Analysis of the effect of spanwise wall oscillations on drag reduction at high $Re$

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66th APS DFD Annual Meeting, Pittsburgh, 2013
Turbulent drag reduction by wall oscillations

\[ W(y = \pm 1, t) = 2\alpha \sin\left(\frac{2\pi}{T} t\right) \]

experiments: Laadhari et al., 94; Choi et al., 98, 02
simulations: Jung et al., 92; Baron, Quadrio, Ricco, 96, 03, 04

↑ Reynolds number  ➞  ↓ Drag reducing ability

Choi, Xu & Sung, 02; Ricco & Quadrio, 08
Touber & Leschziner, 12; Moarref & Jovanović, 12
Model-based approach

- **Eddy viscosity enhanced NS equations**

\[ \nu_T = C_\mu Re_\tau^2 \left( \frac{k^2}{\epsilon} \right) \]

- **Turbulent viscosity**

\[
\begin{align*}
    k &= k_0 + \alpha^2 k_2 + \mathcal{O}(\alpha^4) \\
    \epsilon &= \epsilon_0 + \alpha^2 \epsilon_2 + \mathcal{O}(\alpha^4) \\
    \nu_T &\approx \nu_{T0} + \alpha^2 \nu_{T2}
\end{align*}
\]

\[
\implies \nu_{T2} = \nu_{T0} \left( \frac{2k_2}{k_0} - \frac{\epsilon_2}{\epsilon_0} \right)
\]

Moarref & Jovanović, JFM ’12
Premultiplied energy spectrum of uncontrolled flow

\[ Re_\tau = 186 \]

\[ Re_\tau = 547 \]

\[ Re_\tau = 934 \]

\[ Re_\tau = 2003 \]
Marusic et al., J. Heat & Fluid Flow ’10
Effect of control \( (T^+ = 100) \)

- **2nd-order correction to kinetic energy of fluctuations** - \( k_x k_2(y, k_x) \)

\[
\begin{align*}
R_e \tau &= 186 \\
\lambda_x^+ &
\end{align*}
\]

\[
\begin{align*}
R_e \tau &= 547 \\
\lambda_x^+ &
\end{align*}
\]

\[
\begin{align*}
R_e \tau &= 934 \\
\lambda_x^+ &
\end{align*}
\]

\[
\begin{align*}
R_e \tau &= 2003 \\
\lambda_x^+ &
\end{align*}
\]
2nd-order correction to fluctuations’ energy dissipation $- k_x \epsilon_2(y, k_x) / Re^2_\tau$

$Re_\tau = 186$

$Re_\tau = 547$

$Re_\tau = 934$

$Re_\tau = 2003$
Predictions of drag reduction at higher $Re_\tau$

$$
\nu_{T2} = 2 \left[ k_2 \left( \frac{\nu_{T0}}{k_0} \right) - \left( \frac{\epsilon_2}{Re^2_\tau} \right) \left( \frac{\nu_{T0}}{k_0} \right)^2 \right]
$$

$$
f_{DR} = -\frac{1}{2U_B} \int_{-1}^{1} \int_{-1}^{y} \frac{\nu_{T2}(\xi) U'_0(\xi)}{1 + \nu_{T0}} \, d\xi \, dy
$$

- $k_2$ and $\frac{\epsilon_2}{Re^2_\tau}$ collapse with inner (wall) scaling for high Reynolds numbers
- Use corrections from $Re_\tau = 2003$ to extend predictions to higher $Re_\tau$
Drag reduction $\approx \alpha^2 f_{DR}(T^+)$

$\Rightarrow k_0$ from experimental study of Schultz & Flack, POF ’13
Predictions of drag reduction at higher $Re_{\tau}$

- $k_0$ from experimental study of Schultz & Flack, POF ’13
Predictions of drag reduction at higher \( Re_\tau \)

\[
f_{DR} \quad T^+ \quad Re_\tau = 934 \quad Re_\tau = 2003 \quad Re_\tau = 4000, k_0 = uu/2 \quad Re_\tau = 4000, k_0 = (uu + vv)/2
\]

\( k_0 \) from experimental study of Schultz & Flack, POF ’13
Predictions of drag reduction at higher $Re_\tau$

$\cdot k_0$ from experimental study of Schultz & Flack, POF ’13
Predictions of drag reduction at higher $Re_\tau$

- $k_0$ from experimental study of Schultz & Flack, POF ’13
- Better predictions of $k_0$ are needed
Conclusion

- Model-based approach

- Influence of wall oscillations $\rightarrow$ confined to wavelengths that correspond to universal inner-scaled eddies

- Predictions can be made at high Reynolds numbers