Dynamics and control of wall-bounded shear flows

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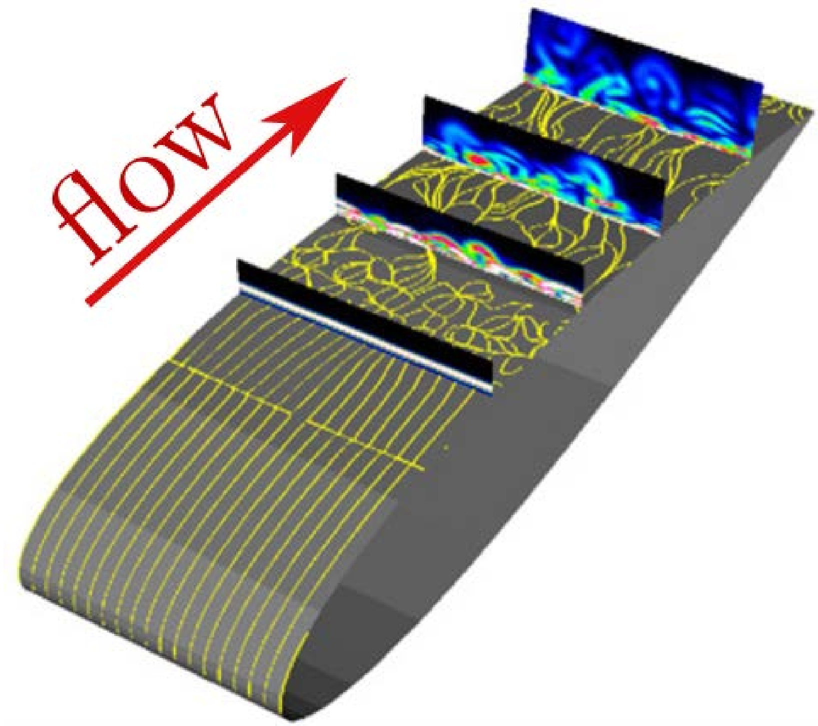
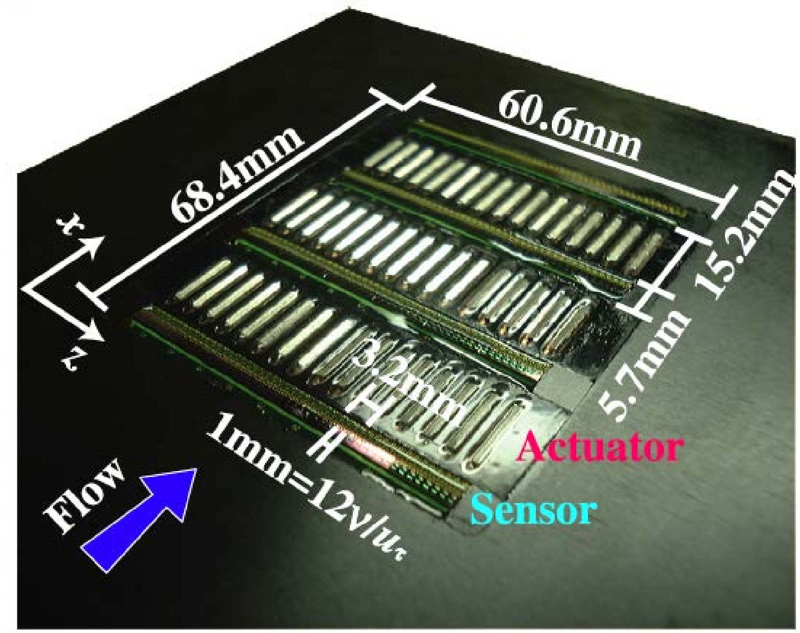
Flow control





technology: shear-stress sensors; surface-deformation actuators

application: turbulence suppression; skin-friction drag reduction

challenge: distributed controller design for complex flow dynamics

Outline

❶ Dynamics and control of wall-bounded shear flows

- **The early stages of transition**
  - ⋆ initiated by high flow sensitivity
- **Controlling the onset of turbulence**
  - ⋆ simulation-free design for reducing sensitivity

Key issue:

high flow sensitivity

❷ Case studies

- **Sensor-free flow control**
  - ⋆ streamwise traveling waves
- **Feedback flow control**
  - ⋆ design of optimal estimators and controllers

❸ Summary and outlook

Transition to turbulence

- LINEAR HYDRODYNAMIC STABILITY: **unstable normal modes**
  - ⋆ **successful in:** Benard Convection, Taylor-Couette flow, etc.
  - ⋆ **fails in:** wall-bounded shear flows (channels, pipes, boundary layers)

- DIFFICULTY 1

  Inability to predict: Reynolds number for the onset of turbulence (Re_c)

  Experimental onset of turbulence: $\begin{cases} \text{much before instability} \\ \text{no sharp value for } Re_c \end{cases}$

- DIFFICULTY 2

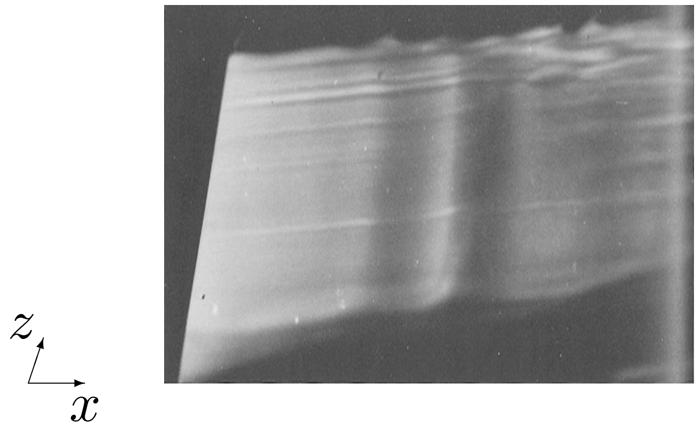
  Inability to predict: flow structures observed at transition

  (except in carefully controlled experiments)

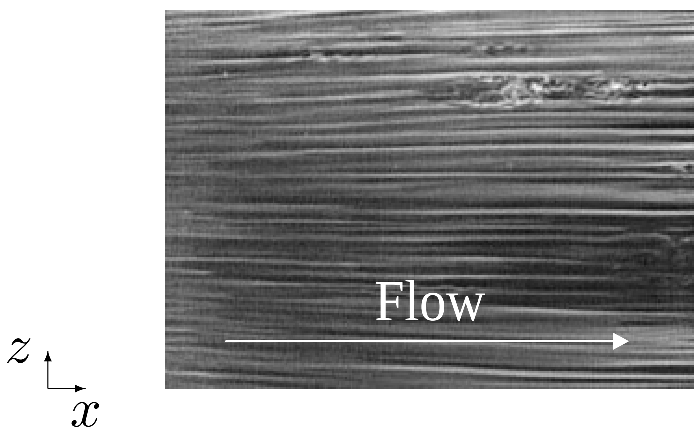
LINEAR STABILITY:

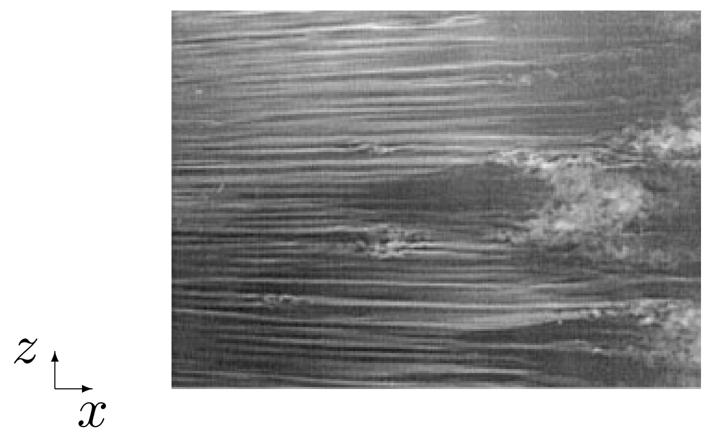
⋆ For $Re \geq Re_c \;\Rightarrow\;$ exp. growing normal modes

corresponding e-functions (TS-waves) $\Big\}$ $=$ exp. growing flow structures



NOISY EXPERIMENTS: **streaky boundary layers and turbulent spots**





Matsubara & Alfredsson, *J. Fluid Mech. '01*

- FAILURE OF LINEAR HYDRODYNAMIC STABILITY
  caused by high flow sensitivity
  - ⋆ large transient responses
  - ⋆ large noise amplification
  - ⋆ small stability margins

TO COUNTER THIS SENSITIVITY: **must account for modeling imperfections**

TRANSITION ≈ STABILITY + RECEPTIVITY + ROBUSTNESS

RECEPTIVITY ↓ **flow disturbances**

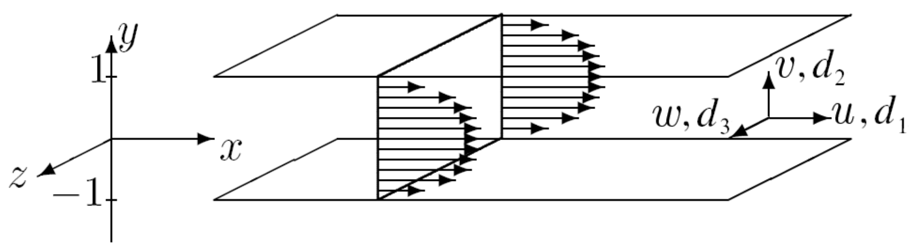
ROBUSTNESS ↓ **unmodeled dynamics**

Farrell, Ioannou, Schmid, Trefethen, Henningson, Gustavsson, Reddy, Bamieh, etc.

Tools for quantifying sensitivity

- INPUT-OUTPUT ANALYSIS: **spatio-temporal frequency responses**

$\mathbf{d}$ { Free-stream turbulence, Surface roughness, Neglected nonlinearities } → **Linearized Dynamics** → { Fluctuating velocity field } $\mathbf{v}$



$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

IMPLICATIONS FOR:

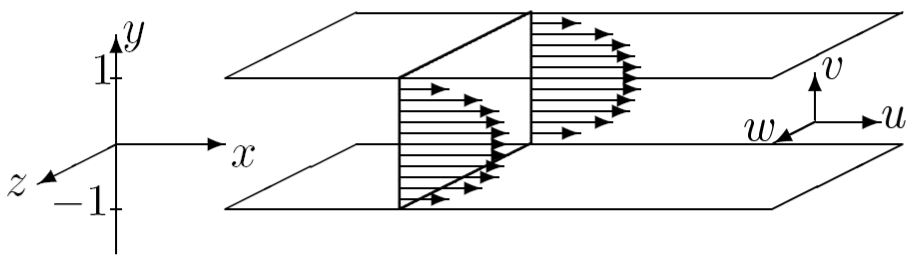
transition: insight into mechanisms

control: control-oriented modeling

Transient growth analysis

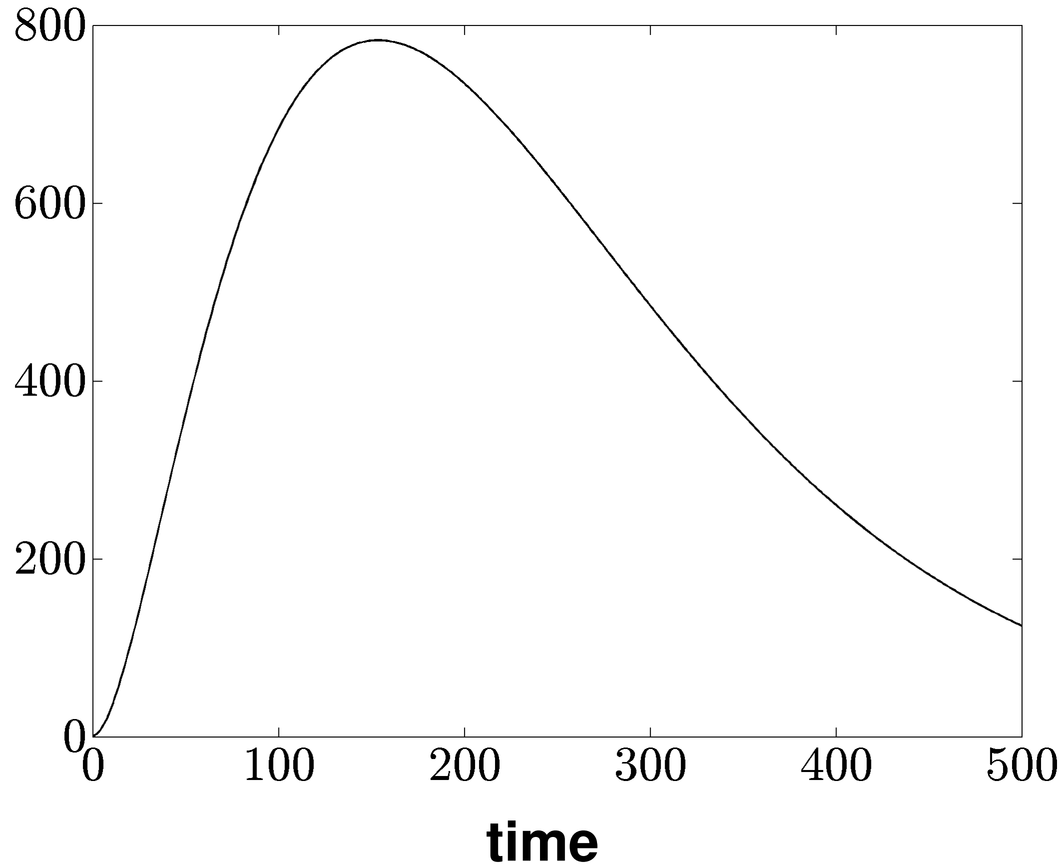
- STUDY TRANSIENT BEHAVIOR OF FLUCTUATIONS' ENERGY

$$Re < 5772$$



$$\psi_t = A\,\psi$$

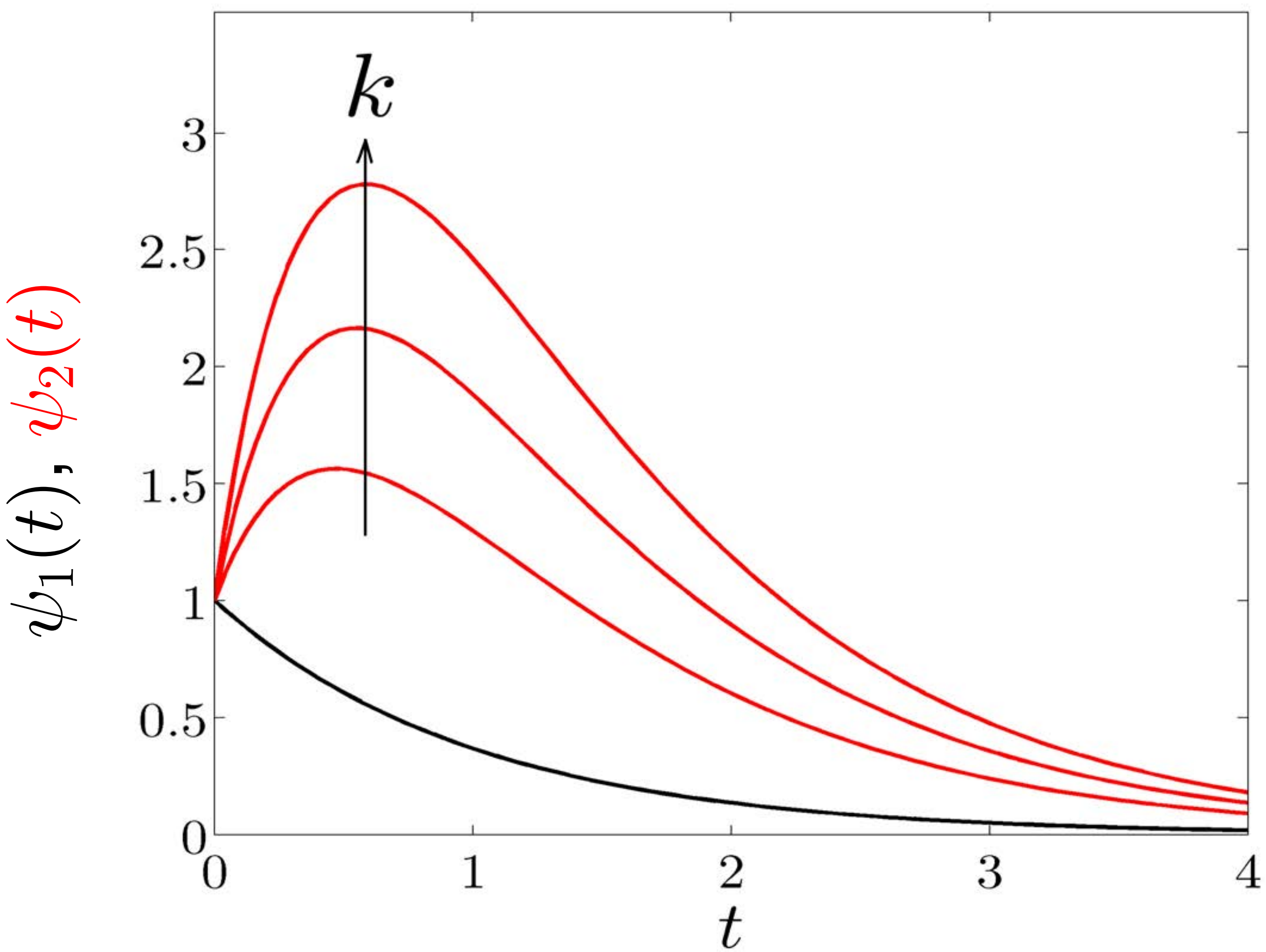
kinetic energy:



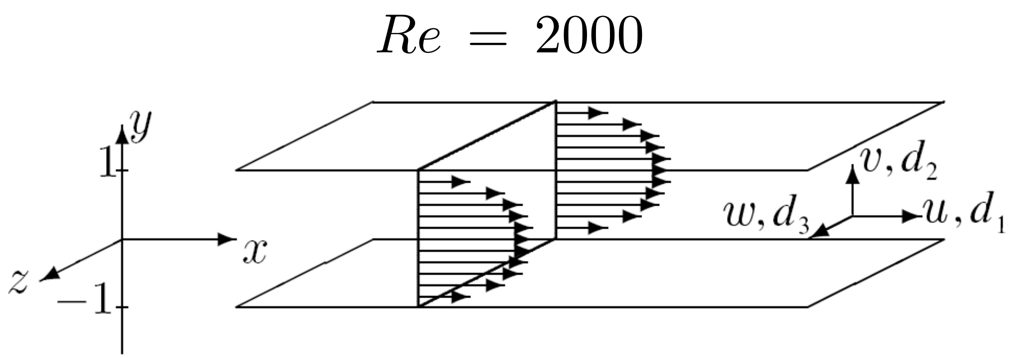
☞ **E-values: misleading measure of transient response**

