1) [15 points] Consider discrete-time LTI system depicted below.

![Discrete-time LTI system](image)

Suppose that \( H(z) = (1 - 2z^{-1})(1 - \frac{1}{2}z^{-1}) \) with ROC \(|z| > 0\). Find the impulse response \( g[n] \) of a stable system \( G(z) \) such that \( w[n] = x[n] - 2 \).

**Hint:** Recall the following z-Transform relations, which you may find useful:

\[
\begin{align*}
    x[n] = a^n u[n] & \quad \leftrightarrow \quad X(z) = \frac{1}{1 - az^{-1}}, \quad \text{ROC: } |z| > |a| \\
    x[n] = -a^n u[-n-1] & \quad \leftrightarrow \quad X(z) = \frac{1}{1 - az^{-1}}, \quad \text{ROC: } |z| < |a|.
\end{align*}
\]
2) [10 points] Recall that the discrete-time convolution operation satisfies several standard arithmetic properties. Namely, for all functions $f[n], g[n], \text{ and } h[n]$, we have:

- Commutativity: $f[n] \ast g[n] = g[n] \ast f[n]$
- Associativity: $f[n] \ast (g[n] \ast h[n]) = (f[n] \ast g[n]) \ast h[n]$
- Distributivity: $f[n] \ast (g[n] + h[n]) = (f[n] + g[n]) \ast (f[n] + h[n])$

Note, however, that for arbitrary functions $g[n], h[n]$, the relationship

$$f[n] \ast (g[n] \ast h[n]) = (f[n]g[n]) \ast (f[n]h[n])$$

(1)

does not hold for all $f[n]$, though it does hold for the trivial cases $f[n] = 1$ and $f[n] = 0$.

Besides the trivial examples given above, find another function $f[n]$ for which (1) holds for all $g[n], h[n]$. 

3) Consider the system shown below.

![System Diagram]

The anti-aliasing filter $H_c(j\Omega)$ is an ideal low-pass continuous-time filter with unit gain and cutoff frequency $\pi/T$. The frequency response of the LTI discrete time system between the converters is given by:

$$H_d(e^{j\omega}) = j\omega e^{-j\omega/3}, \quad |\omega| < \pi.$$

Assume that the C/D and D/C converters are ideal.

a) [5 points] Find the impulse response $h_d[n]$ of the discrete-time LTI system.

b) [5 points] What is the effective continuous-time frequency response of the overall system, $H(j\Omega)$?

c) [5 points] Choose the most accurate statement:

(i) $y(t) = x(t - 1/3)$

(ii) $y(t) = \frac{d}{dt} x(3t)$

(iii) $y(t) = T \frac{d}{dt} x(t - 1/3)$

(iv) $y(t) = T \frac{d}{dt} x(t - T/3)$

(v) $y(t) = \frac{d}{dt} x(t - 1/3)$

(vi) $y(t) = \frac{d}{dt} x(t - T/3)$