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Objectives:

(i) to develop high resolution robust spectral analysis and estimation techniques for signal analysis and system identification,

(ii) to develop notions of distance between power spectra and, accordingly, an approximation theory for reconciling new data with prior information, and

(iii) application of advances from (i-ii) in synthetic aperture radar and ultrasound imaging.

Status of effort

Spectral analysis of a time series often reveals information about the underlying mechanism generating the process. For instance, the spectral content of the echo of a radar provides important clues about the nature, location, and motion of potential targets. Similarly, cross correlation between signals can help identify models for their interaction.

Nonlinear techniques and the use of second order statistics allow a certain improvement in robustness and resolution compared to classical linear Fourier analysis. Most important are the so-called modern nonlinear methods (Maximum entropy, Capon, MUSIC, ESPRIT, etc.) which rely primarily on exploiting the structure of Toeplitz matrices and their connection to the autocorrelation function. The research under the present grant has led to a natural evolution of these methods with substantial benefits. Our current aim is to study limits of resolution and robustness in spectral analysis and, in parallel, to develop reliable high resolution methods.

The starting point has been to consider statistics other than the ordinary partial autocorrelation sequence. It turns out that, in the most general form, such statistics can be cast in the form of the state covariance of a linear filter driven by the stochastic process. We have shown that a state covariance has a very special structure and can be identified with the Pick matrix of an associated analytic interpolation problem. The fertile connection between generalized analytic interpolation and stochastic processes allows an elegant theory for identifying power spectra consistent with such estimated statistics (i.e. solving the relevant inverse problem). In turn, proper selection of statistics and dynamics of a filtering apparatus (which may incorporate

sensor dynamics) results in a substantial improvement in resolution as compared to pre-existing methods.

Besides tapping into techniques and tools of classical generalized analytic interpolation and of H^∞-theory, we have introduced certain natural convex entropy-like functionals for quantifying distances between spectra and between statistics. These have allowed us to resolve certain long standing issues. In particular, we have been able to provide a constructive characterization of all spectra of a given complexity which are consistent with the data. We have also been able to reconcile estimation of statistics with the rigid underlying structure which must be inherited from the dynamics of measuring/processing apparatus. In each case, the underlying approximation problem is convex, leading naturally to computational methods.

On the application side, our efforts have focused in the area of synthetic aperture radar (SAR), ultrasound imaging, and more recently, speech processing and speaker identification.

Accomplishments

Let $\{u_k : k = 0, 1, \ldots\}$ be a zero-mean stationary stochastic process. In the most general form, second order statistics of u_k can be cast in the form of a state covariance $\Sigma = E\{x_k x_k^*\}$ of a linear dynamical system $x_k = A x_{k-1} + B u_k$ driven by u_k ($A \in \mathbb{C}^{n \times n}$, $B \in \mathbb{C}^{n \times m}$ being a controllable pair of matrices). From this point on, Σ is thought of as data, while the above linear system incorporates sensor dynamics and appropriate filtering.

The structure of state covariances is rather special. In [2,7,8] we have shown that $\Sigma \geq 0$ is a state covariance if and only if the matrix equation

$$\Sigma - A\Sigma A^* = BH + H^* B^* \tag{1}$$

is solvable for $H \in \mathbb{C}^{m \times n}$. In the same publications we have shown how to formulate, with the help of A, B, Σ, H, a generalized analytic interpolation problem whose solutions produce all power spectra for u_k that are consistent with Σ. Thus, we treat spectral analysis as an inverse problem and the size of the family of solutions represents the uncertainty. Suitable selection of the filtering parameters in A, B, prior to estimating the statistics Σ, can limit the uncertainty and, thereby, significantly improve resolution and robustness of the spectral estimates. A variety of algorithms, which correspond to certain extremal elements of the family of solutions, have been devised, studied, and used in [2,8].

In order to develop a suitable approximation theoretic framework we have introduced certain information theoretic notions distance between "positive objects." More specifically, if $\hat{\Sigma}$ and Σ are two positive definite matrices,

$$\mathbb{S}(\Sigma, \hat{\Sigma}) := \text{trace}\left(\hat{\Sigma}(\log \hat{\Sigma} - \log \Sigma)\right)$$

represents a notion of distance between them (though not a metric). This expression is known in quantum mechanics as the *von Neumann relative entropy* and is used to quantify distance and entanglement between states of a quantum system. In our context, we formulate the *convex minimization problem*

$$\Sigma := \operatorname{argmin} \left\{ \mathbb{S}(\Sigma, \hat{\Sigma}) \; : \; \text{equation (1) is solvable} \right\}$$

as a means to correct errors of a statistical nature and to identify a legitimate state covariance Σ from an estimated sample covariance $\hat{\Sigma}$.

Similarly, the *Shannon-Kullback-Leibler relative entropy* expression

$$\mathbb{S}(f, \hat{f}) := \int_0^{2\pi} \left(\hat{f}(\theta)(\log \hat{f}(\theta) - \log f(\theta)) \right) d\theta$$

serves as a distance measure between spectral densities. In joint work with Anders Lindquist we have recently shown that, for any "prior" spectral estimate $\hat{f}$ and any datum $\Sigma \geq 0$, there is unique f which is consistent with the hypothesis that it is the spectral density for the input u_k with state covariance Σ, and f is closest to $\hat{f}$ in the sense of $\mathbb{S}(f, \hat{f})$. Furthermore, the solution of the optimal choice f is rather simple and computationally attractive. The use of $\mathbb{S}$ is a natural evolution of earlier ideas in joint work with Chris Byrnes and Anders Lindquist, e.g., see [6]. The above viewpoint leads to potentially fertile connections between spectral analysis and hypothesis testing and has immediate application to areas such as speech analysis and speaker identification.