A toy example

$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -1 & \mathbf{0} \\ k & -2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$



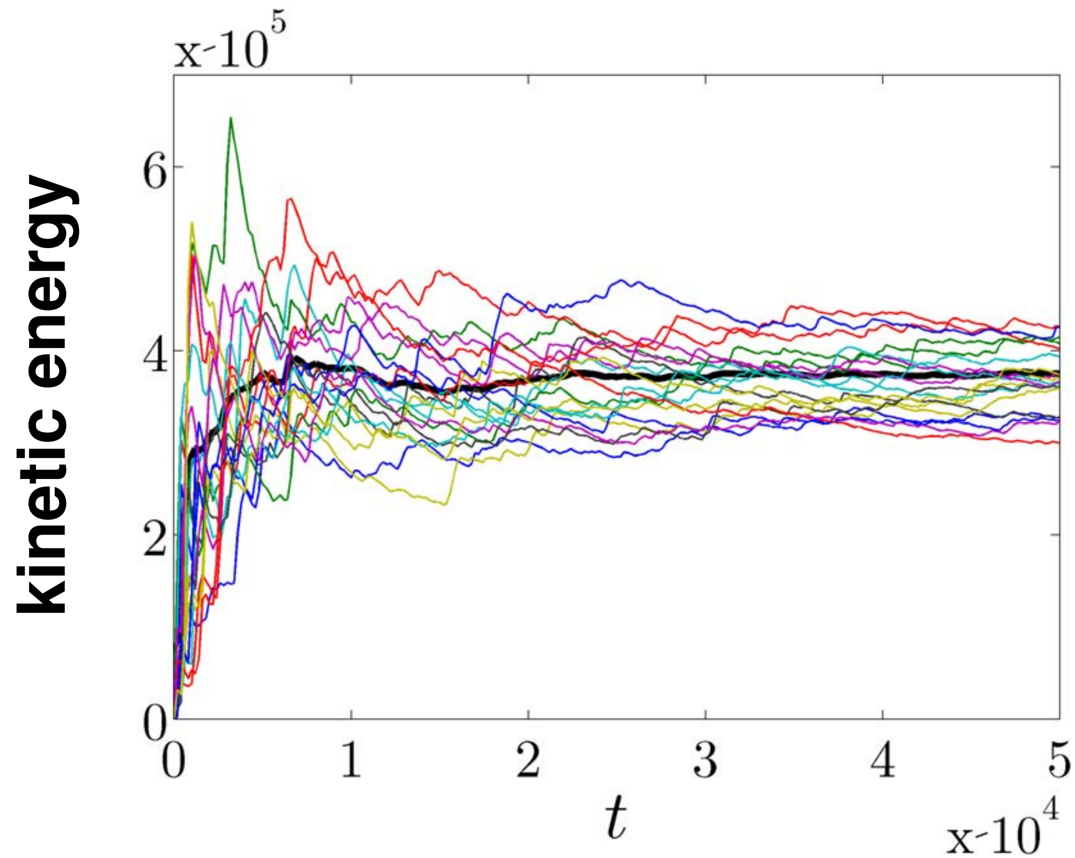
Response to stochastic forcing



forcing:

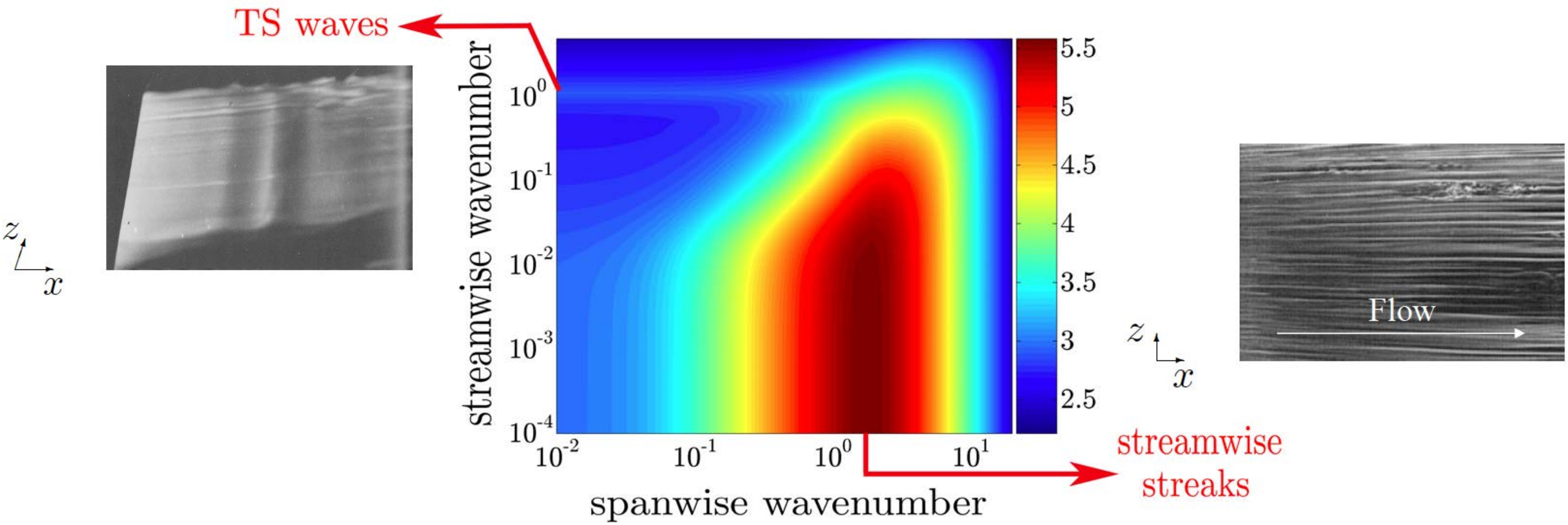
white	in t and y
harmonic	in x and z

$$\mathbf{d}(x, y, z, t) \;=\; \hat{\mathbf{d}}(k_x, y, k_z, t)\, \mathrm{e}^{\mathrm{i}(k_x x \,+\, k_z z)}$$



Ensemble average energy density

channel flow with $Re = 2000$:



- **Dominance of streamwise elongated structures**
  streamwise streaks!

Farrell & Ioannou, Phys. Fluids A '93

Jovanović & Bamieh, J. Fluid Mech. '05

Schmid, Annu. Rev. Fluid Mech. '07

Gayme et al., J. Fluid Mech. '10

Analysis of linear dynamical systems

Schmid, Annu. Rev. Fluid Mech. ’07

State-space representation

state equation: $\dot{\psi}(t) \;=\; A\,\psi(t) \;+\; B\,d(t)$

output equation: $\phi(t) \;=\; C\,\psi(t)$

- **Solution to state equation**

$$\psi(t) \;=\; \mathrm{e}^{At}\,\psi(0) \;+\; \int_0^t \mathrm{e}^{A(t\,-\,\tau)}\,B\,d(\tau)\,\mathrm{d}\tau$$

$\mathrm{e}^{At}\,\psi(0)$ $\longrightarrow$ **unforced response**

$\int_0^t \mathrm{e}^{A(t-\tau)}\,B\,d(\tau)\,\mathrm{d}\tau$ $\longrightarrow$ **forced response**

Transform techniques

$$\dot{\psi}(t) \;=\; A\,\psi(t) \;+\; B\,d(t) \quad \xrightarrow{\text{Laplace transform}} \quad s\,\hat{\psi}(s) \;-\; \psi(0) \;=\; A\,\hat{\psi}(s) \;+\; B\,\hat{d}(s)$$

$$\psi(t) \;=\; \mathrm{e}^{At}\,\psi(0) \;+\; \int_0^t \mathrm{e}^{A(t\,-\,\tau)}\,B\,d(\tau)\,\mathrm{d}\tau$$

$$\Updownarrow$$

$$\hat{\psi}(s) \;=\; (sI \;-\; A)^{-1}\,\psi(0) \;+\; (sI \;-\; A)^{-1}\,B\,\hat{d}(s)$$

Natural and forced responses

- **Unforced response**

matrix exponential	resolvent
$\psi(t) \;=\; \mathrm{e}^{A\,t}\,\psi(0)$	$\hat{\psi}(s) \;=\; (sI \,-\, A)^{-1}\,\psi(0)$

- **Forced response**

impulse response	transfer function
$H(t) \;=\; C\,\mathrm{e}^{A\,t}\,B$	$H(s) \;=\; C\,(sI \,-\, A)^{-1}\,B$

⋆ **Response to arbitrary inputs**

$$\phi(t) \;=\; \int_0^t H(t \,-\, \tau)\, d(\tau)\, \mathrm{d}\tau \;\xrightarrow{\text{Laplace transform}}\; \hat{\phi}(s) \;=\; H(s)\, \hat{d}(s)$$

UNFORCED RESPONSES

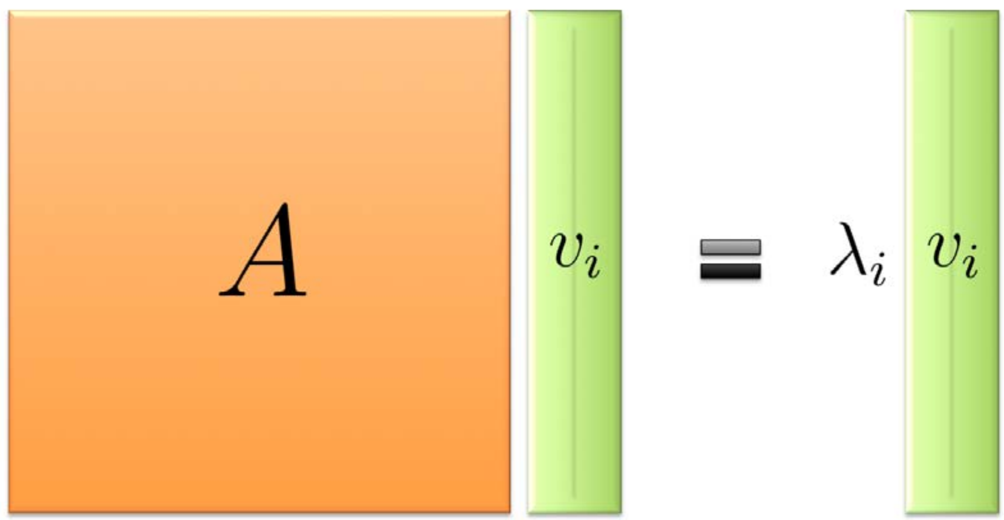
Systems with non-normal A

$$\dot{\psi}(t) \;=\; A\,\psi(t)$$

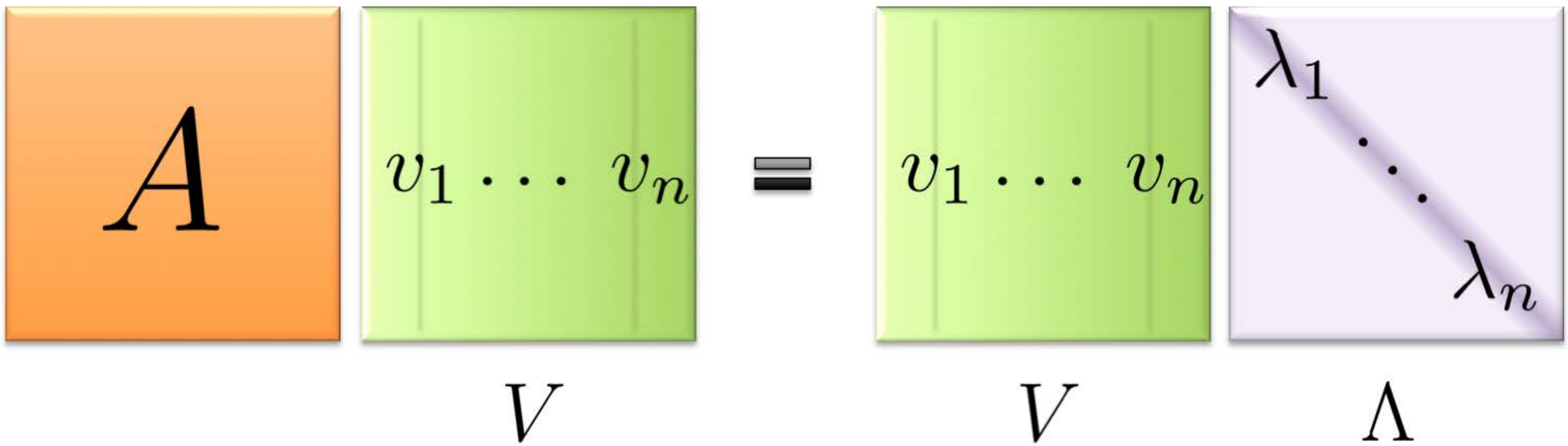
- **Non-normal operator: doesn't commute with its adjoint**

$$A\,A^* \;\neq\; A^* A$$

  - ⋆ **E-value decomposition of A**



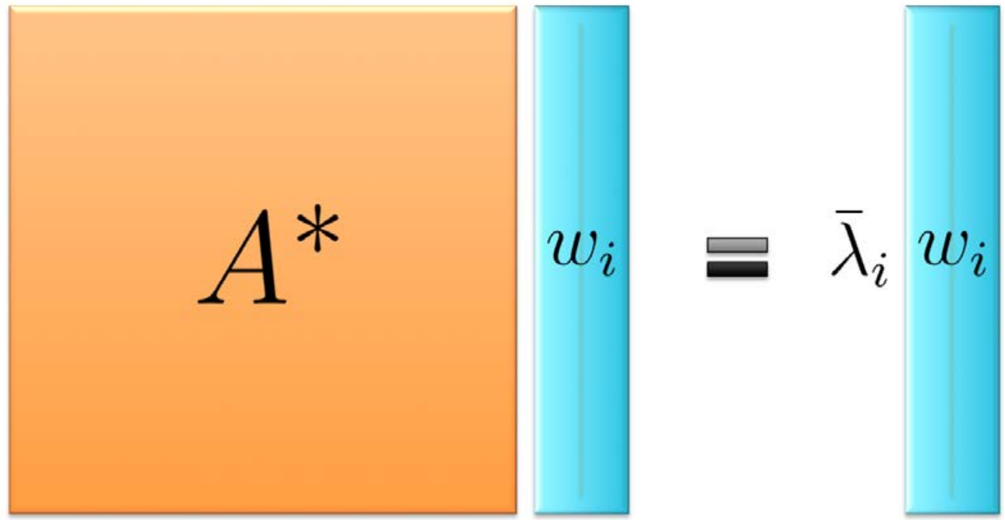
- **Let A have a full set of linearly independent e-vectors**



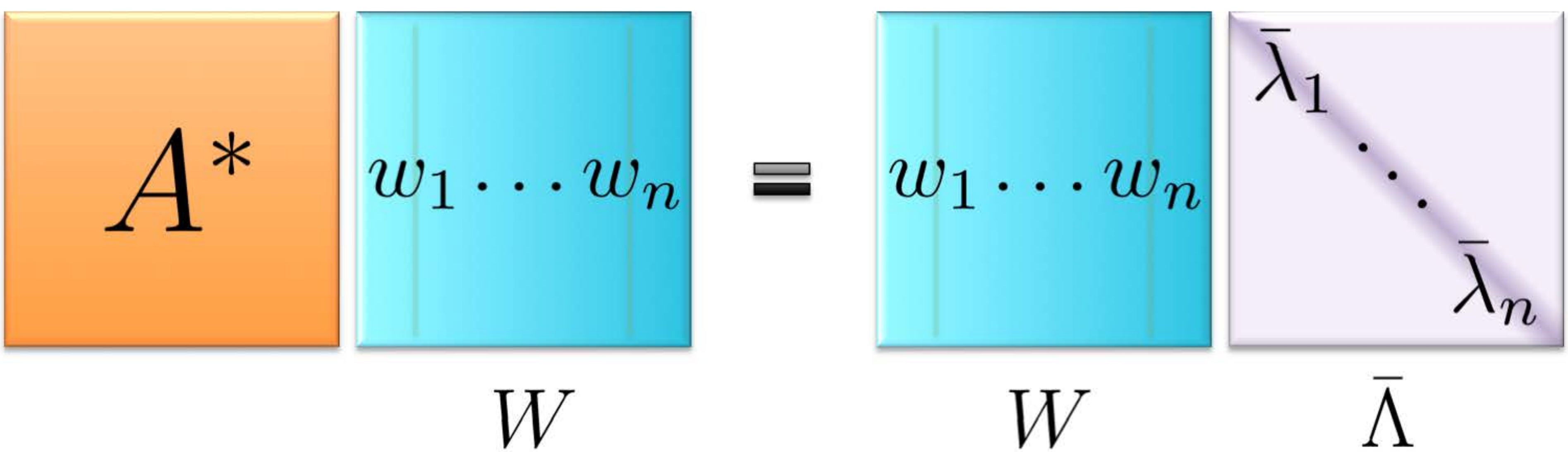
$\star$ **normal A: unitarily diagonalizable**

$$A = V \Lambda V^*$$

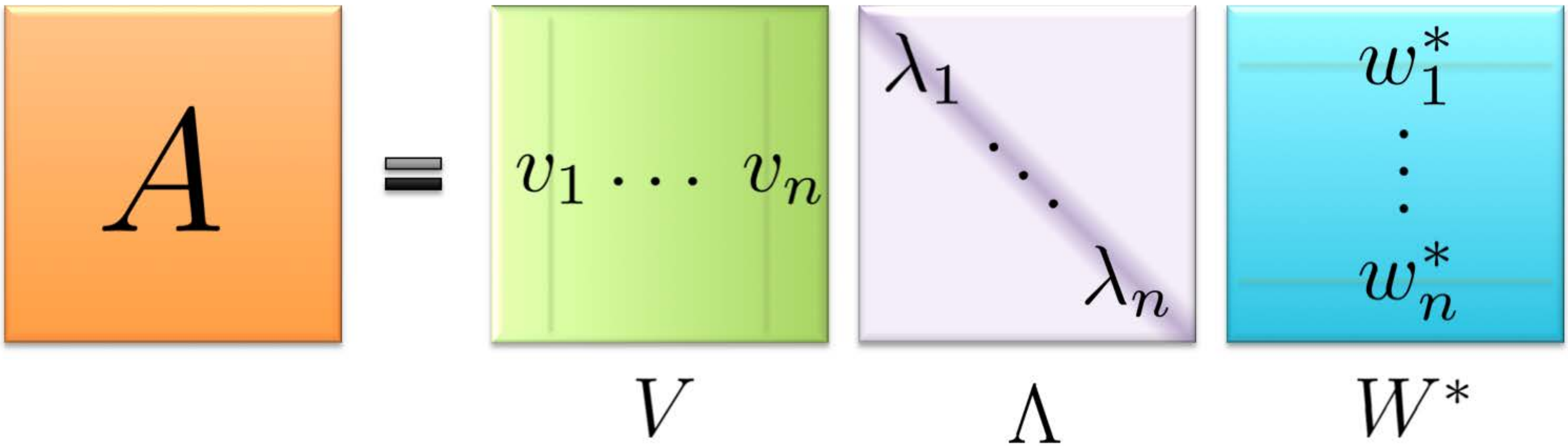
- **E-value decomposition of A^***



choose w_i such that $w_i^* v_j = \delta_{ij}$



- **Use V and W^* to diagonalize A**



⋆ **solution to** $\dot{\psi}(t) \;=\; A\,\psi(t)$

$$\psi(t) \;=\; \mathrm{e}^{A\,t}\,\psi(0) \;=\; \sum_{i\,=\,1}^{n} \mathrm{e}^{\lambda_i t}\, v_i\, \langle w_i, \psi(0) \rangle$$

