Control of vehicular platoons: limitations and tradeoffs

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ELECTRICAL AND COMPUTER ENGINEERING

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Control of vehicular platoons

- ACTIVE RESEARCH AREA FOR $\approx$ 40 YEARS
  (Levine & Athans, Melzer & Kuo, Chu, Ioannou, Varaiya, Hedrick, Swaroop, etc.)

- SPATIO-TEMPORAL SYSTEMS
  signals depend on time & discrete spatial variable n

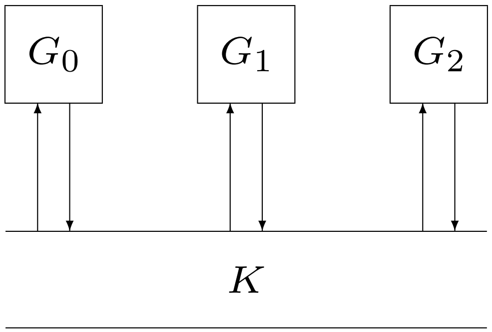
  arrays of micro-cantilevers
  arrays of micro-mirrors
  unmanned aerial vehicle formations
  satellite constellations

- INTERACTIONS CAUSE COMPLEX BEHAVIOR
  'string instability' in vehicular platoons

- SPECIAL STRUCTURE
  every unit has sensors and actuators

Controller architectures: platoons

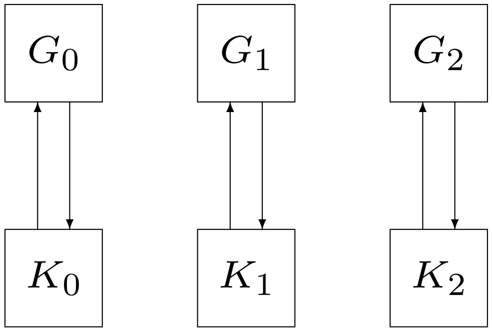
CENTRALIZED:



BEST PERFORMANCE

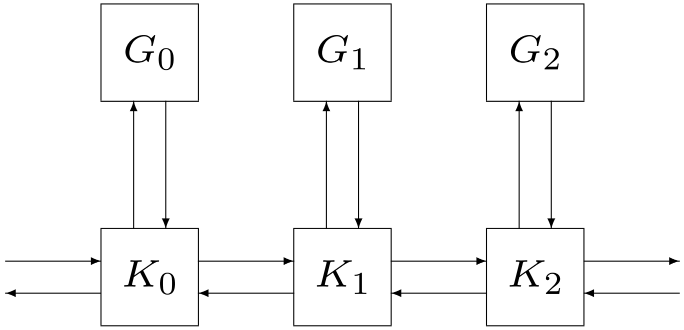
EXCESSIVE COMMUNICATION

FULLY DECENTRALIZED:



NOT SAFE!

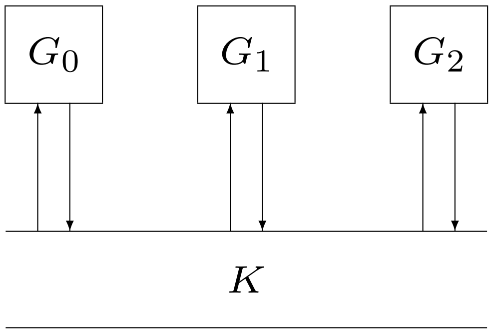
LOCALIZED:



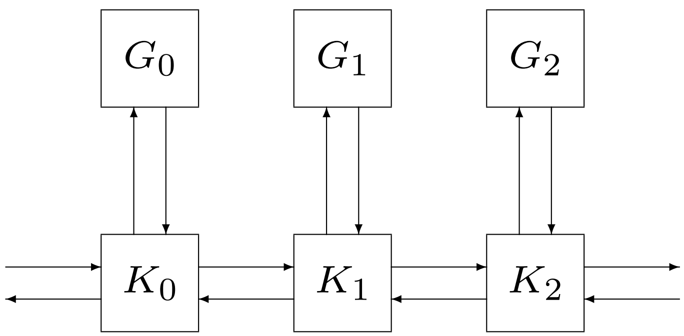
MANY POSSIBLE ARCHITECTURES

Issues

CENTRALIZED:



LOCALIZED:



PERFORMANCE VS. SIZE

FUNDAMENTAL LIMITATIONS/TRADEOFFS

IS IT ENOUGH TO LOOK ONLY
AT NEAREST NEIGHBORS?

