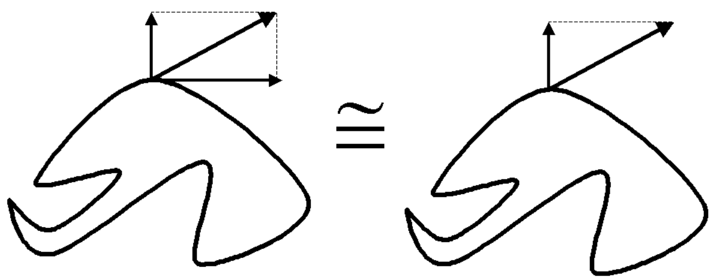

figure 2.1. *Most general form of a curve evolution flow. The velocity is decomposed into its tangential and normal components, the former not affecting the geometry of the flow, just its parametrization.*

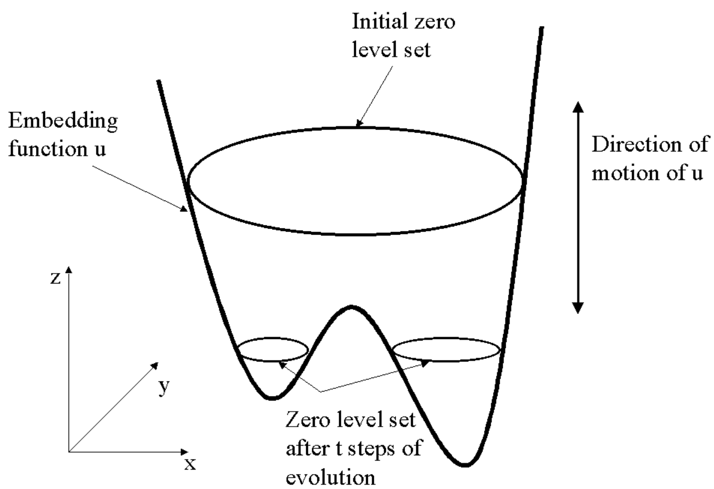

figure 2.2. *Embedding the curve in a higher dimensional function automatically solves topological problems. While the curve can be changing topology, the surface moves up and down, on a fixed coordinate system, without altering its topology. The topological change is just 'discovered' when computing the corresponding level-set of the function.*

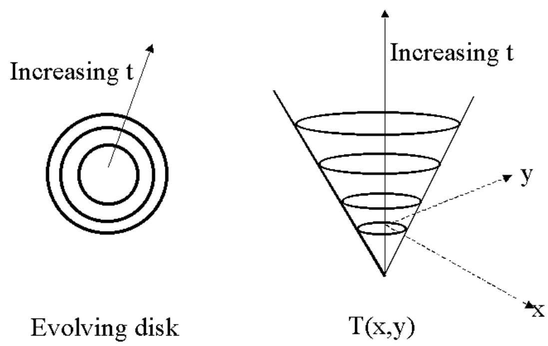

figure 2.3. *A disk evolving with unit speed, together with the corresponding time of arrival function $T(x, y)$.*