- **Right e-vectors**
  - ⋆ **identify initial conditions with simple responses**

$$\psi(t) \;=\; \sum_{i\,=\,1}^{n} \mathrm{e}^{\lambda_i t}\, v_i \,\langle w_i, \psi(0)\rangle$$

$$\Big\downarrow \; \psi(0) \;=\; v_k$$

$$\psi(t) \;=\; \mathrm{e}^{\lambda_k t}\, v_k$$

- E-value decomposition of $A = \begin{bmatrix} -1 & 0 \\ k & -2 \end{bmatrix}$

$$\left\{ v_1 = \frac{1}{\sqrt{1 + k^2}} \begin{bmatrix} 1 \\ k \end{bmatrix},\; v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$\left\{ w_1 = \begin{bmatrix} \sqrt{1 + k^2} \\ 0 \end{bmatrix},\; w_2 = \begin{bmatrix} -k \\ 1 \end{bmatrix} \right\}$$

solution to $\dot{\psi}(t) = A\,\psi(t)$:

$$\psi(t) = \left(\mathrm{e}^{-t}\, v_1\, w_1^* + \mathrm{e}^{-2t}\, v_2\, w_2^*\right) \psi(0)$$

$$\downarrow$$

$$\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{-t}\,\psi_1(0) \\ k\left(\mathrm{e}^{-t} - \mathrm{e}^{-2t}\right)\psi_1(0) + \mathrm{e}^{-2t}\,\psi_2(0) \end{bmatrix}$$

- **E-values: misleading measures of transient response**

Forced responses

Amplification of disturbances

- **Harmonic forcing**

$$d(t) \;=\; \hat{d}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t} \quad \xrightarrow{\textbf{steady-state response}} \quad \phi(t) \;=\; \hat{\phi}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t}$$

⋆ **Frequency response**

$$\hat{\phi}(\omega) \;=\; \underbrace{C\,(\mathrm{i}\omega I \;-\; A)^{-1}\,B}_{H(\omega)}\,\hat{d}(\omega)$$

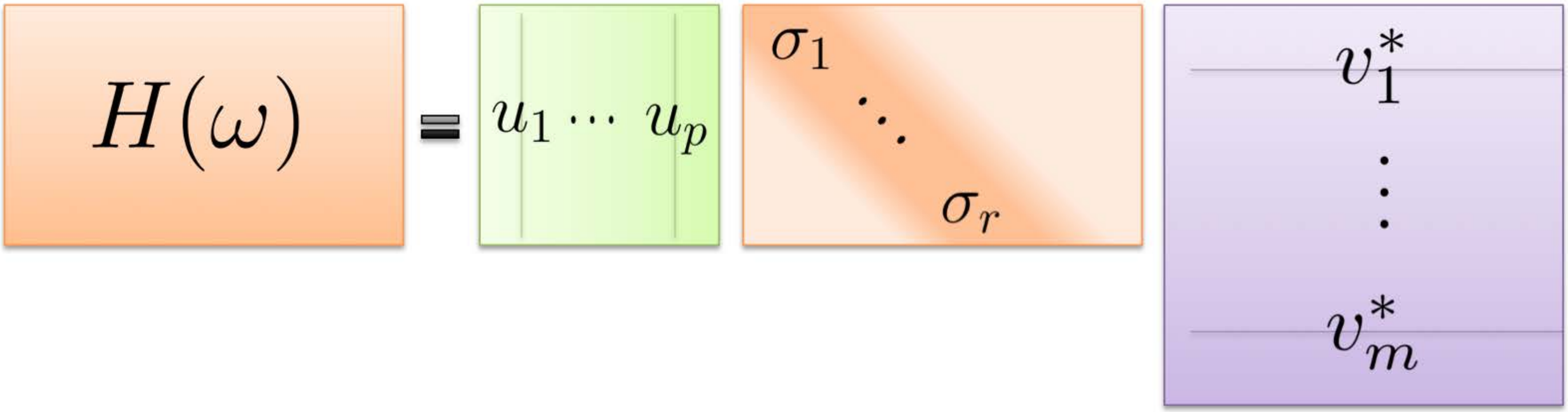
example: 3 **inputs,** 2 **outputs**

$$\begin{bmatrix} \hat{\phi}_1(\omega) \\ \hat{\phi}_2(\omega) \end{bmatrix} \;=\; \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & H_{13}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & H_{23}(\omega) \end{bmatrix} \begin{bmatrix} \hat{d}_1(\omega) \\ \hat{d}_2(\omega) \\ \hat{d}_3(\omega) \end{bmatrix}$$

$H_{ij}(\omega)$ – response from jth input to ith output

Input-output gains

- **Determined by singular values of** $H(\omega)$



left and right singular vectors:

$$
\begin{aligned}
H(\omega) H^*(\omega)\, u_i(\omega) &= \sigma_i^2(\omega)\, u_i(\omega) \\
H^*(\omega) H(\omega)\, v_i(\omega) &= \sigma_i^2(\omega)\, v_i(\omega)
\end{aligned}
$$

$\{u_i\}$ orthonormal basis of output space

$\{v_i\}$ orthonormal basis of input space

- **Action of $H(\omega)$ on $\hat{d}(\omega)$**

$$\hat{\phi}(\omega) \;=\; H(\omega)\,\hat{d}(\omega) \;=\; \sum_{i\,=\,1}^{r} \sigma_i(\omega)\, u_i(\omega) \left\langle v_i(\omega), \hat{d}(\omega) \right\rangle$$

- **Right singular vectors**
  - ⋆ **identify input directions with simple responses**

$$\sigma_1(\omega) \;\geq\; \sigma_2(\omega) \;\geq\; \cdots \;>\; 0$$

$$\hat{\phi}(\omega) \;=\; \sum_{i\,=\,1}^{r} \sigma_i(\omega)\, u_i(\omega) \left\langle v_i(\omega), \hat{d}(\omega) \right\rangle$$

$$\Big\downarrow \; \hat{d}(\omega) \;=\; v_k(\omega)$$

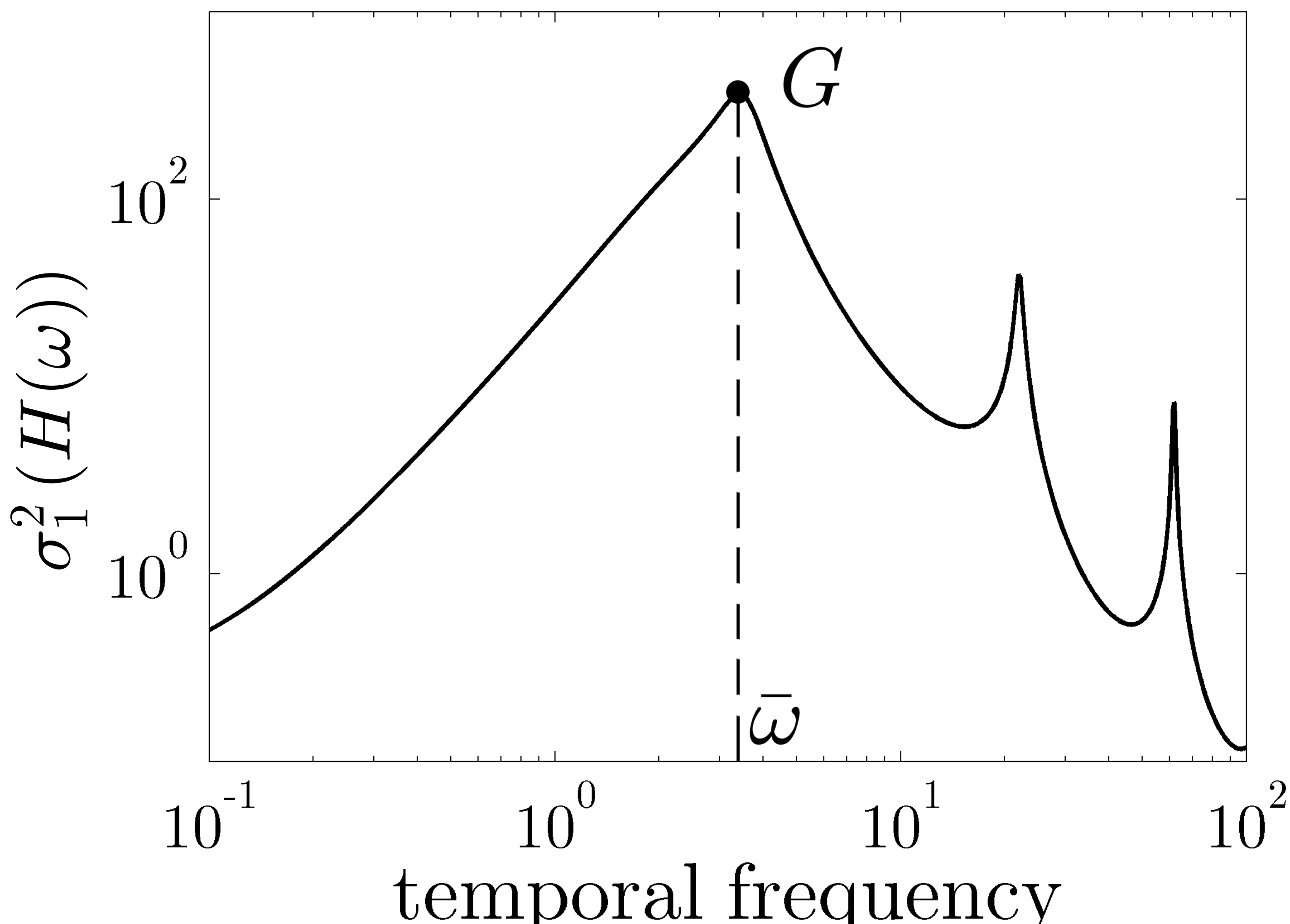
$$\hat{\phi}(\omega) \;=\; \sigma_k(\omega)\, u_k(\omega)$$

$\sigma_1(\omega)$**: the largest amplification at any frequency**

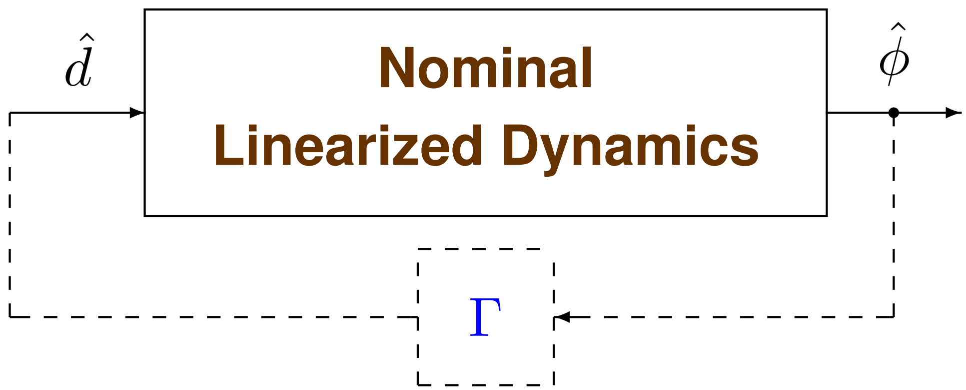
Worst case amplification

- H_∞ **norm: an induced L_2 gain (of a system)**

$$G \;=\; \|H\|_\infty^2 \;=\; \max \frac{\text{output energy}}{\text{input energy}} \;=\; \max_{\omega} \sigma_1^2\,(H(\omega))$$



Robustness interpretation



modeling uncertainty
(can be nonlinear or time-varying)

- **Closely related to pseudospectra of linear operators**

$$\dot{\psi}(t) \;=\; (A \,+\, B\,\Gamma\,C)\,\psi(t)$$

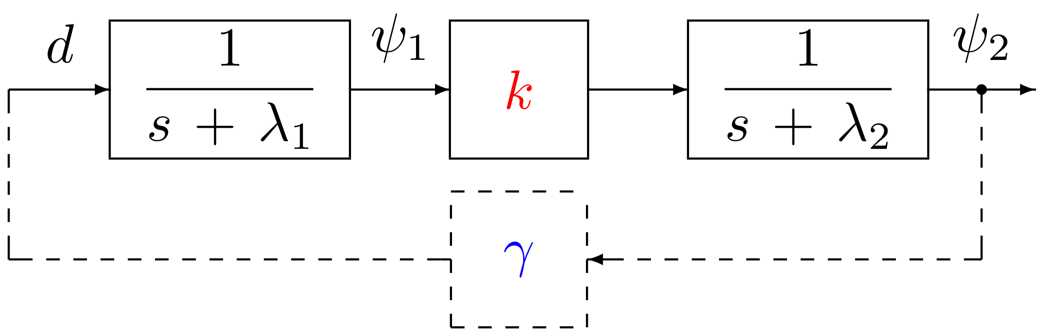
LARGE
worst case amplification $\Rightarrow$ **small stability margins**

Back to a toy example

$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & \mathbf{0} \\ k & -\lambda_2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d$$

$$G = \max_{\omega} |H(\mathrm{i}\omega)|^2 = \frac{k^2}{(\lambda_1 \lambda_2)^2}$$

Robustness



modeling uncertainty

stability

$$\Updownarrow$$

$$\gamma < \lambda_1 \lambda_2 / k$$

$$\det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\lambda_1 & \gamma \\ k & -\lambda_2 \end{bmatrix}\right) = s^2 + (\lambda_1 + \lambda_2)\, s + \underbrace{(\lambda_1 \lambda_2 - \gamma\, k)}_{>0}$$

Response to stochastic forcing

- **White-in-time forcing**

$$\mathcal{E}\left(d(t_1)\, d^*(t_2)\right) \;=\; I\,\delta(t_1 \,-\, t_2)$$

⋆ **Hilbert-Schmidt norm**

power spectral density:

$$\|H(\omega)\|_{\mathrm{HS}}^2 \;=\; \mathrm{trace}\left(H(\omega)\, H^*(\omega)\right) \;=\; \sum_{i\,=\,1}^{r} \sigma_i^2\,(\omega)$$

⋆ H_2 **norm**

variance amplification:

$$\|H\|_2^2 \;=\; \frac{1}{2\,\pi} \int_{-\infty}^{\infty} \|H(\omega)\|_{\mathrm{HS}}^2\, \mathrm{d}\omega \;=\; \int_0^{\infty} \|H(t)\|_{\mathrm{HS}}^2\, \mathrm{d}t$$

Computation of H_2 and H_∞ norms

$$
\begin{aligned}
\dot{\psi}(t) &= A\,\psi(t) \;+\; B\,d(t) \\
\phi(t) &= C\,\psi(t)
\end{aligned}
$$

- H_2 **norm**

  ⋆ **Lyapunov equation**

$$
\mathcal{E}\left(d(t_1)\,d^*(t_2)\right) \;=\; W\,\delta(t_1 \,-\, t_2) \;\;\Rightarrow\;\; \left\{ \begin{array}{rcl} \|H\|_2^2 &=& \mathrm{trace}\left(C\,P\,C^*\right) \\ A\,P \,+\, P\,A^* &=& -\,B\,W B^* \end{array} \right.
$$

- H_∞ **norm**

  ⋆ **E-value decomposition of Hamiltonian in conjunction with bisection**

$$
\|H\|_\infty \geq \gamma \;\;\Leftrightarrow\;\; \begin{bmatrix} A & \frac{1}{\gamma}\,B\,B^* \\ -\frac{1}{\gamma}\,C^*\,C & -A^* \end{bmatrix}
$$
has at least one imaginary e-value

BACK TO FLUIDS

Frequency response: channel flow

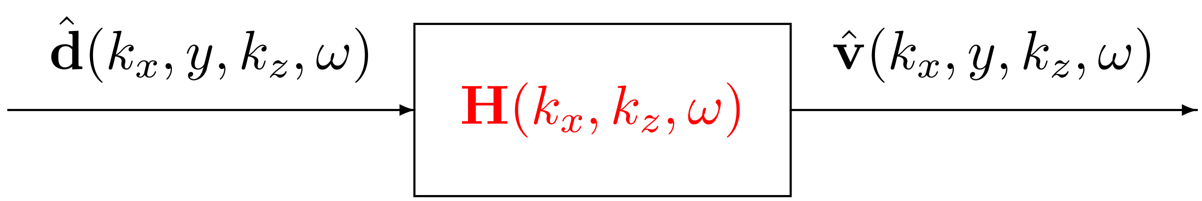
harmonic forcing:

$$\mathbf{d}(x,y,z,t) \;=\; \hat{\mathbf{d}}(k_x,y,k_z,\omega)\,\mathrm{e}^{\mathrm{i}(k_x x \,+\, k_z z \,+\, \omega t)}$$

$\downarrow$ **steady-state response**

$$\mathbf{v}(x,y,z,t) \;=\; \hat{\mathbf{v}}(k_x,y,k_z,\omega)\,\mathrm{e}^{\mathrm{i}(k_x x \,+\, k_z z \,+\, \omega t)}$$

