

EE 8591

FALL 2021

Homework 1 (7 points)**Background on basic concepts of probability theory, statistics and programming***This homework is intended mainly for self-testing your background***Due date: Sep 15, 2021****1. Problem 1 (2 points)**

An e-marketing company generates mass email messages (spam) for its customers. The company is paid a percentage of sales generated within 12 hours after spam has been sent. Based on past history, the probability of at least one sale, in response to a single mass mailing, is 0.6%. The company sends 200 different spam messages each day. What is the probability that it generates (positive) daily revenue?

2. Problem 2 (5 points)

Consider daily price changes of large US technological companies as reflected in daily closing prices of exchange-traded fund (ETF) called QQQ. The daily closing prices of QQQ can be obtained from <http://finance.yahoo.com>, by clicking “get quotes” for symbol QQQ, and then clicking on “historical prices”. (You need to use ‘Adjusted_Close’ prices for this problem).

The daily percentage change of QQQ can be viewed as a random variable

$$X(t) = \frac{Z(t) - Z(t-1)}{Z(t-1)} * 100\% , \text{ where } Z(t) \text{ is the daily closing price of QQQ}(t) \text{ as, and index } t$$

denotes the current trading day. The distribution of this random variable is unknown, however we can obtain its observations, based on the closing prices of QQQ for one year period.

(1) Using historical data for $Z(t)$, obtain observations (or samples) of random variable X during year 2006. Then find the following statistical characteristics of X :

- Empirical distribution, in the form of the histogram of observed X -values, using the range (-2%, +2%), and the bin size 0.2%.
- sample mean and standard deviation of X
- approximate distribution of X using the normal distribution, and show its pdf in a graphical form, using the same scale as for the histogram in part (a). Parameters of the normal distribution are estimated as in (b).

(2) Using the same historical data, calculate 4-day moving average (MA) of the closing prices of $QQQ(t) = Z(t)$, defined as $MA(t) = [Z(t) + Z(t-1) + Z(t-2) + Z(t-3)] / 4$, for the same year 2006.

Then calculate the daily percentage change of MA as $Y(t) = \frac{MA(t) - MA(t-1)}{MA(t-1)} * 100\% .$

For this random variable Y , repeat (a) and (b) of part (1).

(3) Try to find *analytic* relationship between:

- the mean values of X and Y
- the standard deviations of X and Y (*)

(*) For analytic derivations in part (3), assume that X – values are statistically independent.

Then compare your analytic results and (earlier) empirical values of mean and st. deviation for Year 2006.

(4) For additional validation of your theoretical analysis in part (3) estimate the mean and standard deviation for X and Y using empirical data for QQQ for another year (of your choice). Discuss whether (and how well) the analytic relationships between means and st. deviations of X and Y hold for years 2006 and another year. These differences (if any) may suggest some insights about the changes in the stock market – and you can include these insights in your solution.