Consider an npn bipolar transistor at $T = 300$ K that is biased in the common-emitter configuration. The device has the structural parameters and minority/majority carrier profiles as shown in the figure below. The hatched regions indicate the depletion regions. Assume that the diffusion coefficients are the same for all three regions of the device.
The definitions of the parameters in the figure, as well as a few constants, are given below:

\[
\begin{align*}
n_{NE} &= \text{Majority electron concentration in the emitter} \\
p_{NE} &= \text{Minority hole concentration in the emitter} \\
p_{NE0} &= \text{Equilibrium minority hole concentration in the emitter} \\
\Delta p_{NE} &= \text{Excess minority hole concentration in the emitter} \\
p_{PB} &= \text{Majority hole concentration in the base} \\
n_{PB} &= \text{Minority electron concentration in the base} \\
n_{PB0} &= \text{Equilibrium minority electron concentration in the base} \\
\Delta n_{PB} &= \text{Excess minority electron concentration in the base} \\
n_{NC} &= \text{Majority electron concentration in the collector} \\
p_{NC} &= \text{Minority hole concentration in the collector} \\
p_{NC0} &= \text{Equilibrium minority hole concentration in the collector} \\
\Delta p_{NC} &= \text{Excess minority hole concentration in the collector} \\
V_{BE} &= \text{Base-emitter voltage} \\
V_{BC} &= \text{Base-collector voltage} \\
V_{CE} &= \text{Collector-emitter voltage} \\
k_B &= \text{Boltzmann's constant} = 8.617 \times 10^{-5} \text{ eV/K}
\end{align*}
\]

In a bipolar transistor, $\gamma$ is a parameter called the emitter efficiency, and $\alpha_T$ is the base transit factor. These parameters are given by:

\[
\gamma = \frac{I_{EN}}{I_E} \quad \text{and} \quad \alpha_T = \frac{I_{CN}}{I_{EN}},
\]

where $I_E$ is the emitter current, and $I_{EN}$ and $I_{CN}$ are the electron components of the emitter and collector currents, respectively.

The common-emitter DC current gain of bipolar transistor, $\beta_{DC}$, is given by:

\[
\beta_{DC} = \frac{I_C}{I_B},
\]

while the common-base DC current gain of a bipolar transistor, $\alpha_{BC}$, is given by:

\[
\alpha_{BC} = \frac{I_C}{I_E}.
\]
Answer the following questions about this device:

(a) What are the terminal voltages, $V_{BE}$, $V_{BC}$ and $V_{CE}$?

$$V_{BE} = 0.0259 \times \ln(10^9) = 0.537\, \text{V}$$

$$V_{BC} = 0.0259 \times \ln(10^{-7}) = -0.417\, \text{V}$$

$$V_{CE} = V_{BE} - V_{BC} = 0.954\, \text{V}$$

(b) What mode (Saturation, Active, Cutoff or Inverted Active) is this device operating in? Explain your answer in one sentence.

Active. The device is in the active mode because the base-emitter junction is forward biased, and the base-collector junction is reverse biased.

(c) What are the intrinsic carrier concentrations in the emitter, base and collector?

$$n_{iE} = \sqrt{n_{NE0} \cdot p_{NE0}} = \sqrt{n_{NE} \cdot p_{NE0}} = \sqrt{10^{20} \cdot 10^{-2}} = \sqrt{10^{18}} = 10^{9}\, \text{cm}^{-3}$$

$$n_{iB} = \sqrt{p_{PB0} \cdot n_{PB0}} = \sqrt{p_{PB} \cdot n_{PB0}} = \sqrt{10^{18} \cdot 10^{2}} = \sqrt{10^{20}} = 10^{10}\, \text{cm}^{-3}$$

$$n_{iC} = \sqrt{n_{NC0} \cdot p_{NC0}} = \sqrt{n_{NC} \cdot p_{NC0}} = \sqrt{10^{17} \cdot 10^{3}} = \sqrt{10^{20}} = 10^{10}\, \text{cm}^{-3}$$

(d) Is this a heterojunction or homojunction bipolar transistor? In other words, are the three device regions made of the same semiconductor or different semiconductors? Explain your answer in two sentences or less.

Heterojunction. The emitter has a different intrinsic carrier concentration than the base and collector. The emitter therefore is made from a wider band gap material than the base and collector.

(e) What is the base transit factor, $\alpha_T$?

$$\alpha_T = \frac{I_{CN}}{I_{EN}} = \frac{qAD_{NC} \frac{d\Delta n_{PB}(x=W)}{dx}}{qAD_{NC} \frac{d\Delta n_{PB}(x=0)}{dx}} = \frac{qAD_{NB} \cdot 0.995 \cdot M \cdot n_{PB0}}{qAD_{NB} \cdot M \cdot n_{PB0}} = 0.995$$

(f) What is the emitter efficiency, $\gamma$?
\[ \gamma = \frac{I_{EN}}{I_{E}} = \frac{I_{EN}}{I_{EN} + I_{EP}} = \frac{qAD_{NB} \frac{d\Delta n_{PB}(x = 0)}{dx}}{qAD_{NB} \frac{d\Delta n_{PB}(x = 0)}{dx} + qAD_{PE} \frac{d\Delta p_{NE}(x = 0)}{dx}} \]

\[ \gamma = \frac{qAD_{NB} \cdot (M \cdot n_{PB0})}{qAD_{NB} \cdot (M \cdot n_{PB0}) + qAD_{NB} \cdot (3M \cdot p_{NE0})} = \frac{n_{PB0}}{n_{PB0} + 3p_{NE0}} = \frac{10^2}{10^2 + 3 \times 10^{-2}} = 0.9997 \]

(g) What is the common-emitter DC current gain, \( \beta_{DC} \)?

First calculate the common-base DC current gain, \( \alpha_{DC} \):

\[ \alpha_{DC} = \frac{I_C}{I_E} \approx \frac{I_{CN}}{I_{EN}} = \frac{I_{CN}}{I_{EN} + I_{EP}} = \alpha_{\gamma} = 0.995 \cdot 0.9997 = 0.9947 \]

Use result to calculate \( \beta_{DC} \)

\[ \beta_{DC} = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{\alpha_{DC}}{1 - \alpha_{DC}} = \frac{0.9947}{1 - 0.9947} = 187.7 \]

(h) While keeping \( I_B \) constant, \( V_{CE} \) is increased to 1.6 V. This leads to a reduction in the base width, \( W \), by 1%. Calculate the Early voltage, \( V_A \), which is defined below. Assume that \( \alpha_t \) remains unchanged.
$V_A = \left( \frac{\Delta V_{CE}}{\Delta I_C} \right) I_C - V_{CE}$

$V_A = \frac{\Delta V_{CE}}{\Delta I_C} = \left[ \frac{(1.6 - 0.954) \text{ V}}{(qAD_{NB} \cdot \alpha_f \cdot 1.0101M \cdot n_{PB0}) - (qAD_{NB} \cdot \alpha_f M \cdot n_{PB0})} \right] \cdot (qAD_{NB} \cdot 1.0101 \alpha_f M \cdot n_{PB0}) - 1.6 \text{ V}$

$V_A = \frac{\Delta V_{CE}}{\Delta I_C} = \left[ \frac{0.646 \text{ V}}{(1.0101) - 1} \right] \cdot (1.0101) - 1.6 \text{ V} = 63 \text{ V}$