

EE5585: HOMEWORK 2
DUE: MAR 26, START OF CLASS.

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All problems carry equal points

- (1) Consider the following compression scheme for binary (0 and 1) sequences. We divide the binary sequences into blocks of size 12. For each block if the number of zeros is greater than or equal to 10 we store a 0, otherwise we store a 1. If the sequence is random with probability of zero 0.9, compute the rate and average distortion (Hamming metric). Compare your result with the corresponding value of rate distortion function for binary sources.
- (2) Consider the following sampling matrix Φ :

$$\begin{bmatrix} 1 & 0 & 0.5 \\ -3 & 1 & 1 \end{bmatrix}$$

We observe samples of an 1-sparse vector x :

$$y = \Phi x = \begin{bmatrix} 4.5 \\ 9 \end{bmatrix}.$$

Find out x .

Next, suppose we observe:

$$z = \Phi x = \begin{bmatrix} 10.1 \\ -30.1 \end{bmatrix}.$$

For a general vector x . Find out x using ℓ_1 -minimization (basis pursuit). Hint: One can do this calculation by hand. Simply assume $x = [x_1 \ x_2]'$, and then perform a constrained optimization.

- (3) Consider the network of Figure 1. There is one link that has delay associated with it. Other nodes transfer information instantaneously. You are allowed to measure the total delay of any path of the network. What is the minimum number of measurements that you can make to identify the delayed link and the amount of the delay? Write down the paths you measure.
- (4) (Encryption and Compression) Suppose you have a 4×4 image as below.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

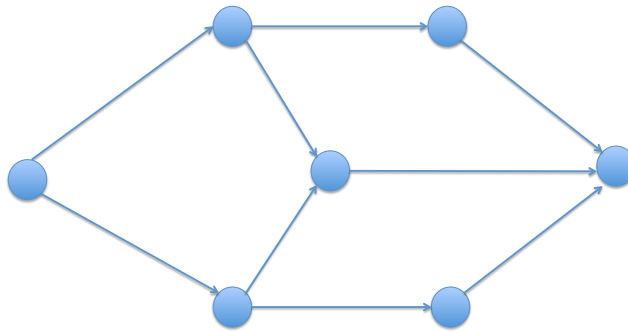


FIGURE 1. A network.

The pixels of the image can take values $\{-2, -1, 0, 1, 2\}$ with the following probabilities:

$$p(0) = 1/2; p(1) = p(-1) = 3/16; p(2) = p(-2) = 1/16$$

Design a Huffman code. How many bits do you need to store the image compressed by this Huffman code?

Now encrypt the image in the following way. Randomly and uniformly pick a number from 0 to 4 (can use MATLAB to generate this number). Let this random number is r . For a pixel value x , perform the following operation

$$f(x) = [(x + 2) \times r] \bmod 5 - 2.$$

You should get back a new value between -2 and $+2$. Now use the same Huffman code to compress this encrypted image. How many bits do you end up using?