Case studies in synthetic aperture radar imaging (SAR), with MSTAR data, highlight the potential of the framework:

Figure 1 shows a T-72 tank and a typical SAR image of the vehicle. Each line originates as estimated power spectrum of the radar echo in the relevant "distance bin". Due to the large number of scatterers in the ground modern nonlinear methods have had limited applicability since they tend to generate high order models, and thus, numerically unreliable. As a consequence fft-based techniques have so-far remained the standard. In contrast, our techniques, via use of specialized statistics, allow a degree of "focusing" and lead to low order and more reliable models.

In Figure 2, from left to right, the first image is constructed via nonlinear superposition (pasting) of high resolution reconstruction of adjacent sectors. The second image shows the result of a small level of (Gaussian) smoothing to connect contiguous scatters, while the third displays edge detection based on the previous two. A substantial improvement over fft and classical nonlinear methods is quite apparent.

Finally, in collaboration E. Ebbini (University of Minnesota), we have applied our framework for spectral analysis in non-invasive temperature sensing via ultrasound (intended for

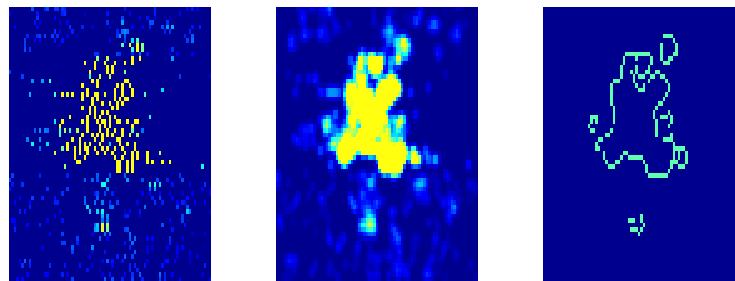Figure 1: T-72 tank and SAR-fft-based reconstruction

Figure 2: High resolution reconstruction and edge detection

controlled heat-treatment in various pathological conditions). The tissue temperature produces a subtle frequency shift of harmonics in the echo—hence the relevance of high resolution analysis.

Personnel Supported

1. Tryphon T. Georgiou (P.I.)

2. Ali Nasiri Amini (M.S./Ph.D. student)

3. Zoran Latinovic (M.S. student)

Interactions

1. Fields Institute, Toronto, workshop on the Calculus of Variations in Physics, Economics, and Geometry, August 2001.

2. NIAAA workshop on Alcohol, Neuroscience, and Bioinformatics, September 2001.

3. University of Illinois, Urbana-Champaign, Seminar-series, April 2002.

4. Lincoln Labs, June 2002.

Transitions

Method and Apparatus for a tunable high-resolution spectral estimator,
Patent No. 6,400,310 (Byrnes/Georgiou/Lindquist).

Honors/Awards

1. *Fellow of the IEEE*, January 2000.

2. *Board of Governors, IEEE Control Systems Society.*
   Elected: January 2002 – December 2005.

3. *Hermes-Luh Professorship in Electrical Engineering,*
   University of Minnesota, 2002—

Journal Publications

1. T.T. Georgiou, "Analytic Interpolation and the Degree Constraint," *Int. J. Applied Mathematics and Computer Science*, Special issue, guest editors M. Fliess and A. El Jai, January 2001, vol. 11, No 1, 101-109.

2. T.T. Georgiou, "Spectral Estimation by Selective Harmonic Amplification," *IEEE Trans. on Automatic Control*, **46(1)**: 29-42, January 2001.

3. S. Varigonda and T.T. Georgiou, "Oscillations in Systems with Relay Hysteresis," *IEEE Trans. on Automatic Control*, **46(1)**: 65-77, January 2001.

4. T.T. Georgiou and M.C. Smith, "Remarks on "Robustness Analysis of Nonlinear Feedback Systems An Input-Output Approach," *IEEE Trans. on Automatic Control*, **46(1)**: 171-172, January 2001.

5. U. Walther, T.T. Georgiou and A. Tannenbaum, "On the Computation of Switching Surfaces in Optimal Control: A Groebner Basis Approach," *IEEE Trans. on Automatic Control*, April 2001.

6. C. Byrnes, T.T. Georgiou, and A. Lindquist, "A generalized entropy criterion for Nevanlinna-Pick interpolation: A convex optimization approach to certain problems in systems and control", *IEEE Trans. on Automatic Control*, **45(6):** 822-839, June 2001.

7. T.T. Georgiou, "The structure of state covariances and its relation to the power spectrum of the input," *IEEE Trans. on Automatic Control*, to appear, July 2002.

8. T.T. Georgiou, "Spectral analysis based on the state covariance: the maximum entropy spectrum and linear fractional parameterization," *IEEE Trans. on Automatic Control*, to appear, October 2002.

Book Chapters

9. I.J. Fialho and T.T. Georgiou, "Chapter 7: Computational Algorithms for Sparse Optimal Digital Controller Realizations," in *Digital Controller Implementation and Fragility: A Modern Perspective*, Robert SH Istepanian & James F Whidborne (editors) Springer-Verlag, 2001.

10. T.T. Georgiou, P.J. Olver, and A. Tannenbaum, Maximal entropy for reconstruction of back projection images, "IMA Volumes in Mathematics and its Applications," *Volume 133: Mathematical methods in computer vision* Springer-Verlag, 2002.

Refereed Conference Publications

11. S. Varigonda, T.T. Georgiou and M. Daoutides, "A flatness based algorithm for optimal periodic control problems," Proceedings of the American Control Conference, Arlington, VA, June 2001.

12. T.T. Georgiou, "On the structure of state covariances," Proceedings of the IEEE Conf. on Decision and Control, Orlando, Florida, December 2001.