There are two Parts and they are assigned 20 points each (for a total of 40/40)

Part #1 (20 points total):
Consider an inertial system modeled as a lumped scalar linear system

\[ \frac{d^2 x(t)}{dt^2} = u(t). \]

Here \( u(t) \) represents a force applied to it by a controller, while \( x(t) \) represents its position at time \( t \in [0, \infty) \). The controller input is taken as the difference \( v(t) = r(t) - x(t) \), between a reference signal \( r(t) \) and the position \( x(t) \). The controller output is the applied force \( u(t) \).

Address the following three independent sub-parts to Part 1:
1. (5 points) Consider the control law \( u(t) = Kv(t) \) and determine whether the feedback system is asymptotically stable for any choice of the gain \( K \). (Recall that a system is asymptotically stable if all of its poles have negative real part.)

2. (5 points) Consider a dynamic control law where

\[ u(t) = v(t) - \int_0^t v(\tau)e^{-2(t-\tau)}d\tau. \]

Determine whether the feedback system is asymptotically stable for this choice of control law. (Prove that it is, or prove that it is not.)

3. (10 points) Design a control law which stabilizes the feedback system and has the additional property that, at steady state, the position follows exactly a sinusoidal reference signal \( r(t) = \sin(t) \).
Part #2 (20 points total):
Consider a dynamical system, described by the delay-differential equation

\[
\frac{d}{dt} x(t) = u(t) - Kx(t - 1) - Kx(t - 2),
\]

where \( x(t) \) represents the state and \( u(t) \) the input. Determine the maximal value of \( K \) for which the system is stable. (Hint: You may use Nyquist's stability criterion.)