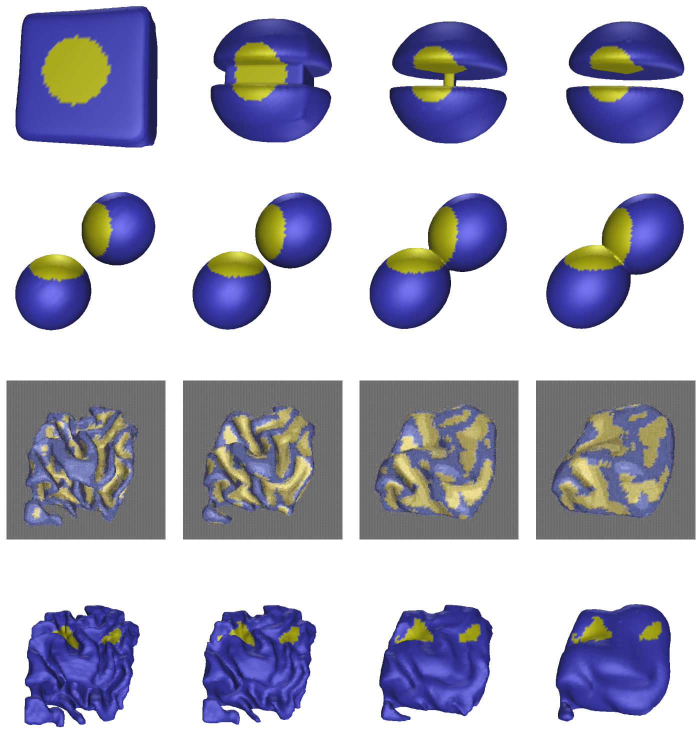

figure 2.4. *Examples of the region tracking algorithm (brighter regions are the ones being tracked). The two top rows show toy examples demonstrating possible topological changes on the tracked region. The next rows show unfolding the cortex, and tracking the marked regions, with a curvature based flow and a 3D morphing one, respectively.*

Interpolation of Two Linked Tori

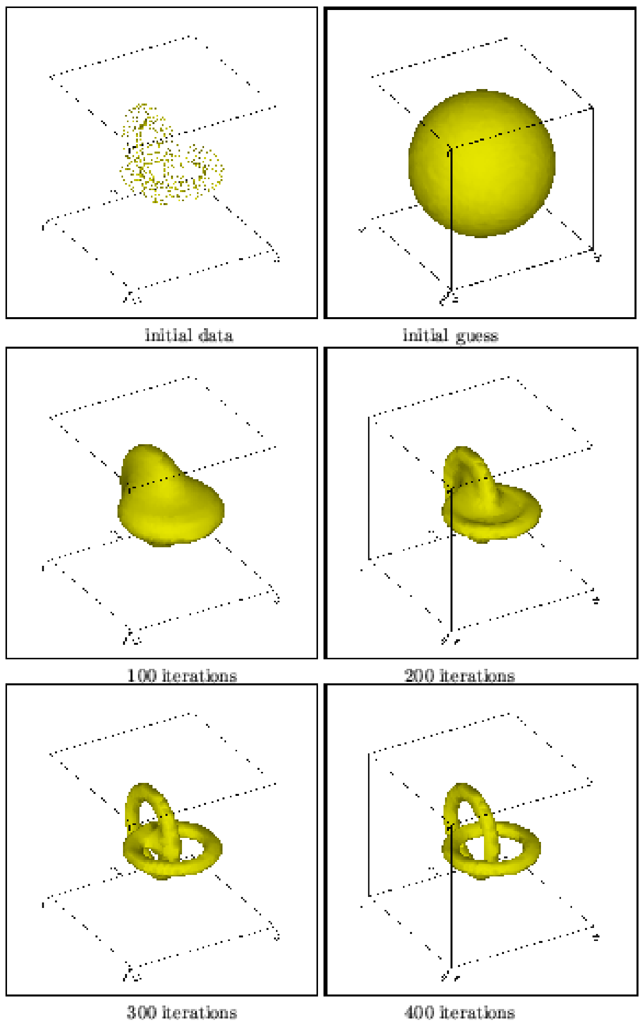

figure 2.5. *Example of the use of variational level-sets for the reconstruction of unorganized points. Observe how the two tori are reconstructed, without any prior knowledge of the connection between the points or the topology of the object.*

