**Problem 9 (a)** A buck converter is to be designed to deliver power from a DC input with voltage ranging between 30 V and 60 V to a 5 V output at a switching frequency of 100 kHz. The load power is expected to vary between 10 W and 200 W.

(i) What choice of inductance will ensure operation in discontinuous conduction mode under all operating conditions (i.e., all possible values of input voltage and load power)?

(ii) What choice of inductance will avoid operation in discontinuous conduction mode under all operating conditions?

**Solution 9 (a)** Denote the critical inductance of the dc-dc buck converter by $L_{\text{crit}}$. Recall that the critical inductance is the minimum inductance required to avoid discontinuous conduction mode (DCM) under all operating conditions. That is, if the chosen inductor for the dc-dc buck converter, $L > L_{\text{crit}}$, then DCM is avoided. On the other hand, if the dc-dc buck converter inductor $L < L_{\text{crit}}$, then the converter always operates in DCM. For $L \geq L_{\text{crit}}$, the input and output voltages are related by

$$V_{\text{out}} = DV_{\text{in}}, \quad (1)$$

where $D$ is the duty cycle. With the active switch turned on, we can write

$$V_{\text{in}} - V_{\text{out}} = L \frac{di}{dt}, \quad (2)$$

where $i$ denotes the instantaneous inductor current. With a straight-line approximation for the inductor current, and recognizing that the output voltage, $V_{\text{out}} = 5$ V, we get

$$V_{\text{in}} - 5 \approx L \frac{\Delta i}{DT} = L \frac{\Delta i}{(5/V_{\text{in}})T}, \quad (3)$$

where $T = f^{-1}$ is the switching period, $\Delta i$ is the current ripple, and the second equality in (3) follows from substituting for the duty cycle, $D$ from (1). Suppose we pick the inductance of the buck converter to be $L_{\text{crit}}$. With this choice, the buck converter operates in the critical conduction mode (i.e., the boundary between continuous and discontinuous conduction modes). In this operating mode, $\Delta i = 2I_{\text{out}}$, where $I_{\text{out}} = P_{\text{out}}/V_{\text{out}} = P_{\text{out}}/5$ is the average output current. Substituting for $\Delta i = 2P_{\text{out}}/5$ in (3), we obtain

$$V_{\text{in}} - 5 = L_{\text{crit}} \frac{2P_{\text{out}}}{5} \frac{1}{(5/V_{\text{in}})T}. \quad (4)$$

Rearranging terms and substituting the switching frequency $f = 100$ kHz, we get,

$$L_{\text{crit}} = \left( \frac{5}{f} - \frac{25}{fV_{\text{in}}} \right) \frac{5}{2P_{\text{out}}} = 1.25 \times 10^{-4} \frac{1 - \frac{5}{V_{\text{in}}}}{P_{\text{out}}} \text{ H.} \quad (5)$$

(i) **Ensuring DCM.** The converter will operate in DCM in all operating cases if $L < L_{\text{crit}}$. From (5), the worst-case condition is obtained for the maximum load power, $P_{\text{out}} = 200$ W and the minimum input voltage, $V_{\text{in}} = 30$ V. Substituting these in (5), we can conclude that DCM can always be ensured with an inductance $L < 0.521 \mu$H.

(ii) **Avoiding DCM.** The converter will avoid DCM in all operating cases if $L > L_{\text{crit}}$. From (5), the worst-case condition is obtained for the minimum load power, $P_{\text{out}} = 10$ W and the maximum input voltage, $V_{\text{in}} = 60$ V. Substituting these values in (5), we can conclude that DCM can always be avoided with an inductance $L > 11.46 \mu$H.
Solution to Part b on Power Systems

The equivalent impedance with only one line is now 0.05 j pu

\[ P_2 + jQ_2 = V_2 (\frac{(V_2 - V_1)}{0.05 j})^* \]

Where \( P_2 = -5.0 \), \( Q_2 = 0 \), \( V_1 = 1 \)

The net real power and reactive power is known for bus 2 Then:

\( -5.0 = \text{Re}(V_2 (\frac{(V_2 - V_1)}{0.05 j})^*) \)
\( 0 = \text{Im}(V_2 (\frac{(V_2 - V_1)}{0.05 j})^*) \)

This results in two simultaneous equations involving \( V_2 \) and \( \theta_2 \)

\[ (-5.0 * 0.05) / V_2 = \sin(\theta_2) \]
and

\[ 0 = V_2 - \cos(\theta_2) \]

Resulting in the solution

\[ V_2 = \sqrt{0.933} = 0.966 \] pu and \( \theta = -15 \) degrees

Opening a line places greater impedance in the circuit, so the bus voltage will be lower and the angle greater for the same load.