Outline

❶ ILL-POSEDNESS OF SEVERAL WIDELY CITED RESULTS

- ⋆ small scale $\xrightarrow{\text{GAP}}$ large scale $\approx$ infinite

FORMULATION OF WELL-POSED CONTROL PROBLEMS

- ⋆ spatially invariant theory

❷ PEAKING IN VEHICULAR PLATOONS

$$\boxed{\begin{array}{c} \text{large platoons} \\ \text{uniform convergence rates} \end{array} \Rightarrow \text{large transient peaks}}$$

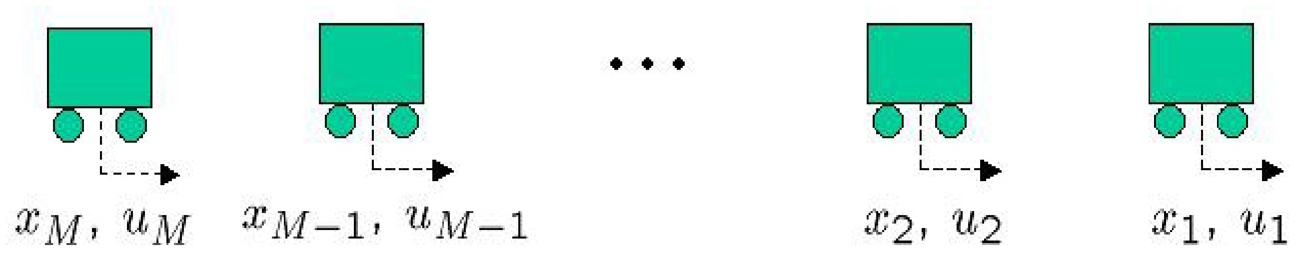
CONTROL IN THE PRESENCE OF SATURATION

- ⋆ explicit constraints on feedback gains to avoid magnitude and rate saturation

