• No book, notes, mobile phones or laptop are allowed in the exam. You can use calculator if needed.

• This exam has two problems:
  – Problem 1 has six parts, and is worth 25 points.
  – Problem 2 has three parts, and is worth 15 points.

• The total number of points (perfect score) is 40.
Problem 1) We have a wireless communication channel, in which the transmitter can send two symbols \( x_1[n] \) and \( x_2[n] \) at each time instance \( n \), and the received signal is

\[
y[n] = h_1 x_1[n] + h_2 x_2[n] + z[n],
\]

where \( z[n] \) is an additive white Gaussian noise, with \( z[n] \sim \mathcal{N}(\mu = 0, \sigma^2 = 100) \). We can send symbols \( x_i[n] \in \{-1, +1\} \) for \( i = 1, 2 \) over this channel.

(a) Assume the transmitter uses the same symbol for both \( x_1 \) and \( x_2 \), i.e., \( x_1[n] = x_2[n] = s \) (we refer to this scheme as scheme-1). Find the signal-to-noise ratio (SNR) for detecting \( s \) at the receiver. [3 points]

(b) Now, consider scheme-2 as follows: in order to communicate two symbols \( s_1 \) and \( s_2 \), the transmitter sends

\[
\begin{align*}
x_1[1] &= s_1, & x_2[1] &= s_2, \\
x_1[2] &= s_2, & x_2[2] &= -s_1,
\end{align*}
\]

in two time instances. Combine \( y[1] \) and \( y[2] \) so that \( s_2 \) gets eliminated, and find \( s_1 \). [5 points]

(c) What is the SNR of this detection rule for decoding \( s_1 \)? [4 points]

(d) Repeat parts (b) and (c) for \( s_2 \). [2 points]

(e) Which one of scheme-1 or scheme-2 do you recommend in order to get a better SNR? (your answer may depend on values of \( h_1 \) and \( h_2 \)) [4 points]

(f) Now assume \( h_1 \) and \( h_2 \) are two independent random variables, each distributed as

\[
P_{h_1}(x) = P_{h_2}(x) = \begin{cases} 
0.2 & x = -10 \\
0.3 & x = -1 \\
0.3 & x = 1 \\
0.2 & x = 10 \\
0 & \text{otherwise.}
\end{cases}
\]

Hence SNR will be also a random variable for both schemes. Find the probability that the (random) SNR is at least 1 (SNR \( \geq 1 \)) for each of scheme-1 and scheme-2. [7 points]
Problem 2) Consider a discrete communication channel modeled as

\[ y[n] = h[n]x[n] \]

where \( x[n] \in \{-1, +1\} \) is the channel input, and \( h[n] \) is the random channel gain with

\[ h[n] = \begin{cases} 
0 & \text{with probability 0.2} \\
1 & \text{with probability 0.8} 
\end{cases} \]

Moreover, \( h[n] \) and \( h[n'] \) are independent from each other for \( n \neq n' \). We say a transmit symbol \( x[n] \) is missed whenever the corresponding channel gain is zero (\( h[n] = 0 \)).

We need to communicate an integer number \( m \) from the set \( M = \{0, 1, 2, \ldots, 255\} \), and we are only allowed to use the channel for ten times. To this end, we first map \( m \) to its 8-digit binary expansion \( s_1s_2s_3 \cdots s_8 \), and then send \( s_1, \ldots, s_8 \) along with

\[ s_o = s_1 \oplus s_3 \oplus s_5 \oplus s_7 \]
\[ s_e = s_2 \oplus s_4 \oplus s_6 \oplus s_8 \]

over the channel in ten time slots, using the mapping 0 \( \mapsto \) +1 and 1 \( \mapsto \) −1. For example if \( m = 97 \) with binary representation 01100001, we first find \( s_o = 0 \oplus 1 \oplus 0 \oplus 0 = 1 \) and \( s_e = 1 \oplus 0 \oplus 0 \oplus 1 = 0 \), and then transmit symbols +1, −1, −1, +1, +1, +1, −1, −1, +1 over the channel corresponding to the binary sequence 0110000110.

(a) How many of the transmitted bits will be missed on average? [2 points]

The receiver observes \( Y = (y[1], y[2], \ldots, y[10]) \), from which he wants to decode \( m \). We say \( m \) can be decoded if we can determine all the bits in its binary expansion, i.e. \( s_1, s_2, \ldots, s_8 \). Otherwise the decoding process fails.

(b) When does this integer decoding process fail (find conditions under which we cannot find all the 8 desired bits)? [7 points]

(c) Find the probability of failure in the decoding process. [6 points]