Problem 9 (a) A buck converter is to be designed to deliver power from a DC input with voltage ranging between 30 V and 60 V to a 5 V output at a switching frequency of 100 kHz. The load power is expected to vary between 10 W and 200 W.

(i) What choice of inductance will *ensure* operation in discontinuous conduction mode under *all* operating conditions (i.e., all possible values of input voltage and load power)?

(ii) What choice of inductance will *avoid* operation in discontinuous conduction mode under *all* operating conditions?

Solution 9 (a) Denote the *critical inductance* of the dc-dc buck converter by L_{crit}. Recall that the critical inductance is the minimum inductance required to avoid discontinuous conduction mode (DCM) under all operating conditions. That is, if the chosen inductor for the dc-dc buck converter, $L > L_{\text{crit}}$, then DCM is avoided. On the other hand, if the dc-dc buck converter inductor $L < L_{\text{crit}}$, then the converter always operates in DCM. For $L \geq L_{\text{crit}}$, the input and output voltages are related by

$$V_{\text{out}} = DV_{\text{in}}, \tag{1}$$

where D is the duty cycle. With the active switch turned on, we can write

$$V_{\text{in}} - V_{\text{out}} = L\frac{di}{dt}, \tag{2}$$

where i denotes the instantaneous inductor current. With a straight-line approximation for the inductor current, and recognizing that the output voltage, $V_{\text{out}} = 5$ V, we get

$$V_{\text{in}} - 5 \approx L\frac{\Delta i}{DT} = L\frac{\Delta i}{(5/V_{\text{in}})T}, \tag{3}$$

where $T = f^{-1}$ is the switching period, Δi is the current ripple, and the second equality in (3) follows from substituting for the duty cycle, D from (1). Suppose we pick the inductance of the buck converter to be L_{crit}. With this choice, the buck converter operates in the critical conduction mode (i.e., the boundary between continuous and discontinuous conduction modes). In this operating mode, $\Delta i = 2I_{\text{out}}$, where $I_{\text{out}} = P_{\text{out}}/V_{\text{out}} = P_{\text{out}}/5$ is the average output current. Substituting for $\Delta i = 2P_{\text{out}}/5$ in (3), we obtain

$$V_{\text{in}} - 5 = L_{\text{crit}}\frac{2P_{\text{out}}}{5}\frac{1}{(5/V_{\text{in}})T}. \tag{4}$$

Rearranging terms and substituting the switching frequency $f = 100$ kHz, we get,

$$L_{\text{crit}} = \left(\frac{5}{f} - \frac{25}{fV_{\text{in}}}\right)\frac{5}{2P_{\text{out}}} = \frac{1.25 \times 10^{-4}}{P_{\text{out}}}\left(1 - \frac{5}{V_{\text{in}}}\right) \text{ H}. \tag{5}$$

(i) **Ensuring DCM.** The converter will operate in DCM in all operating cases if $L < L_{\text{crit}}$. From (5), the worst-case condition is obtained for the maximum load power, $P_{\text{out}} = 200$ W and the minimum input voltage, $V_{\text{in}} = 30$ V. Substituting these in (5), we can conclude that DCM can always be ensured with an inductance $L < 0.521\ \mu$H.

(ii) **Avoiding DCM.** The converter will avoid DCM in all operating cases if $L > L_{\text{crit}}$. From (5), the worst-case condition is obtained for the minimum load power, $P_{\text{out}} = 10$ W and the maximum input voltage, $V_{\text{in}} = 60$ V. Substituting these values in (5), we can conclude that DCM can always be avoided with an inductance $L > 11.46\ \mu$H.

Solution to Part b on Power Systems

The equivalent impedance with only one line is now 0.05j pu

$$P_2 + jQ_2 = V_2((V_2 - V_1)/0.05j)^*$$

Where $P_2 = -5.0$, $Q_2 = 0$, $V_1 = 1$

The net real power and reactive power is known for bus 2 Then:

$$-5.0 = \mathrm{Re}(V_2((V_2 - V_1)/0.05j)^*)$$

$$0 = \mathrm{Im}(V_2((V_2 - V_1)/0.05j)^*$$

This results in two simultaneous equations involving V_2 and θ_2

$$(-5.0 * 0.05) / V_2 = \sin(\theta_2)$$

and

$$0 = V_2 - \cos(\theta_2)$$

Resulting in the solution

$$V_2 = \sqrt{0.933} = 0.966 \text{ pu and } \theta = -15 \text{ degrees}$$

Opening a line places greater impedance in the circuit, so the bus voltage will be lower and the angle greater for the same load.