❸ CONCLUDING REMARKS

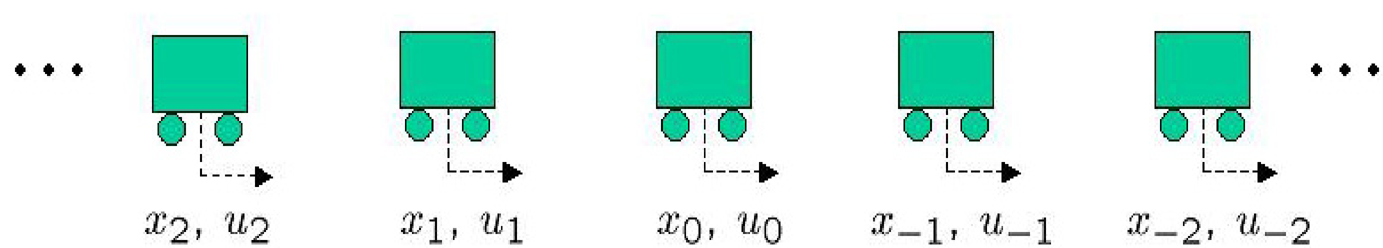
Optimal control of vehicular platoons

- FINITE PLATOONS



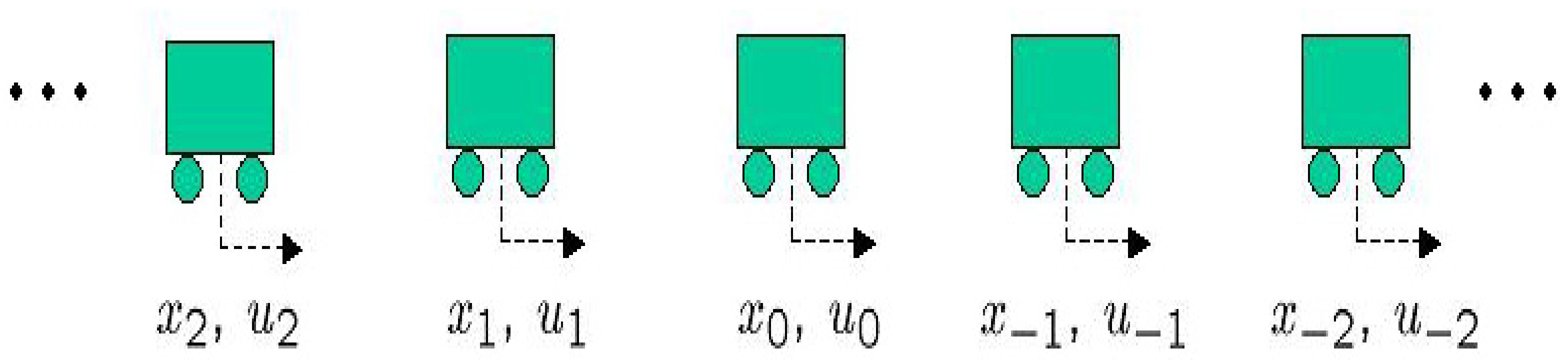
Levine & Athans (LA), IEEE TAC'66
Melzer & Kuo (MK1), IEEE TAC'71

- INFINITE PLATOONS



Melzer & Kuo (MK2), Automatica'71

Control objective



DYNAMICS OF n-TH VEHICLE: $\ddot{x}_n = u_n$

CONTROL OBJECTIVE:

desired cruising velocity	v_d	$:=$	const.
inter-vehicular distance	L	$:=$	const.

☞ COUPLING ONLY THROUGH FEEDBACK CONTROLS

ABSOLUTE DESIRED TRAJECTORY

$$x_{nd}(t) \;:=\; v_d t \;-\; nL$$

Control of finite platoons

$$\left.\begin{array}{lrcl} \text{absolute position error:} & \psi_n(t) & := & x_n(t) \;-\; v_d t \;+\; nL \\ \text{absolute velocity error:} & \phi_n(t) & := & \dot{x}_n(t) \;-\; v_d \end{array}\right\} \quad n \in \{1, \ldots, M\}$$

$$\Downarrow$$

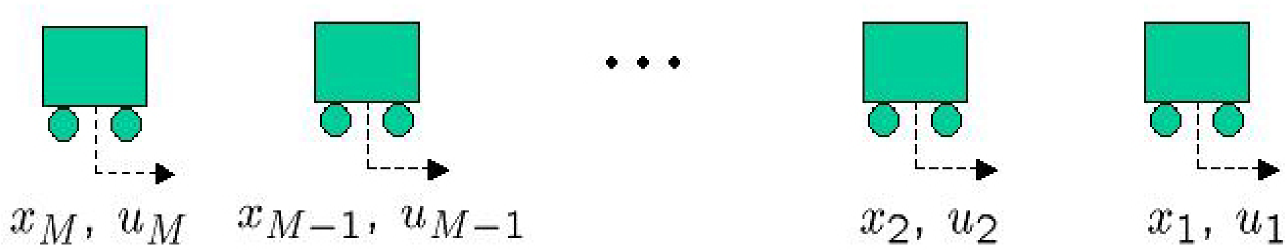
$$\text{MK1:} \quad \begin{bmatrix} \dot{\Psi} \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathrm{U}$$

$$\left.\begin{array}{lrcl} \text{relative position error:} & \zeta_n(t) & := & x_n(t) \;-\; x_{n-1}(t) \;+\; L \\ & & = & \psi_n(t) \;-\; \psi_{n-1}(t) \end{array}\right\} \quad n \in \{2, \ldots, M\}$$

$$\Downarrow$$

$$\text{LA:} \quad \begin{bmatrix} \dot{\mathrm{Z}} \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & \bar{A}_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{Z} \\ \Phi \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathrm{U}$$

