A Si-Ge heterojunction bipolar transistor is used in an amplifier circuit at 2GHz. The equivalent circuit of the input base for a grounded emitter configuration is estimated to have an input resistance of 5Ω in parallel with a capacitance of 1pF. Design a single stub matching circuit to a 50Ω line. Use a Smith chart to design your matching circuit.

\[ Z_L = \frac{1}{\frac{1}{5} + j \frac{2\pi \times 2 \times 10^{-10} \times 1}{2}} \]

\[ Y_L = 0.2 + j 0.0126 \, S \]

\[ Y_L = Y_L \times 50 = 10 + j 0.628 \]

Rotating clockwise, towards generator, the circle cuts the circle at \( g = 1.0 \) to give:

\[ Y'_L = 1.0 - j 2.95 \, S \]

\[ \text{distance} = \frac{0.2985 \lambda}{-0.2481} = 0.0504 \lambda - 50\Omega \]

Stub impedance = \( 2.8 \, S \)

With open circuit stub, stub length at 50Ω:

\[ 0.1962 \lambda \]
A coaxial line operates at 2 GHz and is designed to have an impedance of 50Ω. Assume that the coaxial line is filled with dielectric material whose relative permittivity \( \varepsilon_r \) is 2.25, has an inner copper conductor diameter is 2 mm.

1. Derive the expression for capacitance per unit length using Gauss’s Law.
2. Derive the expression for the inductance using Ampere’s Law.
3. What is inner diameter of the outer conductor to ensure the impedance is 50Ω.
4. What is the phase velocity of this coaxial line?

\[
\text{Capacitance} \quad \frac{2\pi \varepsilon_r \ln \left( \frac{b}{a} \right)}{2\pi} \quad \text{inner radius} \quad a, \text{ outer} \quad b \quad \text{Assume} \quad \text{charge per unit length} \quad \text{inner} \quad \phi, \text{ outer} \quad \psi
\]

\[
V = \oint \mathbf{E} \cdot d\mathbf{r} = \frac{Q}{2\pi \varepsilon_0} \ln \left( \frac{b}{a} \right) \quad C = \frac{Q}{V} = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln \left( \frac{b}{a} \right)} \quad (1)
\]

Similarly, \( \mathbf{H} \cdot d\mathbf{r} = I \), \( \nabla \times \mathbf{H} = \frac{\mu_0 I}{2\pi} \), \( \frac{\mathbf{B}}{\mathbf{H}} = \frac{\mu_0}{2\pi} \)

Total Flux \( \Phi_m \) (fractal flux) \( \Phi = \oint \frac{\mu_0 I}{2\pi} \ln \left( \frac{b}{a} \right) \quad (2) \)

\[
L = \frac{\Phi}{I} = \frac{\mu_0}{2\pi} \ln \left( \frac{b}{a} \right) \quad \text{Z}_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \ln \left( \frac{b}{a} \right) = \frac{2\pi}{\ln \left( \frac{b}{a} \right)}
\]

\[
\eta = \frac{Z_0}{\varepsilon_0} = \gamma_0 = 3.7752
\]

\[
\ln \left( \frac{b}{a} \right) = 5 \quad \text{ln} \left( \frac{b}{a} \right) = \frac{1.5 \times 50 \times \frac{2\pi}{1.5}}{377} = 1.25
\]

\[
\frac{b}{a} = e^{1.25} = 3.49 \quad \text{Hence} \quad b = 3.49 \text{ min}, \text{ diameter} = 6.98 \text{ mm}
\]

\[
\eta \Phi = \frac{C}{\varepsilon_0} \cdot \frac{3\times 10^8}{1.5} = 2 \times 10^8 \text{ V/m}
\]