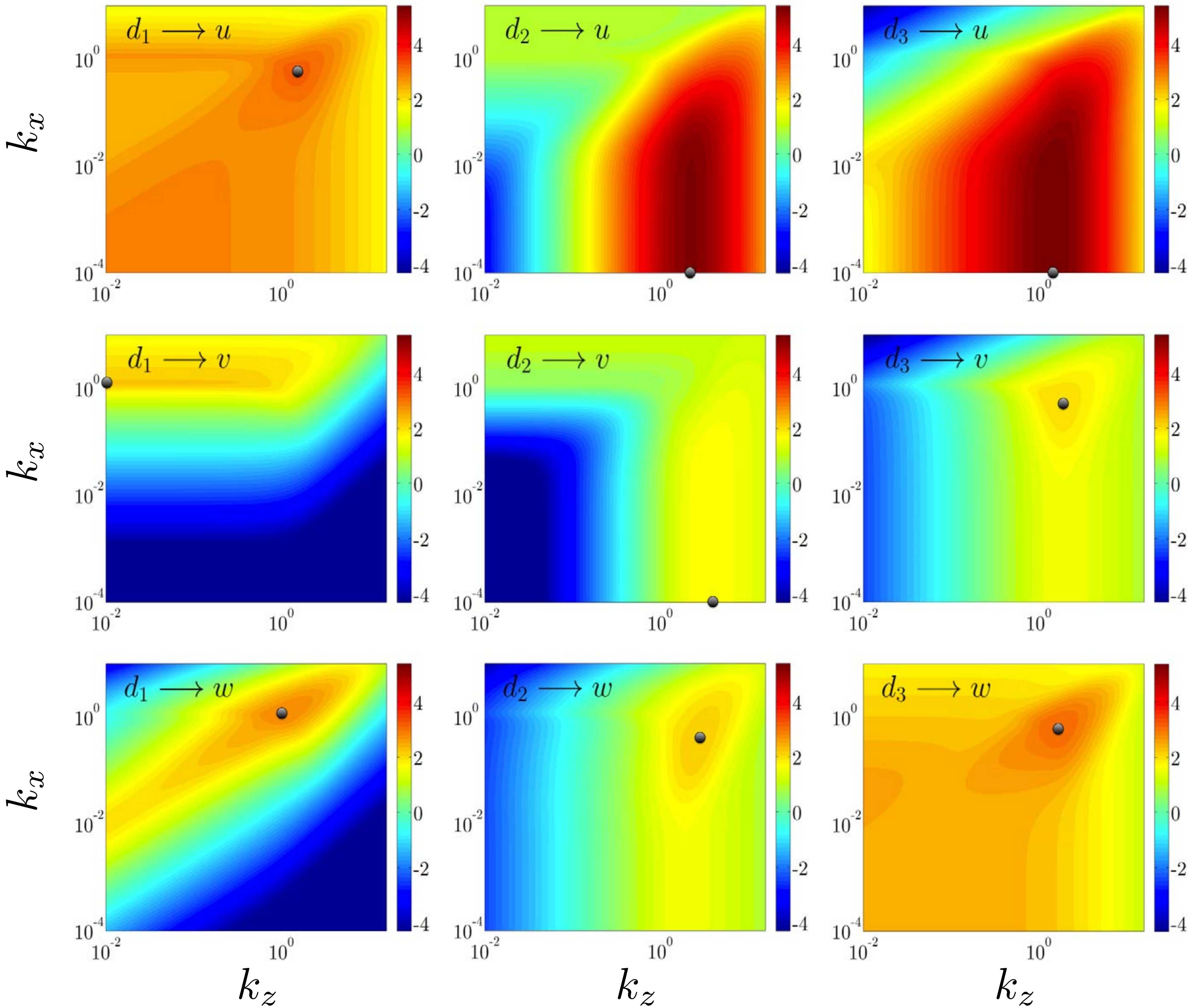
- **Frequency response: operator in y**



⋆ **componentwise amplification**

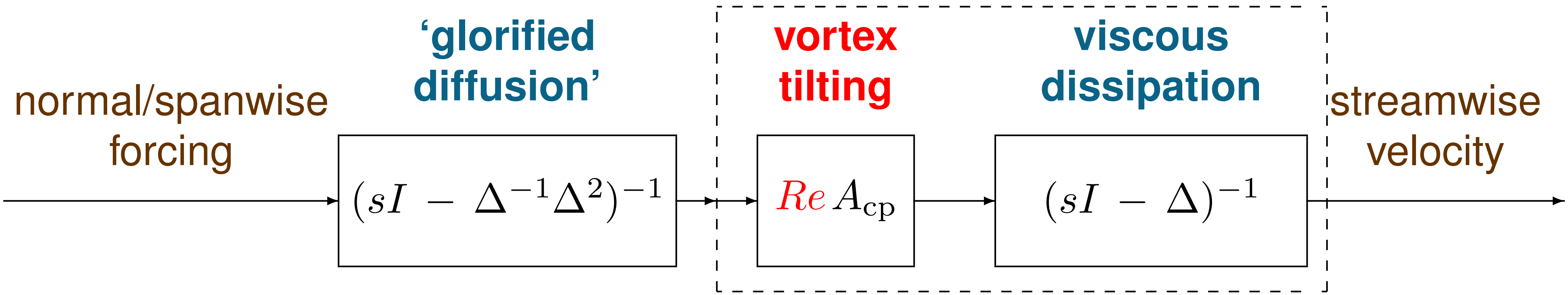
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} H_{u1} & H_{u2} & H_{u3} \\ H_{v1} & H_{v2} & H_{v3} \\ H_{w1} & H_{w2} & H_{w3} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Jovanović & Bamieh, J. Fluid Mech. '05


$d_1 \longrightarrow u$
$d_2 \longrightarrow u$
$d_3 \longrightarrow u$
$d_1 \longrightarrow v$
$d_2 \longrightarrow v$
$d_3 \longrightarrow v$
$d_1 \longrightarrow w$
$d_2 \longrightarrow w$
$d_3 \longrightarrow w$
k_x
k_z
10^{0}
10^{-2}
10^{-4}
4
2
0
-2
-4

Amplification mechanism in flows with high Re

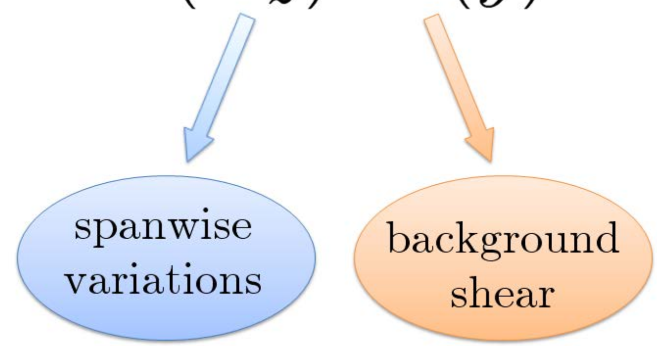
- HIGHEST AMPLIFICATION: $(d_2, d_3) \to u$



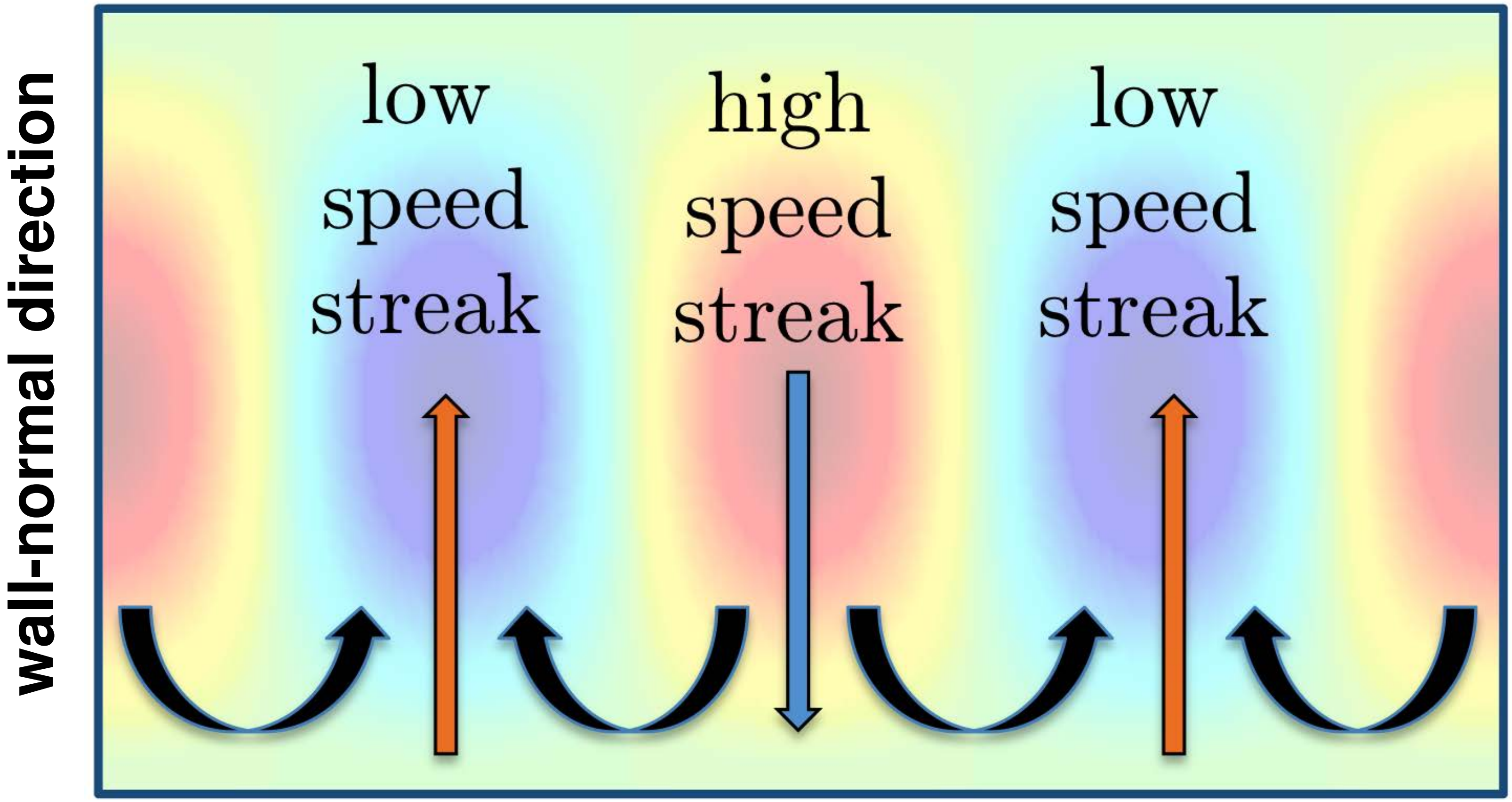
- LINEARIZED DYNAMICS OF NORMAL VORTICITY η

$$\dot{\eta} = \Delta\eta + Re\, A_{cp}\, v \qquad A_{cp} = -(\mathrm{i}k_z)\, U'(y)$$

$Re\, A_{cp}\, v$ ↓ **source**



☞ AMPLIFICATION MECHANISM: **vortex tilting** or **lift-up**



spanwise direction

Linear analyses: Input-output vs. Stability

AMPLIFICATION:

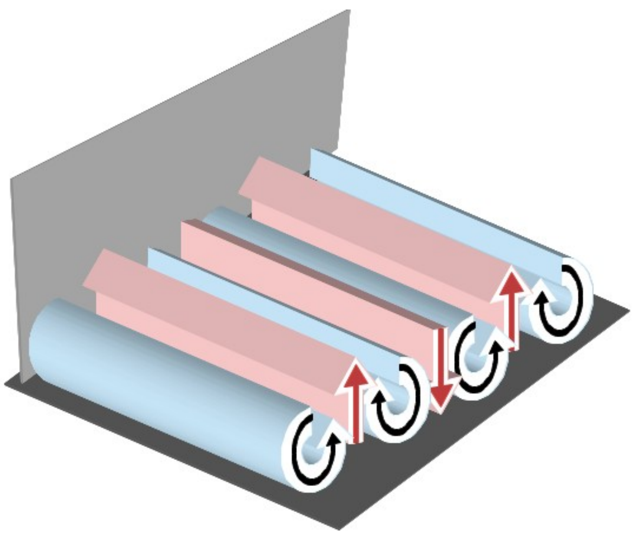
$$\mathbf{v} = H\,\mathbf{d}$$

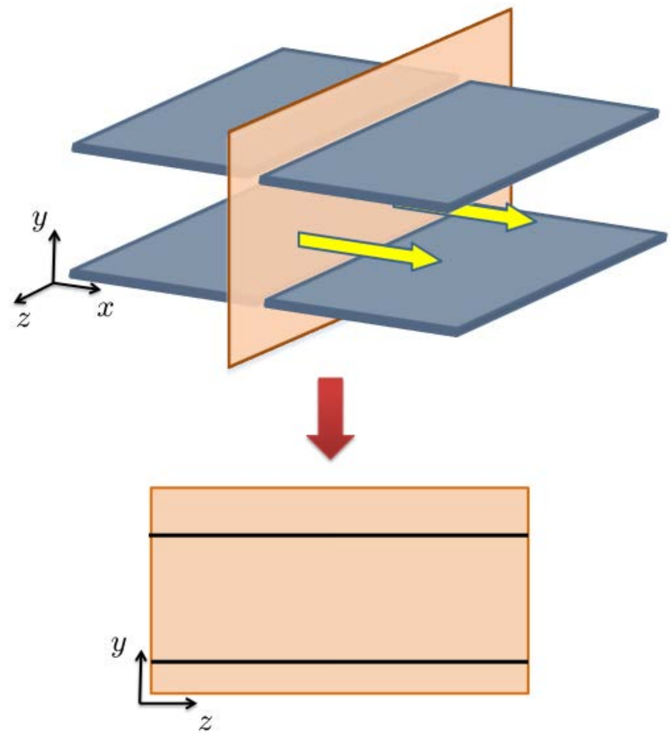
singular values of H

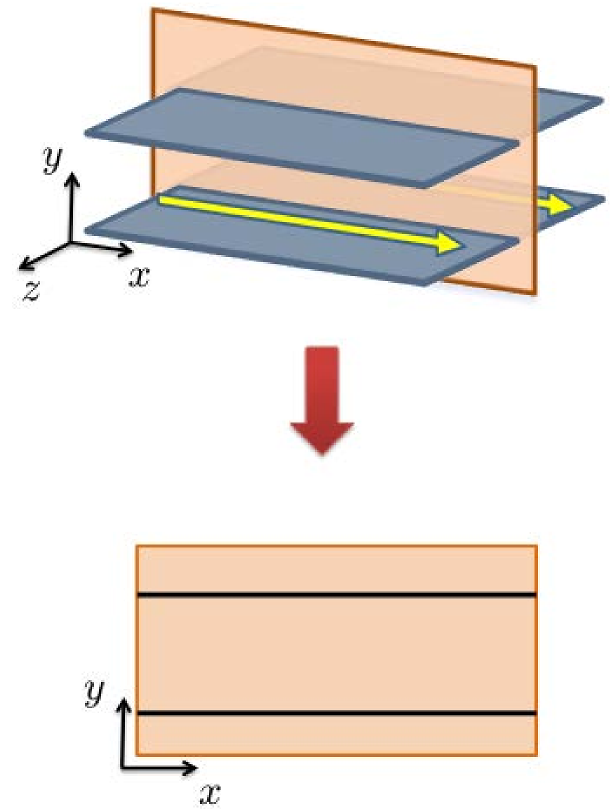
STABILITY:

$$\dot{\psi} = A\,\psi$$

e-values of A







typical structures **cross-sectional dynamics** **2D models**

FLOW CONTROL

- **Objective**
  - ⋆ **controlling the onset of turbulence**
- **Transition initiated by**
  - ⋆ **high flow sensitivity**
- **Control strategy**
  - ⋆ **reduce flow sensitivity**

Sensor-free flow control

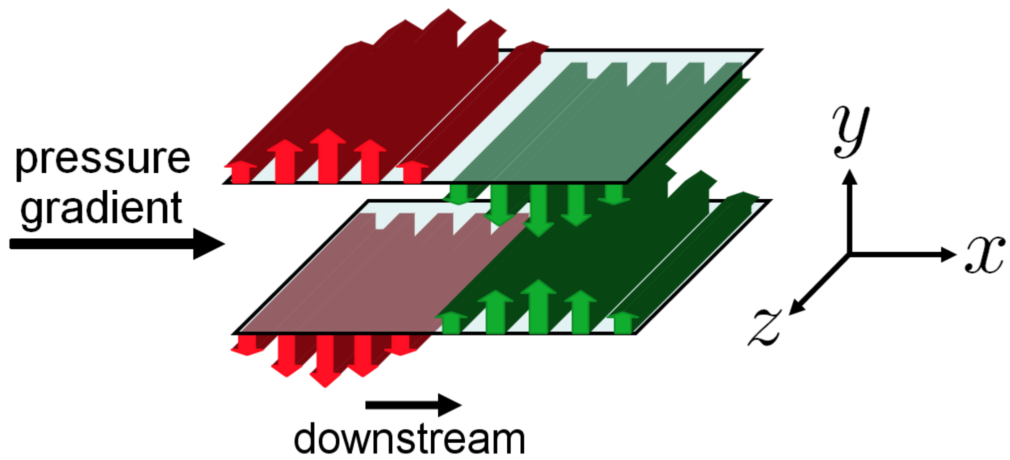
- **Geometry modifications**
  - ⋆ riblets
  - ⋆ surface roughness
  - ⋆ super-hydrophobic surfaces

- **Body forces**
  - ⋆ temporally/spatially oscillatory forces
  - ⋆ traveling waves

- **Wall oscillations**
  - ⋆ transverse wall oscillations

common theme: PDEs with spatially or temporally periodic coefficients

Blowing and suction along the walls



NORMAL VELOCITY: $V(y = \pm 1) \;=\; \mp\, \alpha \cos\left(\omega_x (x \,-\, ct)\right)$

- TRAVELING WAVE PARAMETERS:

spatial frequency: ω_x

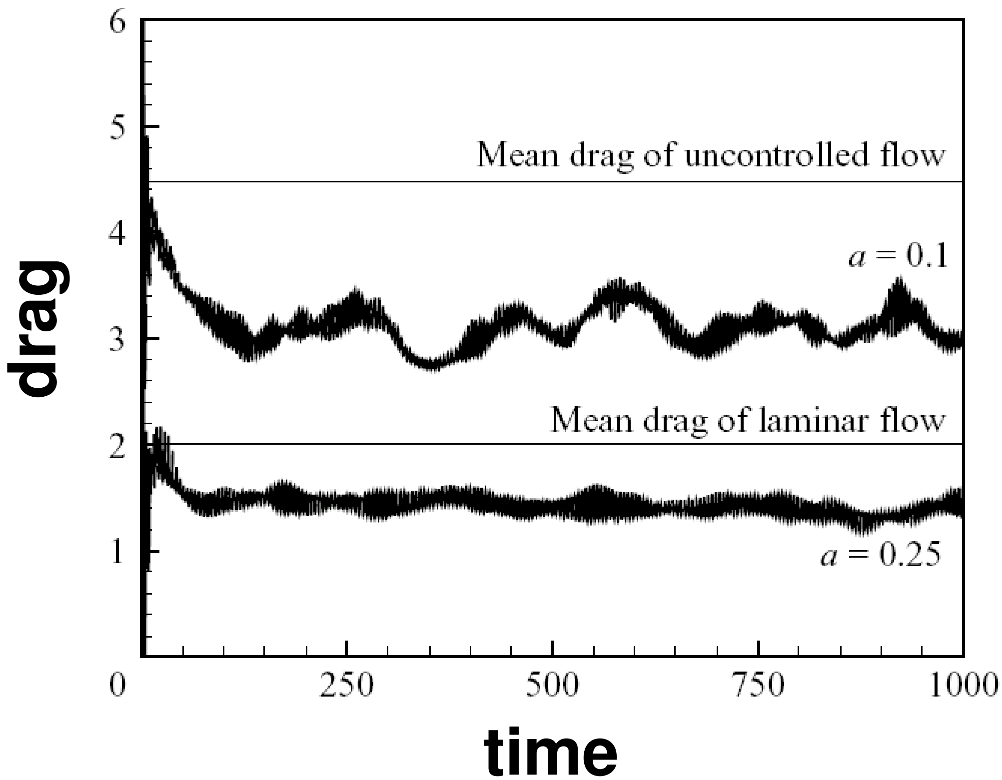
speed: c $\quad \begin{cases} c > 0 & \text{downstream} \\ c < 0 & \text{upstream} \end{cases}$

amplitude: α

- INVESTIGATE THE EFFECTS OF c, ω_x, α ON:
  - ⋆ **base flow**
  - ⋆ **cost of control**
  - ⋆ **onset of turbulence**

Min, Kang, Speyer, Kim, J. Fluid Mech. '06

upstream ⇒ SUSTAINED SUB-LAMINAR DRAG



CHALLENGE: **selection of wave parameters**

- THIS TALK:
  - ⋆ **cost of control**
  - ⋆ **onset of turbulence**

Effects of blowing and suction?

