case}} \quad (R(z) = \infty)$$

$$\therefore \quad q_2(z=f_2) = -\frac{f_2^2}{q_1} = \frac{i f_2^2 \lambda}{\pi n \omega_0'^2}$$

and $\dfrac{1}{q_2(z=f_2)} = \dfrac{-i\pi n \omega_0'^2}{f_2^2 \lambda} = \dfrac{1}{R_2(z)} - i\dfrac{\lambda}{\pi n \omega_0^2(z)}$

$$\Rightarrow \quad \frac{\pi n \omega_0'^2}{f_2^2 \lambda} = \frac{\lambda}{\pi n \omega_0^2(z=f_2)} \quad \text{(Since index is air, } n=1\text{)}$$

$$\therefore \quad \boxed{f_2 = \frac{\pi \omega_0 \omega_0'}{\lambda}}$$