There are two Parts, assigned 2 point each, for a total of 4 points.

**Part I (2 points):**
A feedback system is shown in Figure 1 where $P(s)$ is a system model and $e^{-\tau s}$ represents transmission delay in the feedback path. The system is open loop stable and its transfer function can be fairly accurately modeled over the range of frequencies that are relevant to stability analysis by

$$G(s) = \sqrt{\frac{\sqrt{2}}{s + 1}}$$

for $s = j\omega$ with $\omega$ measured in radians/sec. It is noted that the system model does have a rational transfer function, and hence it does not have a finite-dimensional state representation.

**Complete/answer the following:**

i) Draw the Bode plot for the open loop transfer function (approximately).

ii) Draw the Nyquist plot.

iii) Explain why the closed loop system is stable for zero time-delay $\tau = 0$.

iv) What is the maximal interval $[0, \tau_{\text{max}})$ for the time-delay in the feedback loop for which the closed loop system remains stable.
Controls Problem 2  
Spring 2014

**Part II (2 points):**
Consider two \( n \times n \) real matrices \( A \) and \( \Delta \).

i) Consider the autonomous linear dynamical system
\[
\dot{x}(t) = (A + \Delta)x(t),
\]
with initial conditions \( x(0) = x_0 \). Show that provided \( A\Delta = \Delta A \), the solution is given by
\[
e^{\Delta t}e^{At}x_0.
\]

ii) Give an example of two matrices \( A, \Delta \) such that \( A\Delta = \Delta A \).

iii) Let \( A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \) and \( \Delta = \begin{bmatrix} 0 & \delta_2 \\ \delta_1 & 0 \end{bmatrix} \).

- Determine the conditions that the positive scalars \( \delta_1 \) and \( \delta_2 \) have to satisfy to guarantee stability of the above system.
- Set \( \delta_2 = 0 \) and find the solution to the above system for the initial condition \( x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \).
- Set \( \delta_2 = 0 \) and consider the system with the input \( u \) and the output \( y \)
\[
\dot{x}(t) = (A + \Delta)x(t) + Bu(t) \\
y(t) = Cx(t)
\]
where \( B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) and \( C = [ 0 \ 1 ] \).

How does the peak value on the Bode magnitude plot of the transfer function (from \( u \) to \( y \)) depend on \( \delta_1 \)? At what frequency does this peak value take place?