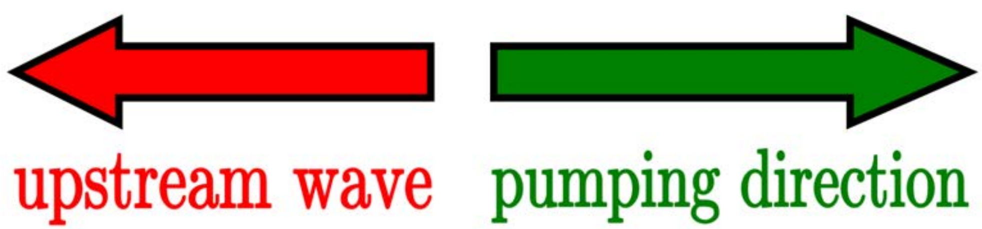
- DESIRED EFFECTS OF CONTROL:
  - ⋆ **bulk flux** $\nearrow$
  - ⋆ **net efficiency** $\nearrow$
  - ⋆ **fluctuations' energy** $\searrow$

TRAVELING WAVE

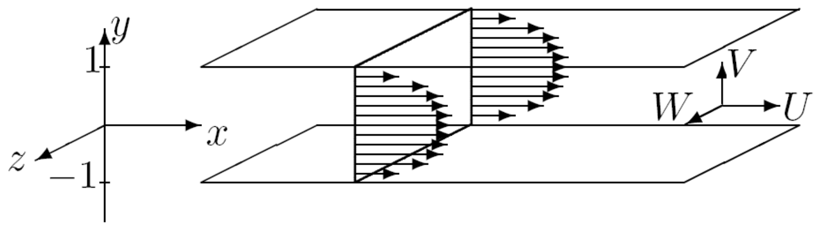
- ⋆ **induces a bulk flux (pumping)**

PUMPING DIRECTION

- ⋆ **opposite to a traveling wave direction**



Nominal velocity



$$V(y=\pm 1) = \mp\alpha \cos(\omega_x(x - ct)) = \mp\alpha \cos(\omega_x \bar{x})$$

$\Rightarrow$ $\bar{\mathbf{u}} = (U(\bar{x}, y),\ V(\bar{x}, y),\ 0)$

steady in a traveling wave frame

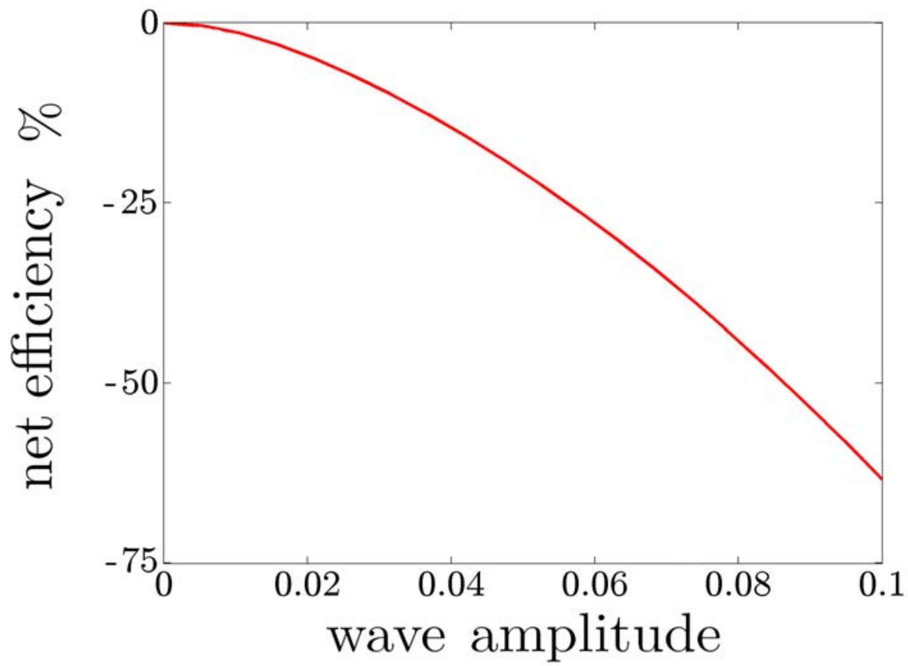
periodic in $\bar{x}$

- SMALL AMPLITUDE BLOWING/SUCTION
  weakly-nonlinear analysis

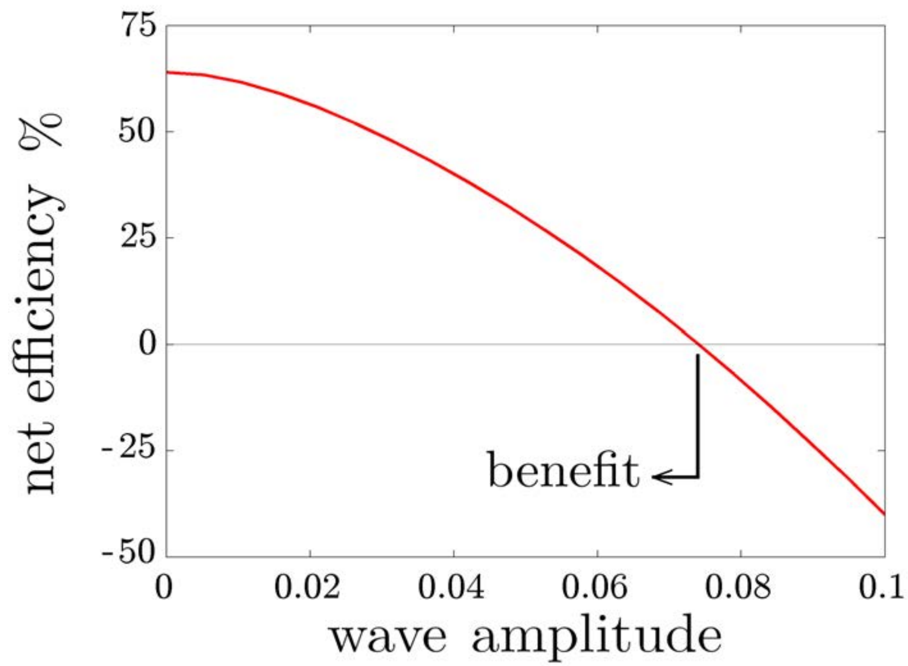
$$U(\bar{x}, y) = \overbrace{U_0(y)}^{\text{parabola}} + \alpha^2 \overbrace{U_{2,0}(y)}^{\text{mean drift}} + \alpha \overbrace{\left(U_{1,-1}(y)\, e^{-i\omega_x \bar{x}} + U_{1,1}(y)\, e^{i\omega_x \bar{x}}\right)}^{\text{oscillatory: no mean drift}}$$
$$+ \alpha^2 \left(U_{2,-2}(y)\, e^{-2i\omega_x \bar{x}} + U_{2,2}(y)\, e^{2i\omega_x \bar{x}}\right)$$
$$+ O(\alpha^3)$$

Best-case scenario for net efficiency

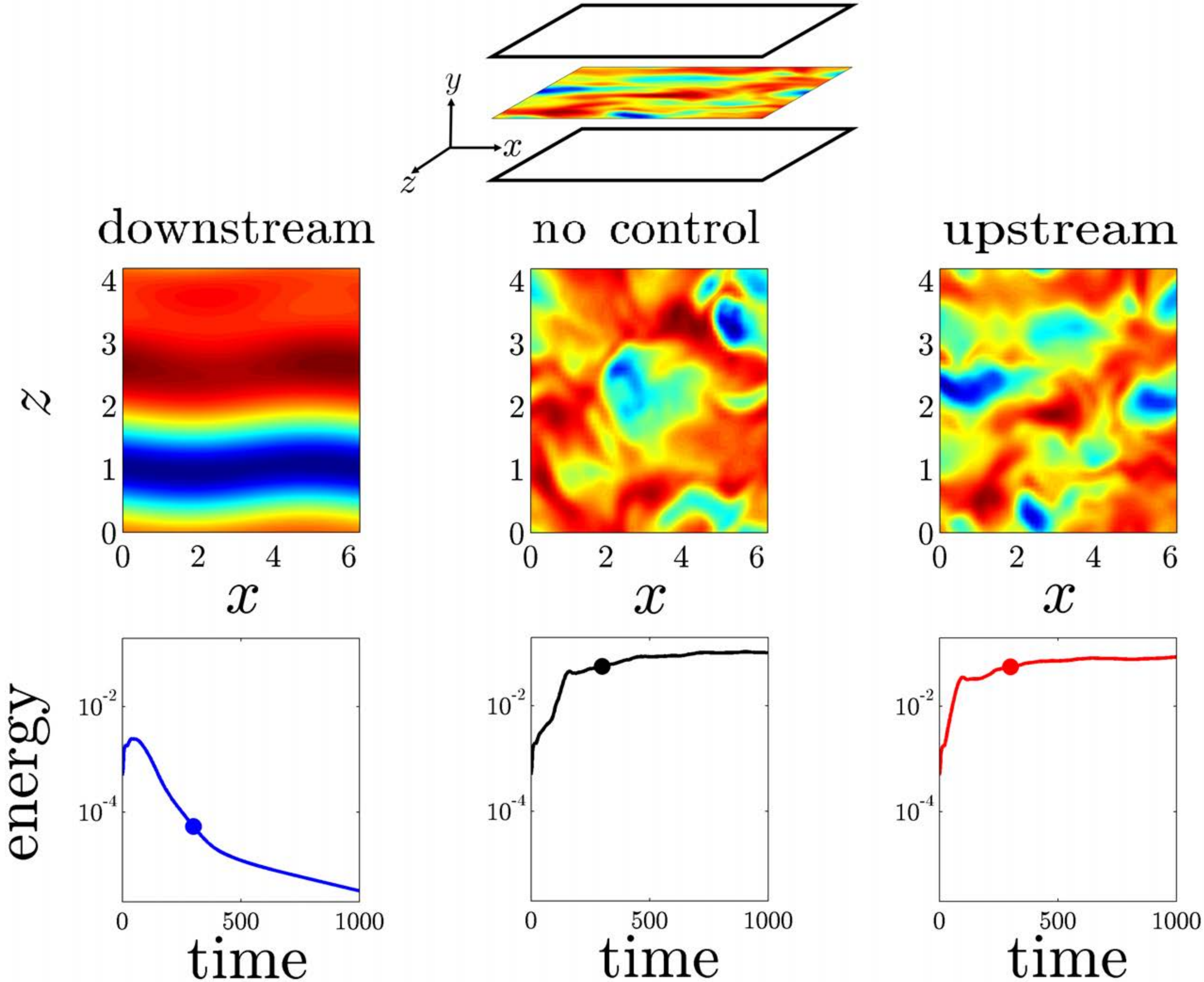
ASSUME:
- no control: laminar
- with control: laminar



ASSUME:
- no control: turbulent
- with control: laminar

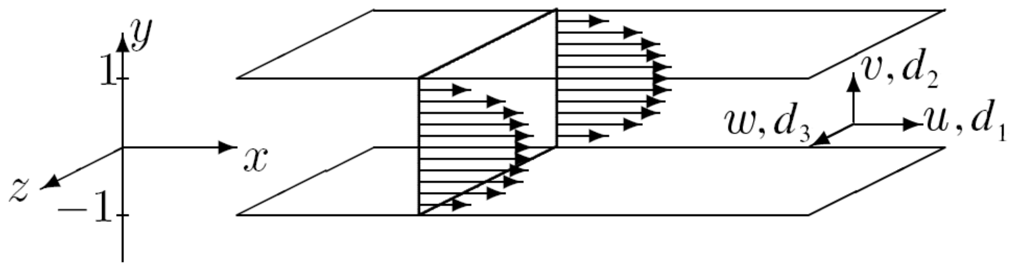


Velocity fluctuations: DNS preview



Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Ensemble average energy density: controlled flow



EVOLUTION MODEL: **linearization around** $(U(\bar{x}, y),\ V(\bar{x}, y),\ 0)$

⋆ **periodic coefficients in** $\bar{x} = x - ct$

$$\left.\begin{aligned} \boldsymbol{\psi}_t &= A\,\boldsymbol{\psi} + B\,\mathbf{d} \\ \mathbf{v} &= C\,\boldsymbol{\psi} \end{aligned}\right\}$$

$$\begin{aligned} \mathbf{d} &= \mathbf{d}(\bar{x}, y, z, t) & \rightsquigarrow &\quad \text{stochastic body forcing} \\ \mathbf{v} &= (u,\ v,\ w) & \rightsquigarrow &\quad \text{velocity fluctuations} \\ \boldsymbol{\psi} &= (v,\ \eta) & \rightsquigarrow &\quad \text{normal velocity/vorticity} \end{aligned}$$

- **Simulation-free approach to determining energy density**

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effect of small wave amplitude:

$$\underbrace{\text{energy density}}_{\text{with control}} = \underbrace{\text{energy density}}_{\text{w/o control}} + \underbrace{\alpha^2}_{\text{small}} E_2(\theta, k_z; Re; \omega_x, c) + O(\alpha^4)$$

$(\theta,\, k_z) \rightsquigarrow$ spatial wavenumbers

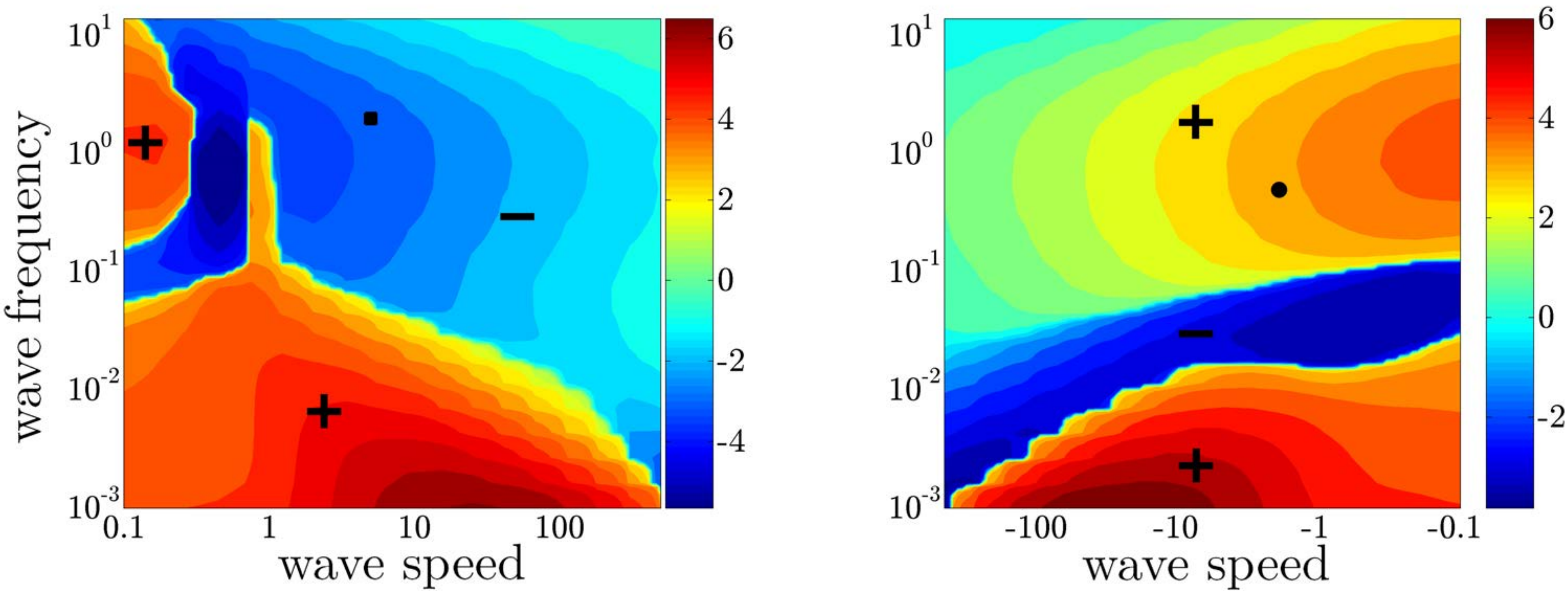
Energy amplification: controlled flow with $Re = 2000$

explicit formula:

$$\frac{\text{energy density with control}}{\text{energy density w/o control}} \approx 1 + \alpha^2 g_2(\theta, k_z; \omega_x, c)$$

- $(\theta = 0,\ k_z = 1.78)$: **most energy w/o control**

g_2, downstream: g_2, upstream:



Recap

facts revealed by perturbation analysis:

Blowing/Suction Type	Nominal flow analysis	Energy amplification analysis
Downstream	reduce bulk flux	reduce amplification ✓
Upstream	increase bulk flux ✓	promote amplification