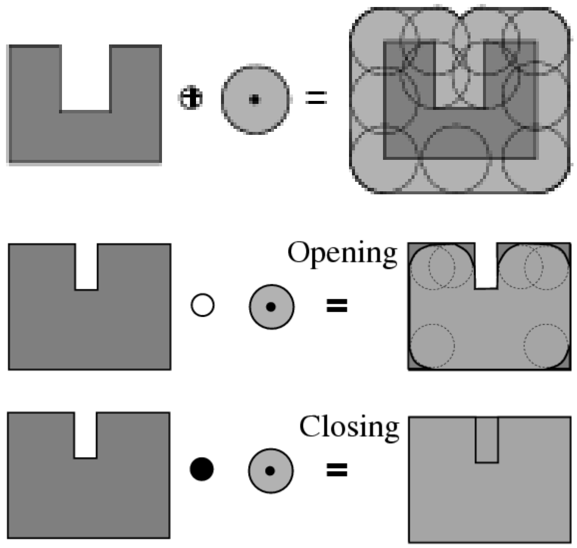

figure 2.6. *Examples of dilation (first row), opening, and closing.*

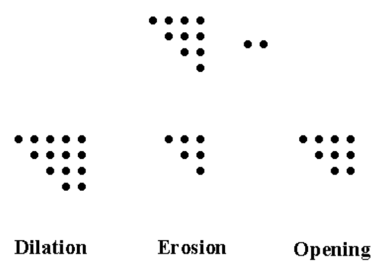

figure 2.7. *Examples of basic discrete morphological operations.*

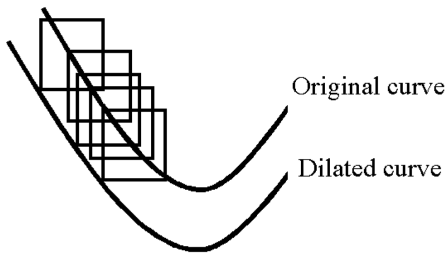

figure 2.8. *Generalized Hüygens principle.*

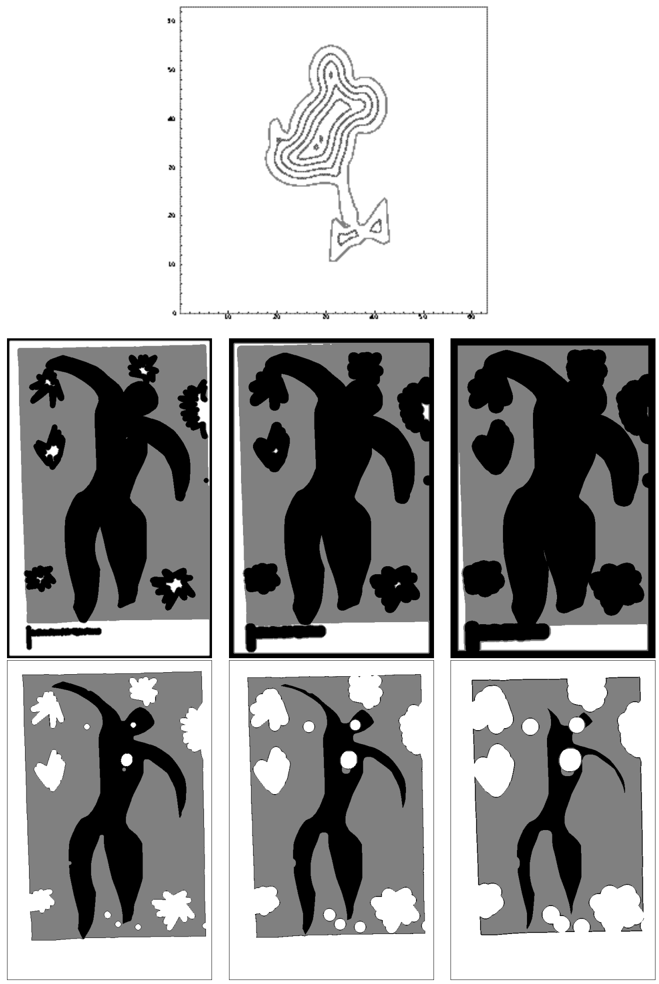

figure 2.9. *Examples of continuous morphology via curve evolution. The erosion of a curve is shown in the first row. Steps of image erosion (with a disk) are shown in the second row and steps of image dilation in the third. Note of course that when the bright objects are eroded, the dark objects (background) are dilated, and viceversa. Also note that several objects are eroded/dilated at the same time, since the process is performed on the gray-valued image, which is equivalent to the level-sets process (binary erosion/dilation).*