Optimal control of finite platoons

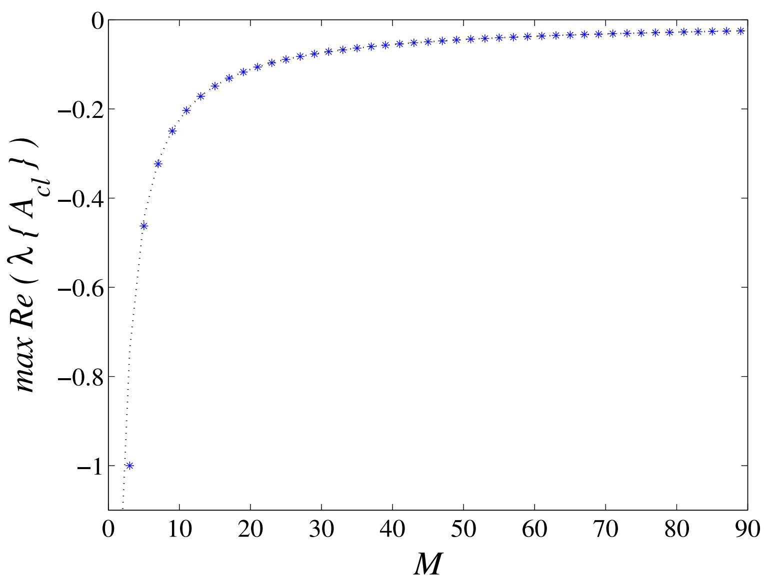


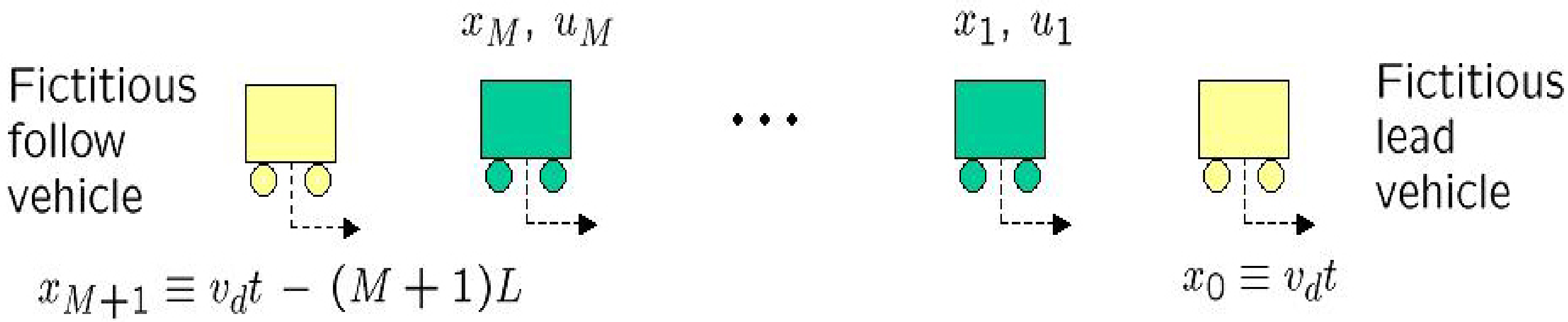
LA:

$$\begin{bmatrix} \dot{\mathrm{Z}} \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & \bar{A}_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{Z} \\ \Phi \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathrm{U}$$

$$J := \frac{1}{2}\int_0^\infty \left(\mathrm{Z}^T(t)\mathrm{Z}(t) + \Phi^T(t)\Phi(t) + \mathrm{U}^T(t)\mathrm{U}(t)\right) \mathrm{d}t$$

