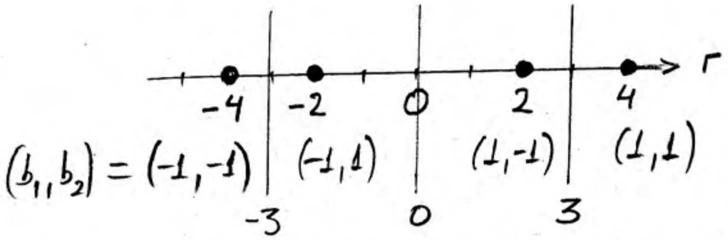
Problem #1:

a) & b):



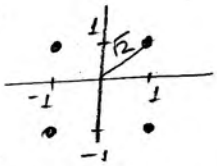
c) $BER_1 = \frac{1}{2} Q(4) + \frac{1}{2} Q(2)$, w/ $Q(z) := \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\pi}} dx$

d) $BER_2 = \frac{1}{2} \left[Q(1) - Q(4) + Q(7) \right] + \frac{1}{2} \left[Q(1) + Q(2) - Q(5) \right]$

e) Most significant / least significant bit, or any application where b_1 is more important than b_2, e.g., different alarms.

Problem #2

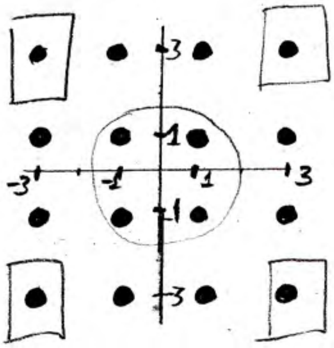
a)



$E_s = 2E_b = 2 \Rightarrow$ coordinates ± 1 to yield arc length $\sqrt{2}$

$$\boxed{d_{min}^{4QAM} = 2}$$

b)



$\bar{E}_s = \frac{1}{16} \cdot \left[4 \cdot 2 + 4 \cdot 18 + 8 \cdot 10 \right] =$

$= \frac{160}{16} = 10.$

Here $\bar{E}_s = 4\bar{E}_b$, so $\bar{E}_s$ should be 4, but it is 10; $\Rightarrow$ must scale both axes by $\frac{1}{\sqrt{2.5}}$, then $\bar{E}_s$ will be scaled by $\frac{1}{2.5}$.

$\frac{1}{\sqrt{2.5}} = \sqrt{0.4} \cong 0.6325$

Hence $\boxed{d_{min}^{16\text{-}QAM} = 2 \cdot 0.6325 = 1.2649 < 2}$

c) Since every symbol carries 2 bits, $E_s = 2E_b = 2$; and by orthogonality:

$$\begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

and $\boxed{d_{min}^{4\text{-}1} = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = 2}$

d) Here $E_s = 4E_b = 4$, thus

$$\begin{bmatrix}\sqrt{4}\\0\\\vdots\\0\end{bmatrix}, \begin{bmatrix}0\\\sqrt{4}\\0\\\vdots\\0\end{bmatrix}, \dots$$

and $\boxed{d_{min}^{16-\perp} = \sqrt{8} = 2\sqrt{2}} > 2$

e) For fixed E_b, as constellation order (and thus bits per symbol, and, for fixed T_s, also rate in bits per second) increases, d_{min} goes down for QAM, up for $\perp$-modulation. Since d_{min} is the primary factor affecting the probability of error, we see that the latter is reduced in $\perp$-modulation, increased in QAM. The not-so-hidden price paid is that (if T_s is held fixed as above) $\perp$ modulation requires a bandwidth expansion that is linear in the constellation order / exponential in the rate in bits per second.

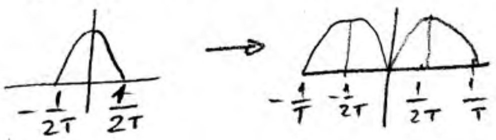
<u>Problem #3</u> Part a): $\int_0^T \cos(2\pi f_c t)\sin(2\pi f_c t)\,dt =$

$$= \frac{1}{2}\int_0^T \cancel{\sin(0)}\,dt + \frac{1}{2}\int_0^T \sin(2\pi 2 f_c t)\,dt$$

This integral is exactly equal to zero when T is an integer multiple of the period of the integrand, i.e., $\frac{1}{2f_c}$: Cond. is $T = \frac{k}{2f_c}$, or $f_c = \frac{k}{2T}$, $k \in \mathbb{Z}_+$

b) The integral is approx. equal to zero when $f_c \gg \frac{1}{T}$, because then the imbalance is $\leq \frac{1}{2f_c}$, compared to energy of I component, which is $\sim \frac{T}{2}$.

[Note: A carrier is required — smallest $f_c = \frac{1}{2T}$, and this does expand the bandwidth relative to baseband transmission



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