Problem 9 (a) A Photovoltaic panel under a given insolation condition operates at its maximum power point with the voltage of 60 V and the current of 20 A. This PV panel supplies power to a 120-V, 60-Hz, residential grid. The PV interface consists of a boost dc-dc converter, followed by a full-bridge, single-phase, dc-ac inverter. Assume all components to be ideal (i.e., zero power loss everywhere for the sake of simplicity). The output of the boost dc-dc converter, operating in a continuous-current-conduction mode at a switching frequency of 250 kHz, is 250 V (dc) across a very large dc-link capacitor such that the ripple in the capacitor voltage can be assumed to be negligible. This capacitor voltage acts as the input to a single-phase, full-bridge, inverter that supplies power to the grid at a unity power factor; assume the ripple in the output current to be negligible. Inverter switches are sine-PWM controlled with the switching frequency of 100 kHz. Assume the inductance on the ac-side of the inverter to be negligible.

(i) Draw the circuit diagram of such an interface and label all the components.

(ii) What is the duty-ratio of the transistor-switch in the boost dc-dc converter?

(iii) In the single-phase inverter, over one switching time-period, calculate the duty-ratios of all the four transistor-switches when the grid voltage is at its positive peak.

(iv) What is the peak value of the current into the residential grid?
Problem 9 (b) A synchronous machine is connected to an infinite bus as shown in the figure below. The machine has the following parameters: synchronous speed, $\omega_s = 120\pi \text{[rad/s]}$, inertia constant, $H = 8 \text{[s]}$, machine damping, $D = 0.004 \text{[rad/s]}$, machine terminal voltage, $E = 0.9 \text{[pu]}$, mechanical power input, $P_M = 1.0 \text{[pu]}$, machine terminal impedance, $X_M = 0.2 \text{[pu]}$, the infinite bus voltage magnitude, $v_\infty = 1.0 \text{[pu]}$, and the line impedance, $X_L = 0.25 \text{[pu]}$. Recall that the swing equations that govern the evolution of the rotor angle, $\delta$, and frequency, $\omega$, are given by:

$$
\frac{d\delta}{dt} = \omega - \omega_s,
$$

$$
\frac{2H}{\omega_s} \frac{d\omega}{dt} = P_M - P_E - D(\omega - \omega_s),
$$

where $P_E$ is the electrical output power sourced by the generator from terminal 1.

![Figure 1: Single line diagram of single machine infinite bus](image)

(i) Write an expression for the electrical output power, $P_E$. Your expression should only be a function of $E$, $v_\infty$, $\delta$, $X_M$, and $X_L$.

(ii) Find the equilibrium points for the dynamical system in (1) in the interval $[0, \pi]$.

(iii) Linearizing the system around the equilibrium point, find the eigenvalues.

(iv) Which equilibrium point is stable?