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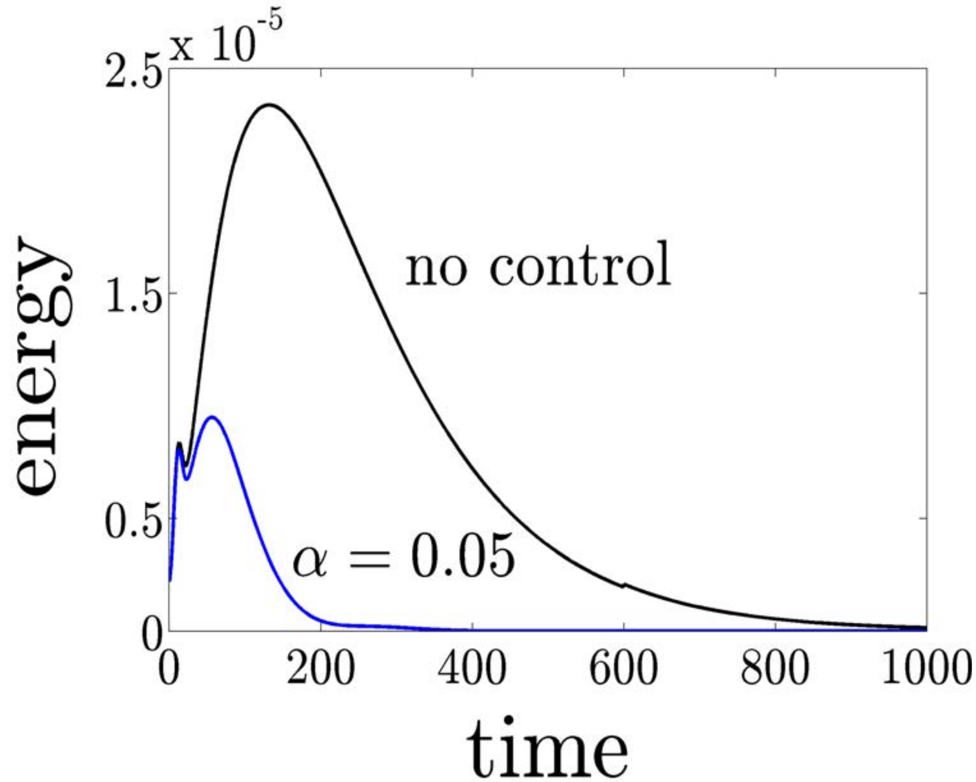
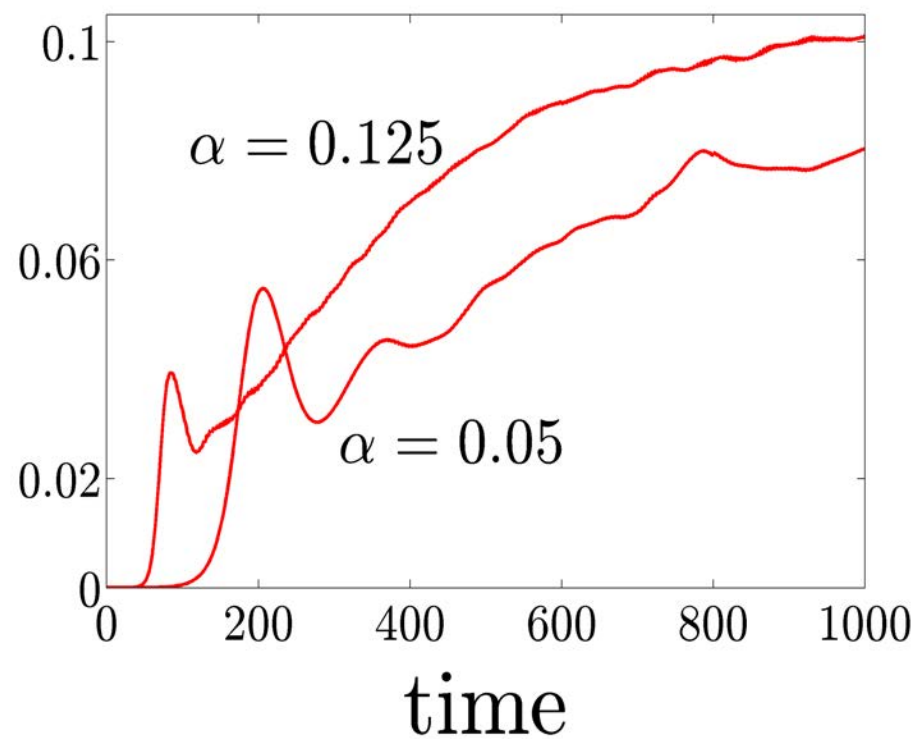
DNS results: avoidance/promotion of turbulence

small initial energy
(flow with no control stays laminar)

DOWNSTREAM: NO TURBULENCE



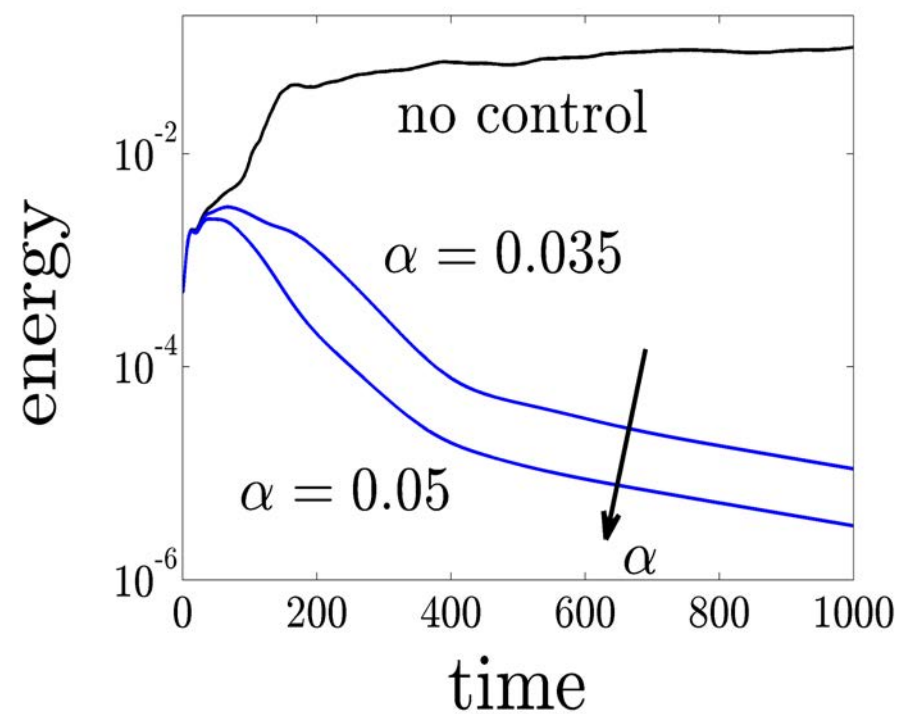
UPSTREAM: PROMOTES TURBULENCE

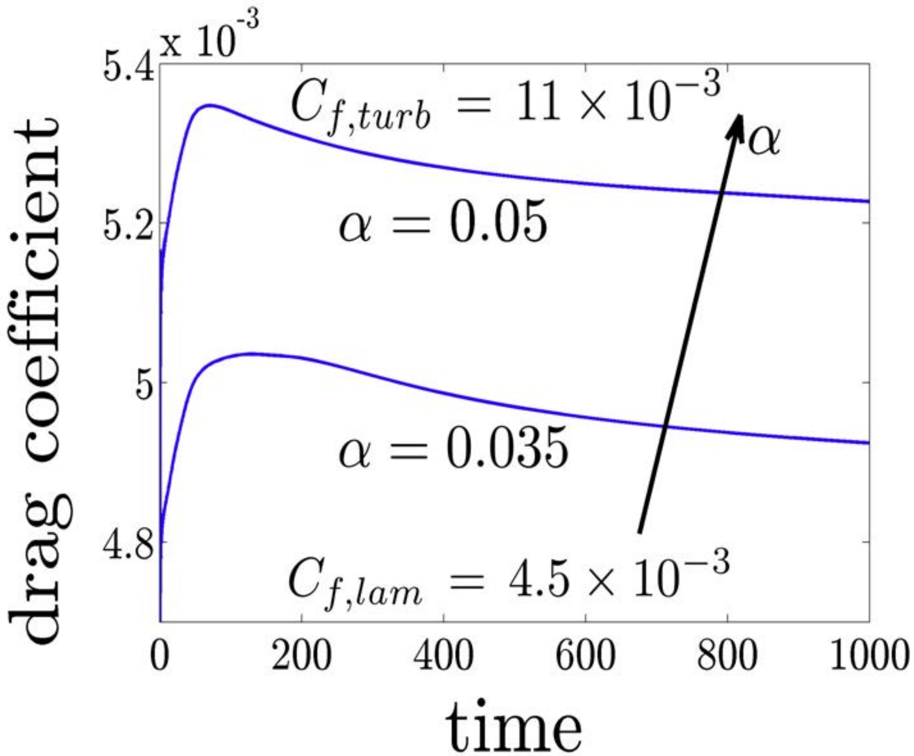


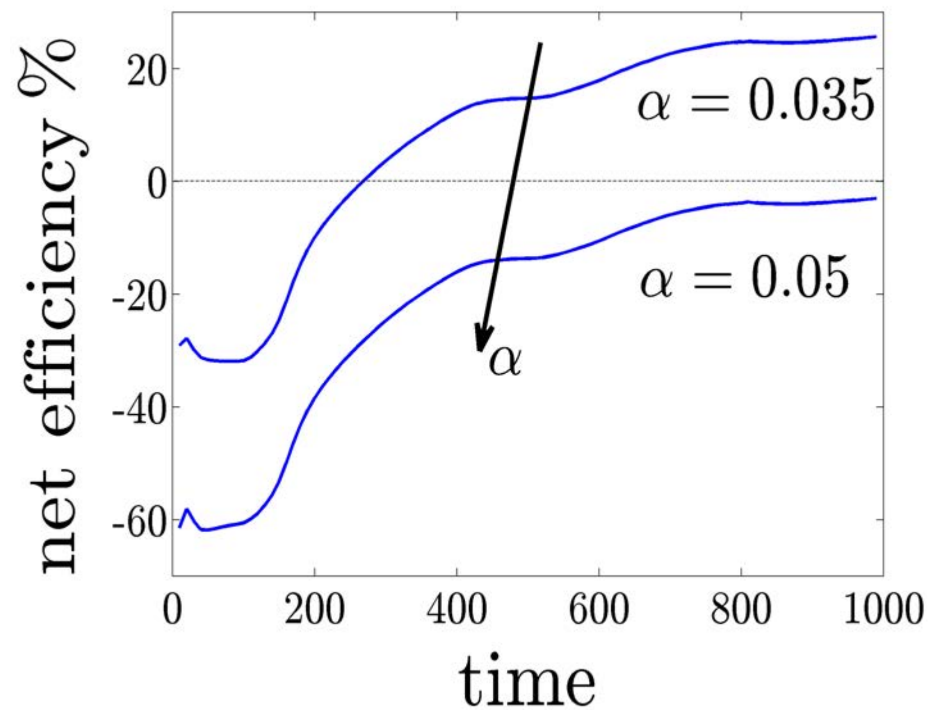
NO TURBULENCE:

DOWNSTREAM

moderate initial energy



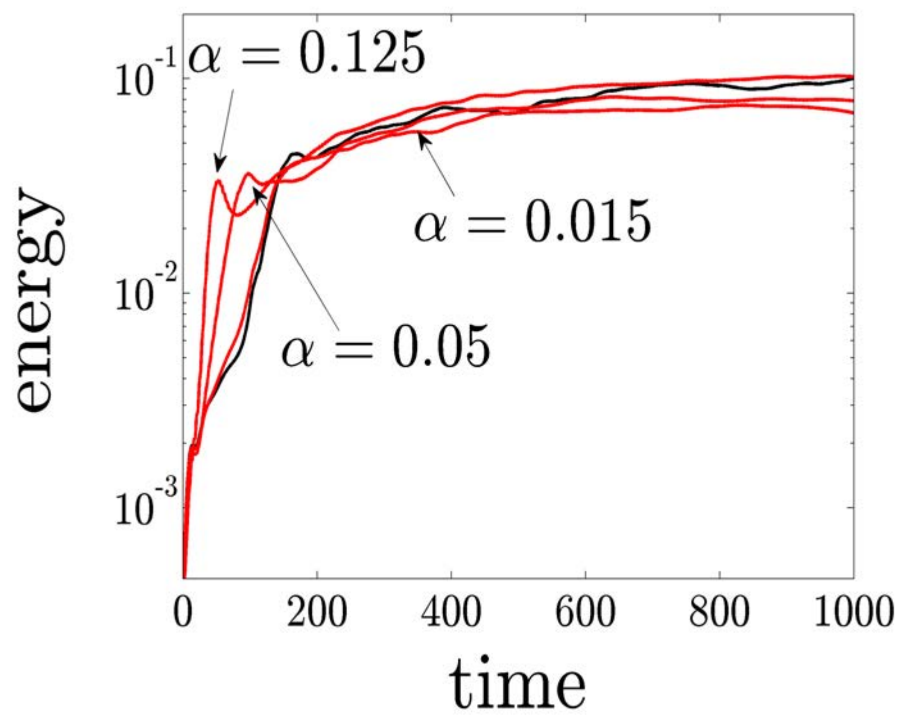


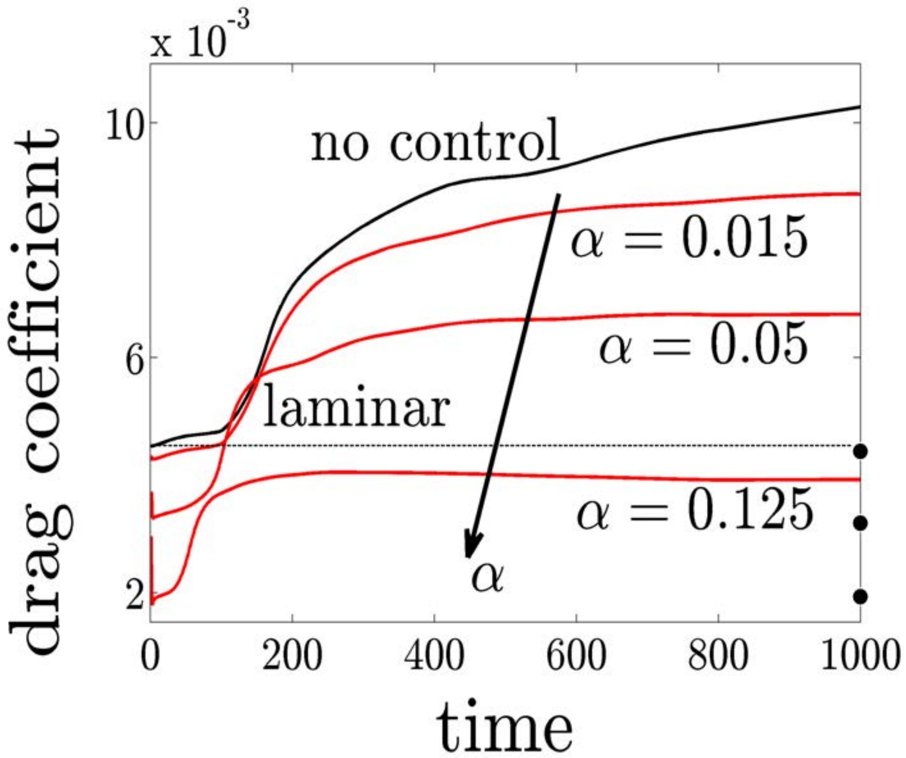


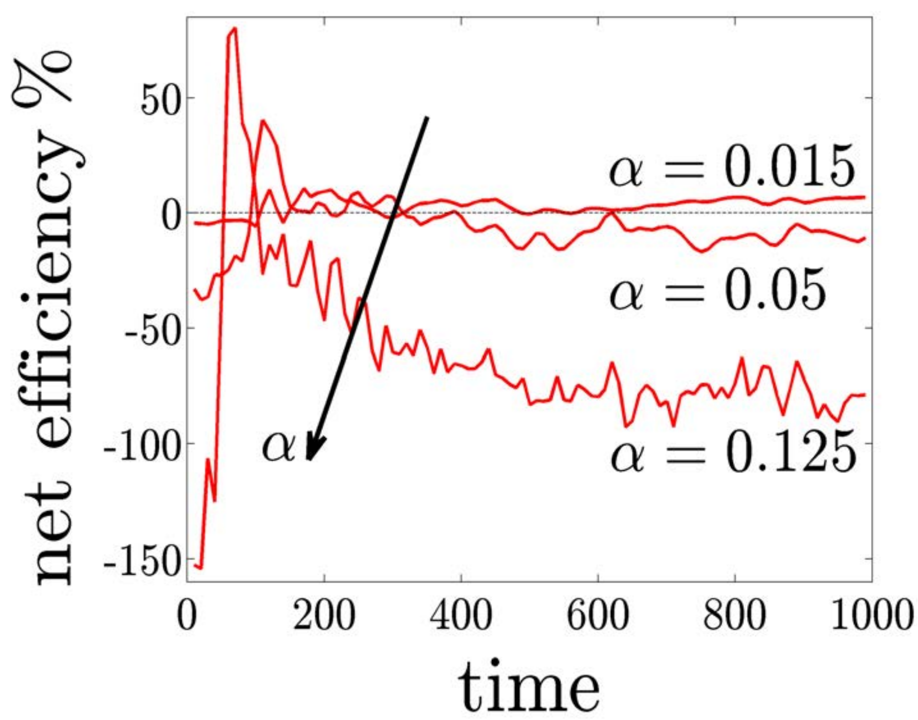
TURBULENCE:

UPSTREAM

moderate initial energy







Optimal control and estimation

Linear Quadratic Regulator (LQR)

- **Minimize quadratic objective subject to linear dynamic constraint**

$$\text{minimize} \quad J(\psi, u) = \frac{1}{2}\int_0^T \left(\langle \psi(\tau), Q\,\psi(\tau)\rangle + \langle u(\tau), R\,u(\tau)\rangle \right) \mathrm{d}\tau \;+\; \frac{1}{2}\langle \psi(T), Q_T\,\psi(T)\rangle$$

$$\text{subject to} \quad A\,\psi(t) \;+\; B\,u(t) \;-\; \dot{\psi}(t) \;=\; 0$$

$$\psi(0) \;=\; \psi_0, \quad t \;\in\; [0,\, T]$$

⋆ **optimization variable is a function**

$$u\colon\; [0,\, T] \;\longrightarrow\; \mathbb{H}_{\mathrm{u}}$$

⋆ **state and control weights**

$$\begin{cases} Q,\; Q_T & \text{self-adjoint, non-negative} \\ R & \text{self-adjoint, positive} \end{cases}$$

⋆ **infinite number of constraints**

• Introduce Lagrangian

$$\mathcal{L}(\psi, u, \lambda) \;=\; J(\psi, u) \;+\; \int_0^T \Big\langle \lambda(\tau), A\,\psi(\tau) \;+\; B\,u(\tau) \;-\; \dot{\psi}(\tau) \Big\rangle \, \mathrm{d}\tau$$