$\max Re(\lambda\{A_{cl}\})$:



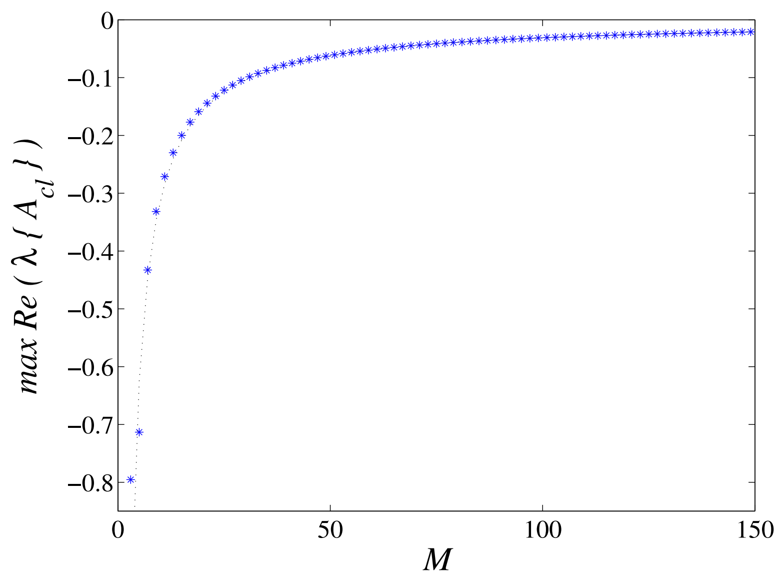


MK1:

$$\begin{bmatrix} \dot{\Psi} \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathrm{U}$$

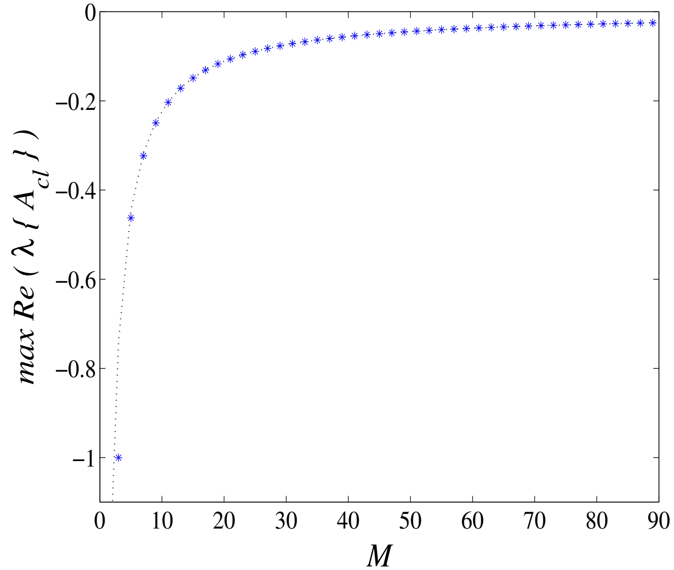
$$J := \frac{1}{2} \int_0^\infty \left(\sum_{n=1}^{M+1} (\psi_n(t) - \psi_{n-1}(t))^2 + \sum_{n=1}^{M} (\phi_n^2(t) + u_n^2(t)) \right) \mathrm{d}t$$

$\max Re(\lambda\{A_{cl}\})$:



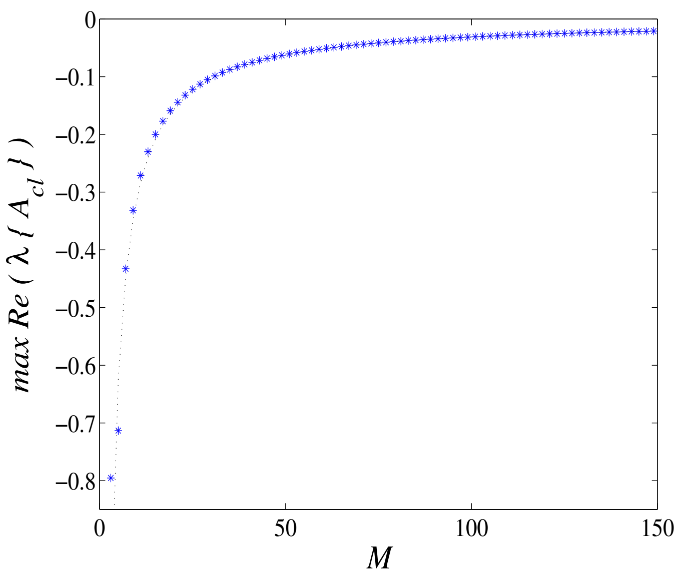
- **LEVINE & ATHANS:**

$$\max Re(\lambda\{A_{cl}\}) :$$



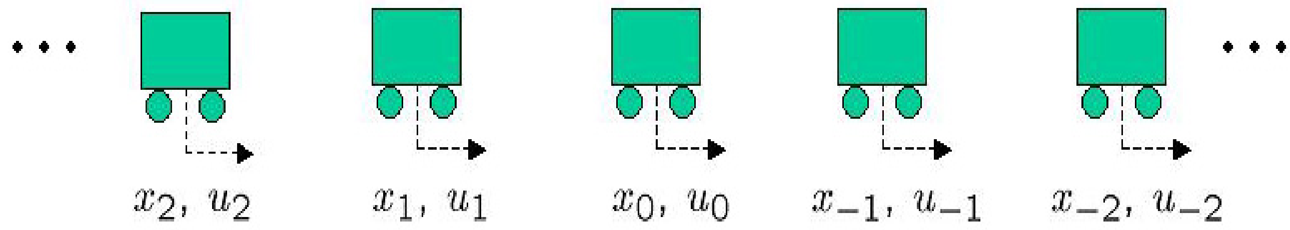
- **MELZER & KUO:**

$$\max Re(\lambda\{A_{cl}\}) :$$



Optimal control of infinite platoons

☞ GOOD APPROXIMATION OF LARGE-BUT-FINITE PLATOONS



MAIN IDEA: EXPLOIT SPATIAL INVARIANCE

LA:

$$\begin{bmatrix} \dot{\zeta}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 - T_{-1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n, \quad n \in \mathbb{Z}$$

$$\Big\downarrow \ \text{SPATIAL } \mathcal{Z}_\theta\text{-TRANSFORM}$$

$$\begin{bmatrix} \dot{\zeta}_\theta \\ \dot{\phi}_\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 - e^{-j\theta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta_\theta \\ \phi_\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_\theta, \quad 0 \le \theta < 2\pi$$

☞ NOT STABILIZABLE AT $\theta = 0$

MK2:

$$\begin{bmatrix} \dot{\psi}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n, \quad n \in \mathbb{Z}$$

$$J := \frac{1}{2} \int_0^\infty \sum_{n \in \mathbb{Z}} \left((\psi_n(t) - \psi_{n-1}(t))^2 + \phi_n^2(t) + u_n^2(t) \right) \mathrm{d}t$$

$$\Big\downarrow \text{ SPATIAL } \mathcal{Z}_\theta\text{-TRANSFORM}$$

$$A_\theta = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad Q_\theta = \begin{bmatrix} 2(1 - \cos\theta) & 0 \\ 0 & 1 \end{bmatrix}, \quad 0 \le \theta < 2\pi$$