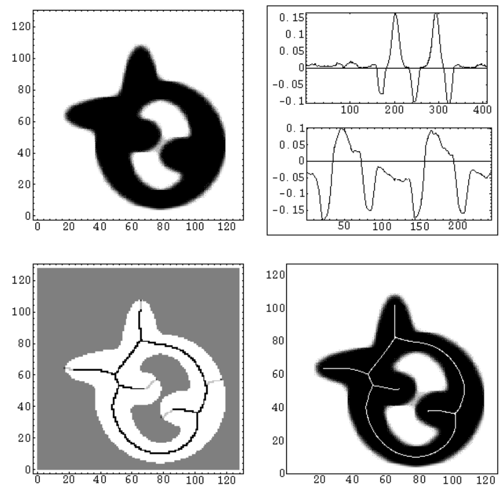

figure 2.10. *Examples of the skeleton constructed with the constant velocity curve flow. The first row shows the original binary shape and its curvature (for both inner and outer contours). the second row shows a discrete computation (left) and the curve evolution one (right).*

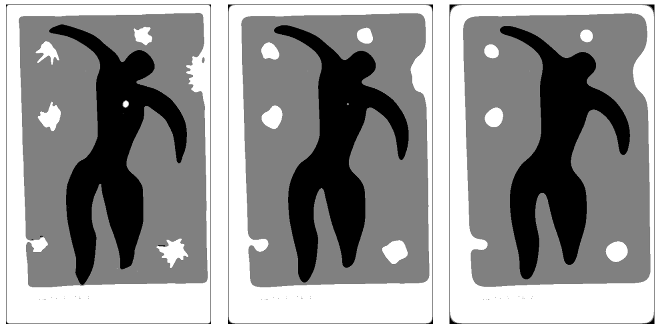

figure 2.11. *Steps of the Euclidean geometric heat flow. Note how the objects are getting smoother.*

β	Volume Preserving Flow	Area Preserving flow
1	$\mathcal{S}_t = \left(1 - \frac{\rho \mathbf{A}}{3\mathbf{V}_0}\right)\vec{\mathcal{N}}$	$\mathcal{S}_t = \left(1 - \frac{\rho \iint \mathbf{H} d\mu}{2\mathbf{A}_0}\right)\vec{\mathcal{N}}$
$\mathbf{H}$	$\mathcal{S}_t = \left(\mathbf{H} - \frac{\rho \iint \mathbf{H} d\mu}{3\mathbf{V}_0}\right)\vec{\mathcal{N}}$	$\mathcal{S}_t = \left(\mathbf{H} - \frac{\rho \iint \mathbf{H}^2 d\mu}{2\mathbf{A}_0}\right)\vec{\mathcal{N}}$
$\mathbf{K}$	$\mathcal{S}_t = \left(\mathbf{K} - \frac{\rho 4\pi}{3\mathbf{V}_0}\right)\vec{\mathcal{N}}$	$\mathcal{S}_t = \left(\mathbf{K} - \frac{\rho \iint \mathbf{K}\mathbf{H} d\mu}{2\mathbf{A}_0}\right)\vec{\mathcal{N}}$
$\mathbf{K}_+^{1/4}$	$\mathcal{S}_t = \left(\mathbf{K}_+^{1/4} - \frac{\rho \iint \mathbf{K}_+^{1/4} d\mu}{3\mathbf{V}_0}\right)\vec{\mathcal{N}}$	$\mathcal{S}_t = \left(\mathbf{K}_+^{1/4} - \frac{\rho \iint \mathbf{K}_+^{1/4}\mathbf{H} d\mu}{2\mathbf{A}_0}\right)\vec{\mathcal{N}}$

figure 2.12. *Area and volume preserving flows for a number of 3D geometric evolution equations frequently used.*

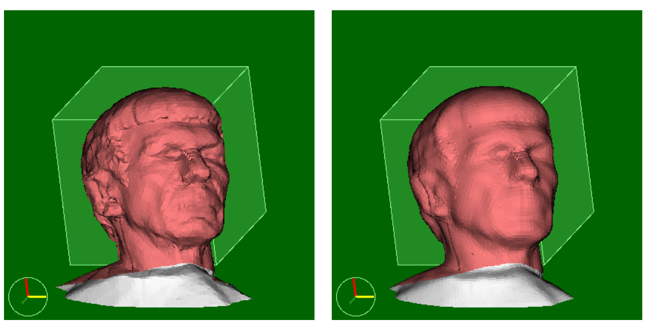

figure 2.13. *Geometric smoothing with a volume preserving flow.*