⋆ form variations wrt ψ, u, λ

$$\mathcal{L}(\psi, u + \tilde{u}, \lambda) \;-\; \mathcal{L}(\psi, u, \lambda) \;=\; \int_0^T \langle R\,u(\tau) \;+\; B^*\lambda(\tau), \tilde{u}(\tau) \rangle \, \mathrm{d}\tau \;=\; 0$$

$$\downarrow$$

$$u(t) \;=\; -R^{-1} B^* \lambda(t), \quad t \;\in\; [0,\, T]$$

necessary conditions for optimality:

wrt λ $\Rightarrow$	$\dot{\psi}(t) \;=\; A\,\psi(t) \;+\; B\,u(t),$	$\psi(0) \;=\; \psi_0$
wrt ψ $\Rightarrow$	$\dot{\lambda}(t) \;=\; -Q\,\psi(t) \;-\; A^*\,\lambda(t),$	$\lambda(T) \;=\; Q_T\,\psi(T)$
wrt u $\Rightarrow$	$u(t) \;=\; -R^{-1} B^* \lambda(t),$	$t \;\in\; [0,\, T]$

Solution to finite horizon LQR

two-point boundary value problem:

$$
\begin{aligned}
\begin{bmatrix} \dot{\psi}(t) \\ \dot{\lambda}(t) \end{bmatrix} &= \begin{bmatrix} A & -B\,R^{-1}B^* \\ -Q & -A^* \end{bmatrix} \begin{bmatrix} \psi(t) \\ \lambda(t) \end{bmatrix} \\
\begin{bmatrix} \psi_0 \\ 0 \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi(0) \\ \lambda(0) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ Q_T & -I \end{bmatrix} \begin{bmatrix} \psi(T) \\ \lambda(T) \end{bmatrix}
\end{aligned}
$$

$$u(t) \;=\; -R^{-1}B^*\lambda(t)$$

- **Differential Riccati Equation**

can show: $\lambda(t) \;=\; X(t)\,\psi(t)$

$$
\begin{aligned}
-\dot{X}(t) &= A^*\,X(t) \;+\; X(t)\,A \;+\; Q \;-\; X(t)\,B\,R^{-1}B^*X(t) \\
X(T) &= Q_T
\end{aligned}
$$

⋆ **optimal controller: determined by state-feedback**

$$
\begin{aligned}
u(t) &= -K(t)\,\psi(t) \\
K(t) &= R^{-1}B^*X(t)
\end{aligned}
$$

Infinite horizon LQR

$$\text{minimize} \quad J \;=\; \frac{1}{2}\int_0^\infty \big(\langle \psi(\tau), Q\,\psi(\tau)\rangle \,+\, \langle u(\tau), R\,u(\tau)\rangle \big)\,\mathrm{d}\tau$$

$$\text{subject to} \quad \dot\psi(t) \;=\; A\,\psi(t) \,+\, B\,u(t)$$

- **Optimal controller:** $\left\{ \begin{array}{rcl} u(t) & = & -K\,\psi(t) \\ K & = & R^{-1}B^*X \end{array} \right.$

$\star$ $X = X^*$ – **non-negative solution to Algebraic Riccati Equation (ARE)**

$$A^*X \,+\, X\,A \,+\, Q \,-\, X\,B\,R^{-1}B^*X \;=\; 0$$

$$\left. \begin{array}{ll} (A,B) & \text{stabilizable} \\ (A,Q) & \text{detectable} \end{array} \right\} \;\Rightarrow\; \text{stability of} \;\; \dot\psi(t) \;=\; (A \,-\, B\,K)\,\psi(t)$$

Scalar example

$$\dot{\psi} \;=\; a\,\psi \,+\, u$$

$$J \;=\; \frac{1}{2}\int_0^\infty \big(q\,\psi^2(\tau) \,+\, r\,u^2(\tau)\big)\,\mathrm{d}\tau$$

- **Optimal controller**

$$k_{\mathrm{lqr}} \;=\; a \,+\, \sqrt{a^2 \,+\, \frac{q}{r}} \;\;\Rightarrow\;\; \psi(t) \;=\; \exp\left(-\sqrt{a^2 \,+\, \frac{q}{r}}\;t\right)\psi(0)$$

tradeoff:

	large q/r	small q/r
convergence rate	fast ✓	slow
control effort	large	low ✓

State-feedback H_2 controller

minimize $\quad \lim_{t \to \infty} \mathcal{E}\big(\langle \psi(t), Q\,\psi(t) \rangle \, + \, \langle u(t), R\,u(t) \rangle \big)$

subject to $\quad \dot{\psi}(t) \, = \, A\,\psi(t) \, + \, B_d\,d(t) \, + \, B_u\,u(t)$

$$\mathcal{E}\left(d(t_1)\,d^*(t_2)\right) \, = \, W_d\,\delta(t_1 \, - \, t_2)$$

- **Minimum variance controller**

state-feedback controller:

$$\begin{aligned} u(t) &= -K\,\psi(t) \\ K &= R^{-1}B_u^*\,X \\ 0 &= A^*\,X \, + \, X\,A \, + \, Q \, - \, X\,B_u\,R^{-1}B_u^*\,X \end{aligned}$$

State estimation

state equation: $\dot{\psi}(t) \;=\; A\,\psi(t) \;+\; B_d\,d(t) \;+\; B_u\,u(t)$

measured output: $\varphi(t) \;=\; C\,\psi(t) \;+\; n(t)$

$d(t)$ – process disturbance; $n(t)$ – measurement noise

- **Estimator (observer)**

  ⋆ **copy of the system** $+$ **linear injection term**

$$\dot{\hat{\psi}}(t) \;=\; A\,\hat{\psi}(t) \;+\; 0 \cdot d(t) \;+\; B_u\,u(t) \;+\; L\,(\varphi(t) \;-\; \hat{\varphi}(t))$$
$$\hat{\varphi}(t) \;=\; C\,\hat{\psi}(t) \;+\; 0 \cdot n(t)$$

  ⋆ **estimation error:** $\tilde{\psi}(t) \;=\; \psi(t) \;-\; \hat{\psi}(t)$

$$\dot{\tilde{\psi}}(t) \;=\; (A \;-\; L\,C)\,\tilde{\psi}(t) \;+\; \left[\begin{array}{cc} B_d & -L \end{array}\right] \left[\begin{array}{c} d(t) \\ n(t) \end{array}\right]$$
$$\tilde{\varphi}(t) \;=\; C\,\tilde{\psi}(t) \;+\; n(t)$$

(A, C): detectable $\Rightarrow$ can design L to provide stability of the error dynamics

Kalman filter

$$\dot{\psi}(t) \;=\; A\,\psi(t) \;+\; B_d\,d(t) \;+\; B_u\,u(t)$$
$$\varphi(t) \;=\; C\,\psi(t) \;+\; n(t)$$

$$\mathcal{E}\left(d(t_1)\,d^*(t_2)\right) \;=\; W_d\,\delta(t_1 \,-\, t_2); \quad \mathcal{E}\left(n(t_1)\,n^*(t_2)\right) \;=\; W_n\,\delta(t_1 \,-\, t_2)$$

- **Kalman filter: optimal estimator**
  - ⋆ **minimizes steady-state variance of** $\tilde{\psi}(t) \;=\; \psi(t) \;-\; \hat{\psi}(t)$

Kalman gain:

$$L \;=\; Y\,C^*\,W_n^{-1}$$
$$0 \;=\; A\,Y \;+\; Y\,A^* \;+\; B_d\,W_d\,B_d^* \;-\; Y\,C^*\,W_n^{-1}\,C\,Y$$

Output-feedback H_2 controller

$$\text{minimize} \quad \lim_{t \to \infty} \mathcal{E}\big(\langle \psi(t), Q\, \psi(t) \rangle \, + \, \langle u(t), R\, u(t) \rangle \big)$$

$$\text{subject to} \quad \begin{aligned} \dot{\psi}(t) \; &= \; A\, \psi(t) \, + \, B_d\, d(t) \, + \, B_u\, u(t) \\ \varphi(t) \; &= \; C\, \psi(t) \, + \, n(t) \end{aligned}$$

$$\mathcal{E}\left(d(t_1)\, d^*(t_2)\right) \; = \; W_d\, \delta(t_1 \, - \, t_2); \quad \mathcal{E}\left(n(t_1)\, n^*(t_2)\right) \; = \; W_n\, \delta(t_1 \, - \, t_2)$$

- **Minimum variance controller**

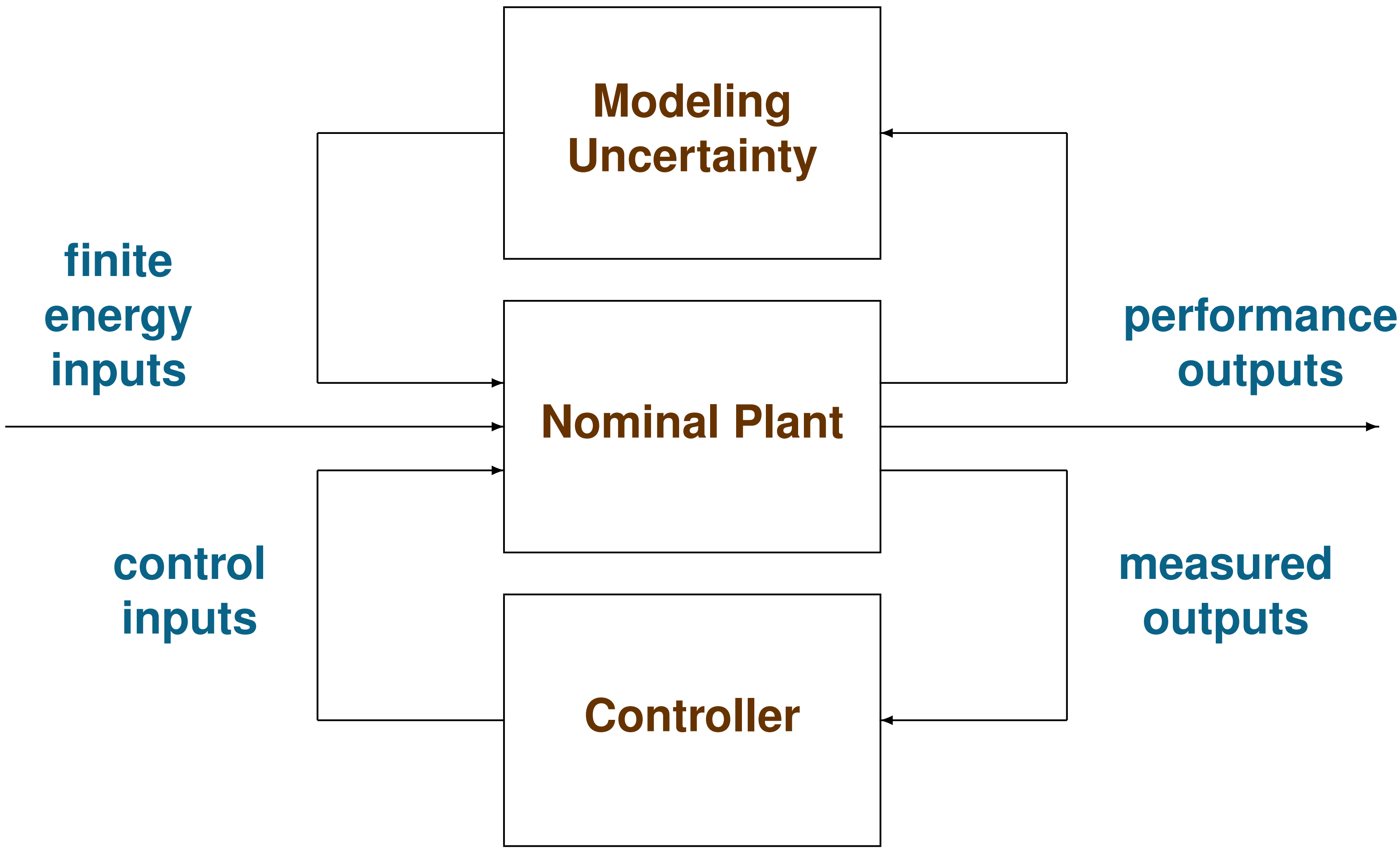
observer-based controller:

$$\begin{aligned} \dot{\hat{\psi}}(t) \; &= \; (A \, - \, L\, C)\, \hat{\psi}(t) \, + \, B_u\, u(t) \, + \, L\, \varphi(t) \\ u(t) \; &= \; -K\, \hat{\psi}(t) \end{aligned}$$

⋆ **feedback and observer gains:** $\begin{cases} K & \text{LQR gain} \\ L & \text{Kalman gain} \end{cases}$

H_∞ controller

- BLENDS CLASSICAL WITH OPTIMAL CONTROL



Boundary actuation

- Example: heat equation

$$\begin{aligned} \phi_t(y,t) &= \phi_{yy}(y,t) + d(y,t) \\ \phi(-1,t) &= u(t) \\ \phi(+1,t) &= 0 \end{aligned}$$

- Problem: control doesn't enter additively into the equation

- Coordinate transformation

$$\psi(y,t) = \phi(y,t) - f(y)\, u(t)$$

  - ⋆ Choose $f(y)$ to obtain $\psi(\pm 1, t) = 0$
  - ⋆ Many possible choices

Conditions for selection of f:

$$\{f(-1) = 1,\ f(1) = 0\} \xrightarrow{\text{simple option}} f(y) = \frac{1 - y}{2}$$

- In new coordinates:

$$\begin{aligned} \phi_t(y,t) &= \phi_{yy}(y,t) + d(y,t) \\ \phi(-1,t) &= u(t) \\ \phi(+1,t) &= 0 \end{aligned}$$

$$\Big\downarrow \; \phi(y,t) = \psi(y,t) + f(y)\, u(t)$$

$$\begin{aligned} \psi_t(y,t) + f(y)\,\dot{u}(t) &= \psi_{yy}(y,t) + f''(y)\,u(t) + d(y,t) \\ \psi(\pm 1,t) &= 0 \end{aligned}$$

- New input: $v(t) = \dot{u}(t)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \psi(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_0 & f'' \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \psi(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} -f \\ I \end{bmatrix} v(t)$$

$$\phi(t) = \begin{bmatrix} I & f \end{bmatrix}\begin{bmatrix} \psi(t) \\ u(t) \end{bmatrix}$$

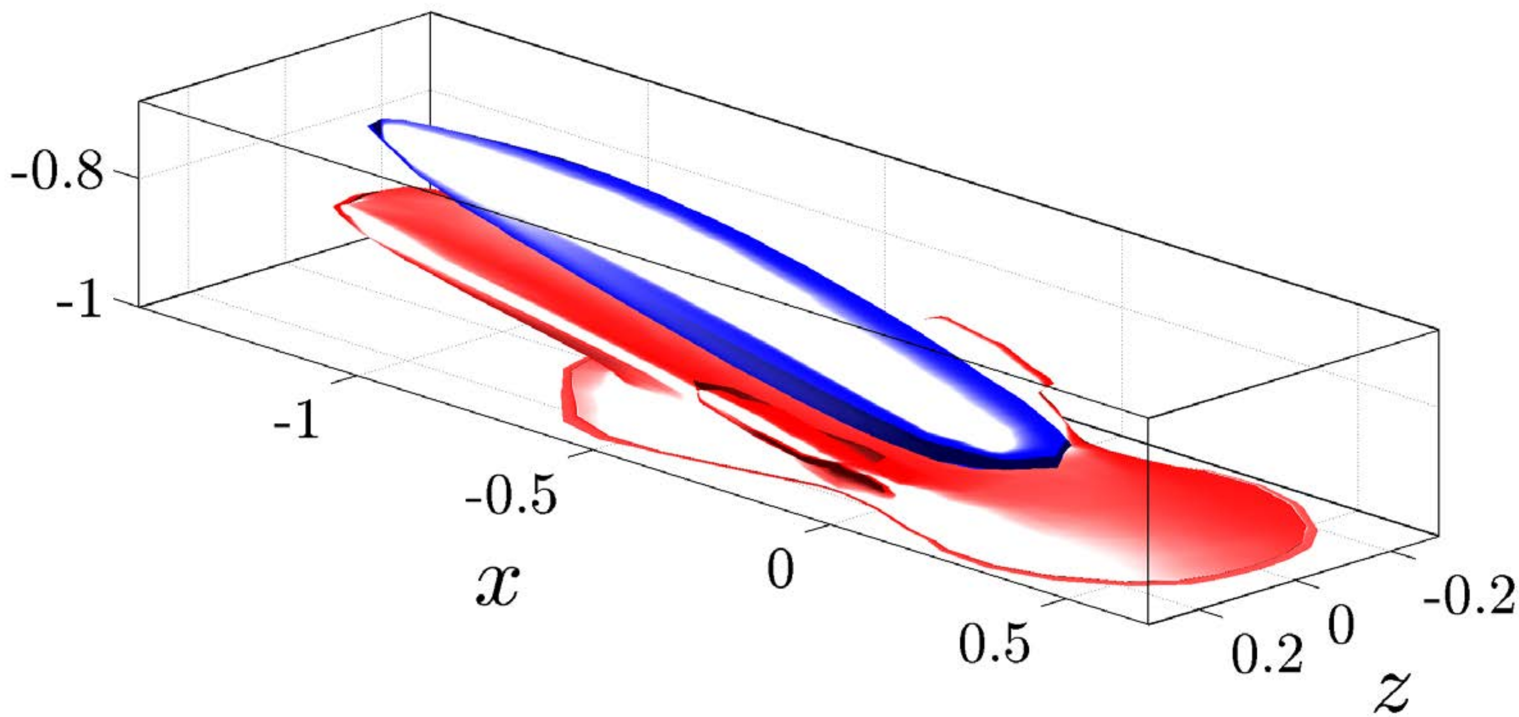
$$A_0 = \frac{\mathrm{d}^2}{\mathrm{d}y^2} \text{ with Dirichlet BCs}$$

Blowing and suction along the walls

$$v(x,\pm 1,z,t) \;=\; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-1}^{1} K_v^{\pm}(x-\xi,y,z-\zeta)\,v(\xi,y,\zeta,t)\,\mathrm{d}y\,\mathrm{d}\xi\,\mathrm{d}\zeta \;+$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-1}^{1} K_\eta^{\pm}(x-\xi,y,z-\zeta)\,\eta(\xi,y,\zeta,t)\,\mathrm{d}y\,\mathrm{d}\xi\,\mathrm{d}\zeta$$

- **Optimal controller: exponentially decaying convolution kernels**

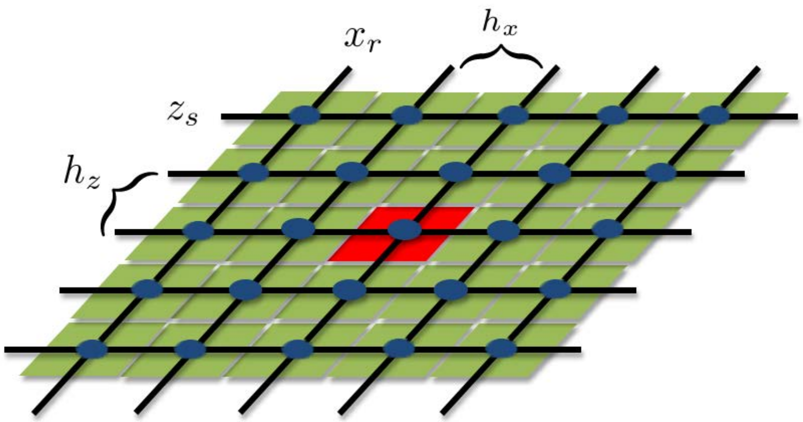
$K_v^{-}(0-\xi,y,0-\zeta)$:

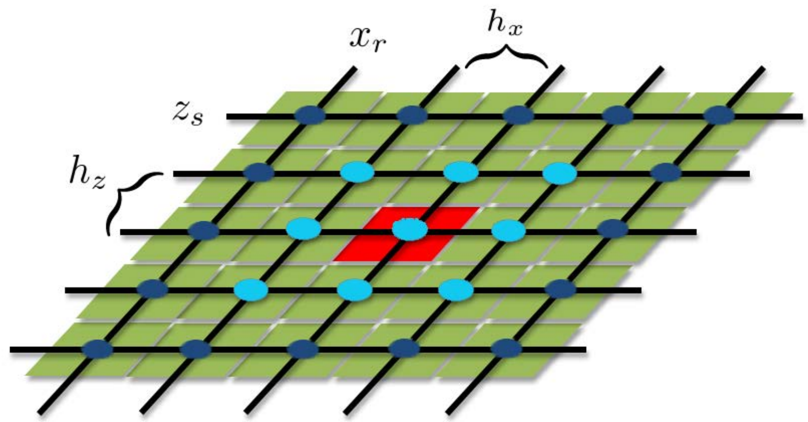


Högberg, Bewley, Henningson, J. Fluid Mech. '03

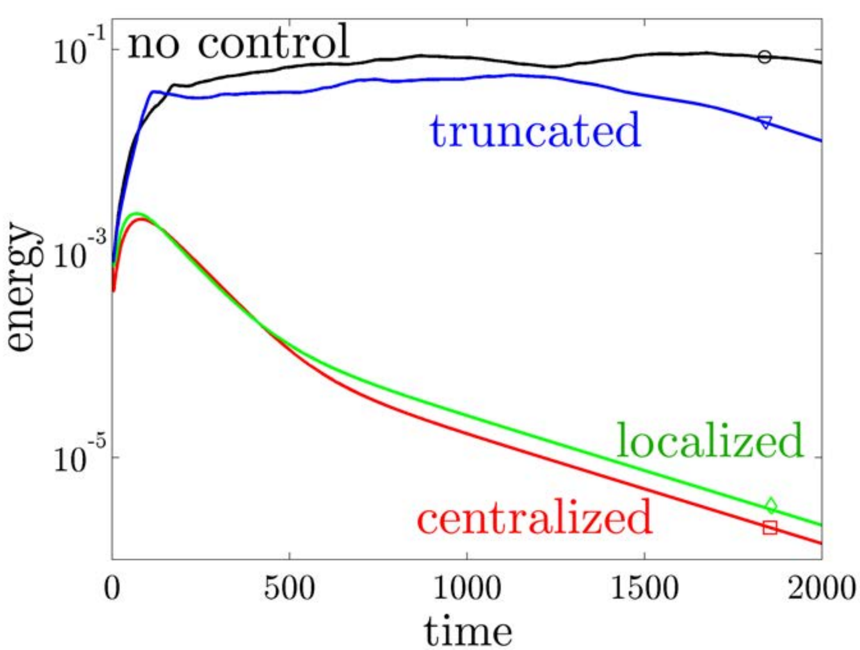
Optimal localized control

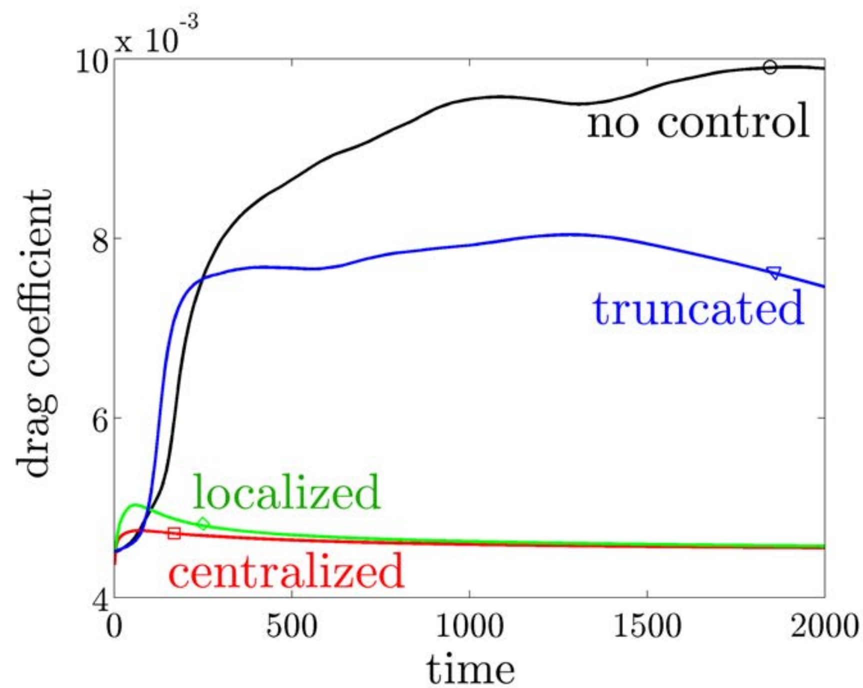
- **Blowing and suction along the discrete lattice**





⋆ **DNS verification**





Moarref, Lieu, Jovanović, CTR Summer Program 2010

Sparsity-promoting optimal control

- **Strike balance between quadratic performance and sparsity of K**

$$\text{minimize} \quad J(K) \quad + \quad \gamma \, \mathbf{card}\,(K)$$

$J(K)$ ↓ **variance amplification**

$\gamma \, \mathbf{card}\,(K)$ ↓ **sparsity-promoting penalty function**

- $\mathbf{card}\,(K)$ – number of non-zero elements of K

$$K = \begin{bmatrix} 5.1 & -2.3 & 0 & 1.5 \\ 0 & 3.2 & 1.6 & 0 \\ 0 & -4.3 & 1.8 & 5.2 \end{bmatrix} \quad \Rightarrow \quad \mathbf{card}\,(K) = 8$$

- $\gamma > 0$ – quadratic performance vs. sparsity tradeoff

Lin, Fardad, Jovanović, IEEE TAC '11 (conditionally accepted; arXiv:1111.6188v1)

SUMMARY AND OUTLOOK

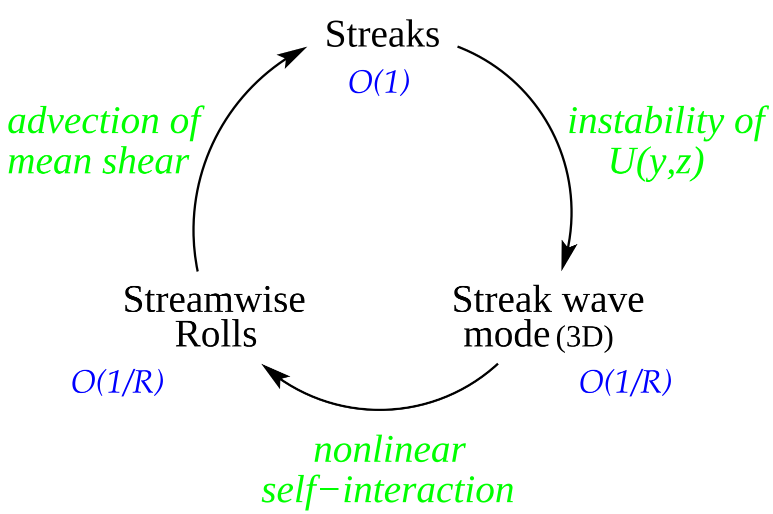
Summary: transition

- INPUT-OUTPUT ANALYSIS
  - ⋆ quantifies flow sensitivity
  - ⋆ reveals distinct mechanisms for subcritical transition

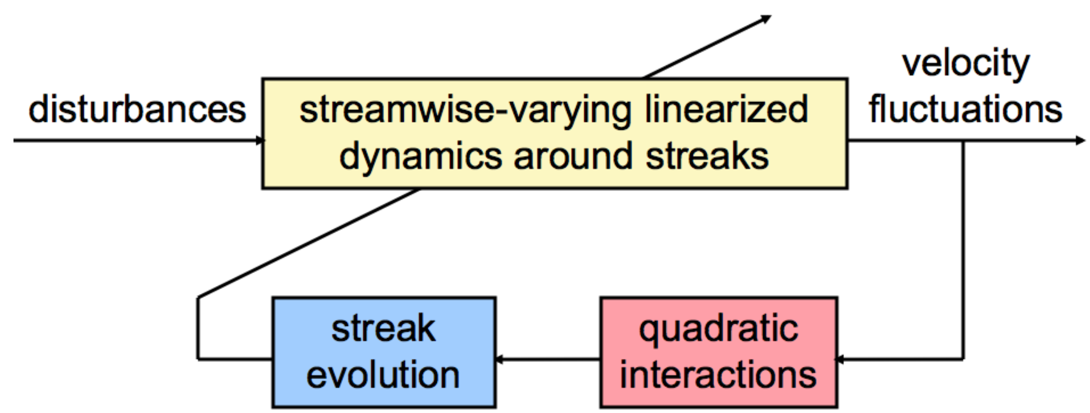
    streamwise streaks, oblique waves, TS-waves
  - ⋆ exemplifies the importance of streamwise elongated flow structures

Jovanović & Bamieh, J. Fluid Mech. '05

- LATER STAGES OF TRANSITION
  - ⋆ **challenge: relative roles of flow sensitivity and nonlinearity**



Waleffe, Phys. Fluids '97



Farrell & Ioannou, CTR Summer Program '12

Summary: sensor-free flow control

- CONTROLLING THE ONSET OF TURBULENCE

facts revealed by perturbation analysis:

Blowing/Suction Type	Nominal flow analysis	Energy amplification analysis
Downstream	reduce bulk flux	reduce amplification ✓
Upstream	increase bulk flux ✓	promote amplification

- POWERFUL SIMULATION-FREE APPROACH TO PREDICTING FULL-SCALE RESULTS
  - ⋆ DNS verification

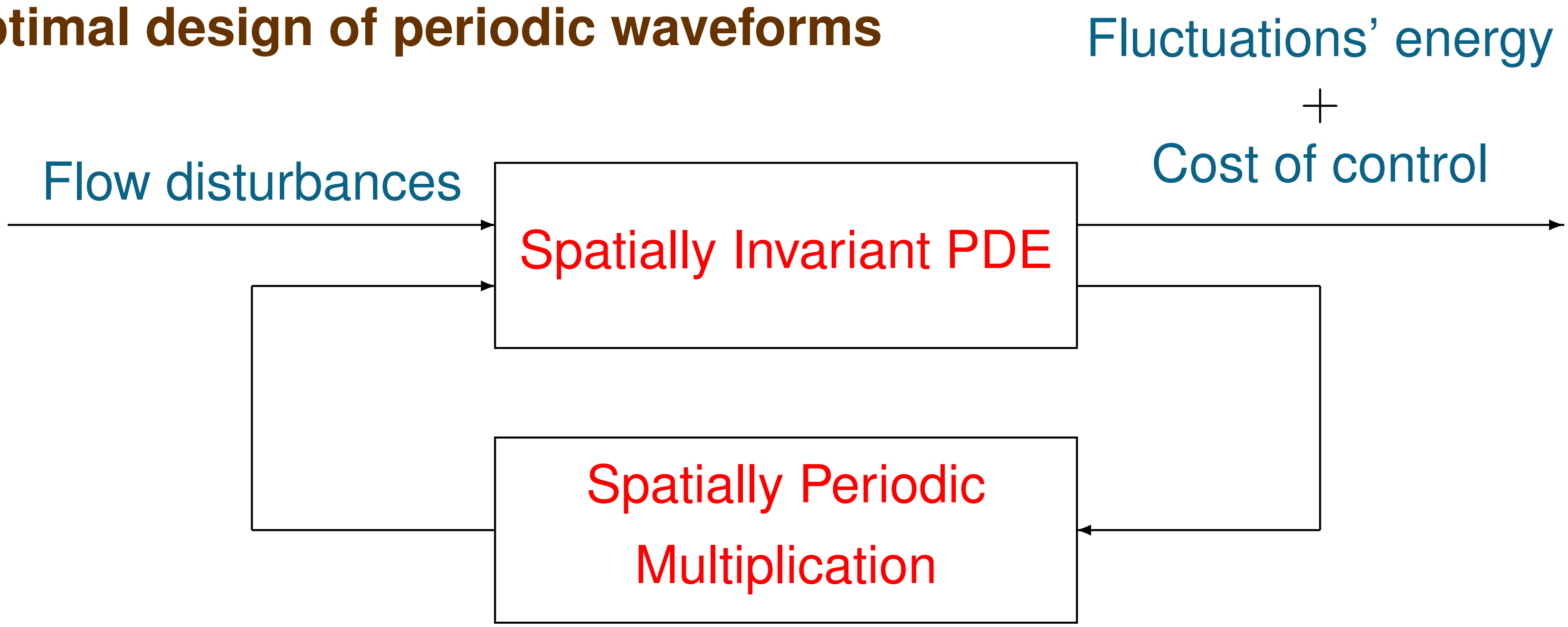
Moarref & Jovanović, J. Fluid Mech. ’10

Lieu, Moarref, Jovanović, J. Fluid Mech. ’10

Outlook: model-based sensor-free flow control

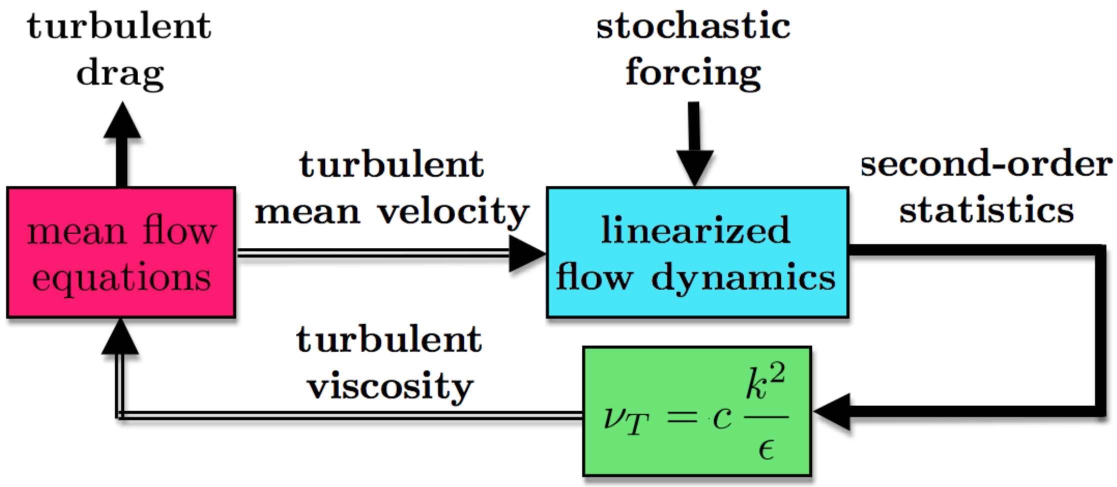
GEOMETRY MODIFICATIONS	WALL OSCILLATIONS	BODY FORCES
riblets super-hydrophobic surfaces	transverse oscillations	oscillatory forces traveling waves

- USE DEVELOPED THEORY TO DESIGN GEOMETRIES AND WAVEFORMS FOR
  - ⋆ control of transition/skin-friction drag reduction
- CHALLENGES
  - ⋆ **control-oriented modeling of turbulent flows**
  - ⋆ **optimal design of periodic waveforms**

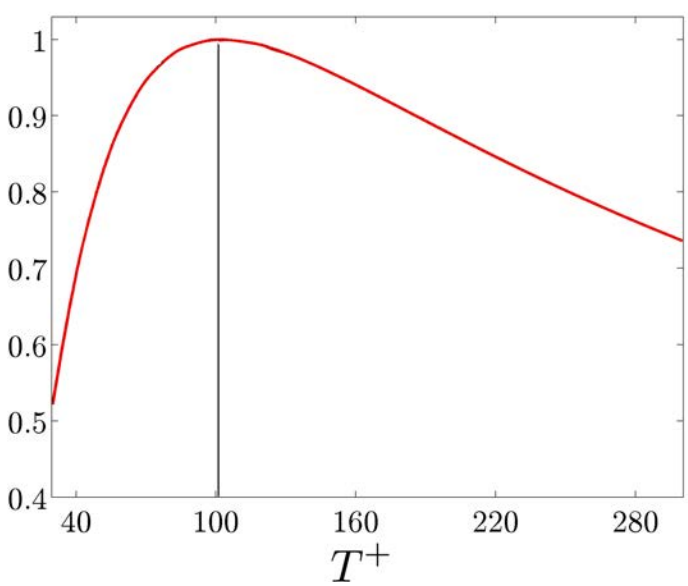


• CONTROL OF TURBULENT FLOWS

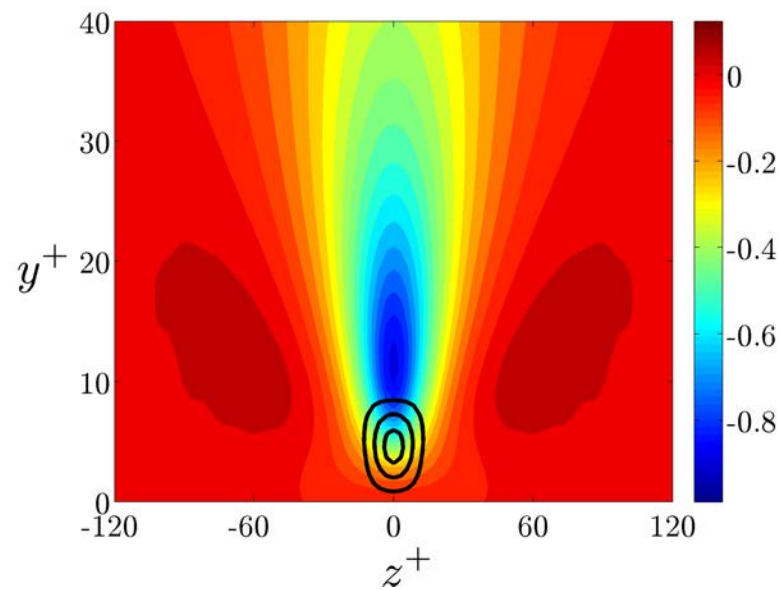
control-oriented modeling



model-based control design



flow structures



Moarref & Jovanović, J. Fluid Mech. '12 (in press; arXiv:1206.0101)

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