(a)

The minimum values of $X$, $Y$ and $P$ are all 0.

The maximum value of $X = 1011101_2 = 64 + 16 + 8 + 4 + 1 = 93$

The maximum value of $Y = 101101_2 = 32 + 8 + 4 + 1 = 45$

The maximum value of $P = (93)(45) = 4185$

A 12-bit unsigned number can represent values in the range $[0, 4095]$, which is not large enough.

A 13-bit unsigned number can represent values in the range $[0, 8191]$, which is large enough.

Therefore, the minimum number of bits needed to represent $P$ is 13.
For each term $x_i y_j$ in the above, include an AND gate as follows:

$$x_i \quad \text{AND} \quad y_j \quad x_i y_j$$
Circuit diagram - Part 1:

- Notation:
  A full adder is:

- Note: Assignment of signals to specific inputs of the full adders within a column is arbitrary.
Circuit diagram - Part 2:
(c)

The minimum value of $Z = 10111000_2 = -128 + 32 = 16 + 8 = -72$

The maximum value of $Z = 01111111_2 = 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$

The minimum value of $W = 0 + (32)(-72) = -2304$

The maximum value of $W = 4185 + (32)(127) = 8249$

A 14-bit signed number can represent values in the range $[-8192, 8191]$, which is not large enough.

A 15-bit signed number can represent values in the range $[-16384, 16383]$, which is large enough.

Therefore, the minimum number of bits need to represent $W$ without any signed overflow is 15.
(d) sign-extend $P$ and 322 to 15-bits:

$$
\begin{align*}
0 & 0 & P_{12} & P_{11} & P_0 & P_9 & P_8 & P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0 \\
Z_7 & Z_7 & Z_7 & Z_6 & 1 & 1 & 1 & Z_2 & Z_1 & Z_0 \\
W_{14} & W_{13} & W_{12} & W_{11} & W_{10} & W_9 & W_8 & W_7 & W_6 & W_5 & W_4 & W_3 & W_2 & W_1 & W_0
\end{align*}
$$

Circuit diagram - part 1:
Circuit diagram - Part 2: