EE5585: HOMEWORK 4
DUE: MAY 6, START OF CLASS.

INSTRUCTOR: A. MAZUMDAR

All problems carry equal points

(1) Following table shows height-weight data of 12 monkeys:

Height	Weight
18	29.5
28	39.2
36	54.5
25	36.0
17	25.4
31	43.8
21	30.4
35	56.1
24	36.0
22	29.9
18	26.9
32	48.2

Find out the Karhunen-Loeve transform matrix to compress this data. Perform PCA.

(2) An $n \times n$ Hadamard Transform matrix exists for n equal to any power of 2. By definition,

$$H_1 = [1].$$

Given H_n, define H_{2n} in the following way:

$$H_{2n} = \begin{pmatrix} H_n & H_n \\ H_n & -H_n \end{pmatrix}.$$

(a) What is H_4?
Consider the following "image":

$$\begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

(b) Perform the two dimensional Hadamard transform of this image by first taking the one dimensional transform of the rows, and then taking the column-by-column transform of the resulting matrix.

(c) Obtain the two dimensional Hadamard transform of this image by first taking the one dimensional transform of the columns, and then taking the row-by-row transform of the resulting matrix.

(3) You are given the following 4-dimensional binary training set:

$$T = \{(1,1,1,1),(1,1,1,0),(1,1,1,0),(0,0,0,1),(1,0,0,1),$$
$$(0,0,0,1),(1,0,0,0),(0,0,0,1),(1,1,0,1)\}.$$

Use K-means with Hamming distortion measure to design a 2-Codeword Vector Quantizer. Use the initial codebook

$$\{(1,1,0,0),(0,0,1,1)\}.$$

No computer is necessary - however you are allowed to use one, and make necessary assumptions.

(a) What is the final codebook that you get?

(b) What is the average Hamming distortion between the training sequence and the final codebook?

(4) A stationary unit variance sequence $\{x_n\}$ has zero mean and autocorrelation function

$$r_k \equiv \mathbb{E}x_n x_{n+k} = (0.5)^{|k|},$$

for all n, k.

Design the optimal linear predictor for $\{x_n\}$ with memory size 2. Predict the value of x_{17} when it is observed that x_{16} is 2.4 and x_{15} is -1.9.