(1) (12 points) Suppose $Z_i, i = 1, \ldots, n$ are i.i.d. random variables with the following distribution:

$$Z_i = \begin{cases} 
-1 & \text{with prob. } p \\
0 & \text{with prob. } 1 - 2p \\
1 & \text{with prob. } p.
\end{cases}$$

Let,

$$Z = \sum_{i=1}^{n} Z_i.$$ 

a. Find $E[Z]$ and $E[Z^2].$

b. Find the characteristic function of $Z.$

Now consider a discrete time signal $X_i$ is transmitted over an additive-noise channel. At the output of the channel we obtain $i = 1, \ldots, n:\n
Y_i = X_i + Z_i.$

If $[X_1, X_2, \ldots, X_n]$ is a Gaussian vector with covariance matrix $= 10I,$ where $I$ is an $n \times n$ identity matrix, then,

c. Find the Signal to Noise ratio for this channel when $\rho = 0.2.$
(2) (8 points) Suppose, you need to sample and transmit the following signal using an 8-bit PCM (pulse code modulation):

\[ X(t) = 32 \cos(8\pi t). \]

a. What is the bits/sec. transfer rate (assume Nyquist rate sampling)? 3
b. What is the mean square quantization error in the PCM? You can assume that the quantization noise is uniformly distributed. 5
(3) (20 points) Consider the following Octal Phase-Shift-Keying (PSK) scheme. Each of the 8 signal waveforms are represented as (below $f_c = 1$ Hz):

$$s_n(t) = \left| t - \frac{1}{2} \right| \cos \left( 2\pi f_c t + \frac{\pi}{4} (n - 1) \right), \quad n = 1, 2, \ldots, 8, \quad 0 \leq t \leq 1.$$ 

a. Compute the energy of the signal waveform $s_n(t)$. 5
b. Write an basis for this set of signals. There should be only two signals in this basis. 5
c. Represent $s_1(t)$ and $s_2(t)$ as a linear combination of above two basis signals. 5
d. How many bits of information can be sent in the interval $0 \leq t \leq 1$? Suppose, signals with adjacent phases can be confused at the receiver. How many bits can still be sent so that information retrieval with certainty is possible? 5