- PAIR (Q_θ, A_θ) NOT DETECTABLE AT $\theta = 0$

- A FIX: PENALIZE ABSOLUTE POSITION ERRORS IN J

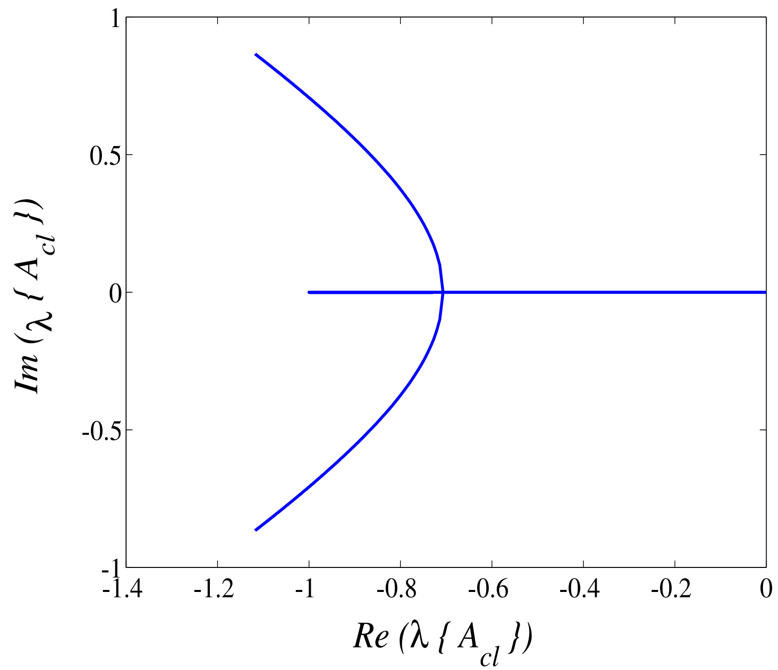
$$J := \frac{1}{2} \int_0^\infty \sum_{n \in \mathbb{Z}} \left(q\psi_n^2(t) + (\psi_n(t) - \psi_{n-1}(t))^2 + \phi_n^2(t) + u_n^2(t) \right) \mathrm{d}t$$

$$\begin{bmatrix} \dot{\psi}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n, \quad n \in \mathbb{Z}$$

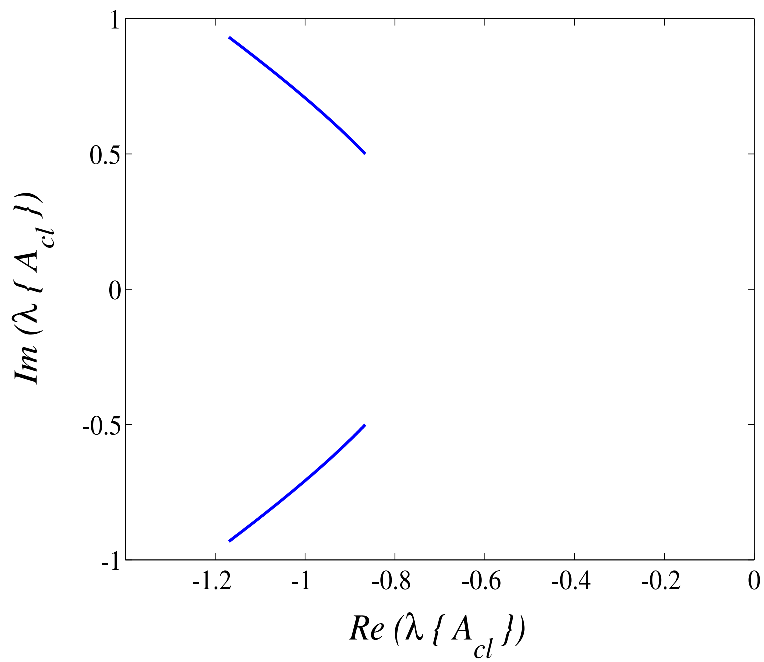
$$J := \frac{1}{2} \int_0^\infty \sum_{n \in \mathbb{Z}} \left(q\psi_n^2(t) + (\psi_n(t) - \psi_{n-1}(t))^2 + \phi_n^2(t) + u_n^2(t) \right) \mathrm{d}t$$

CLOSED-LOOP SPECTRUM:

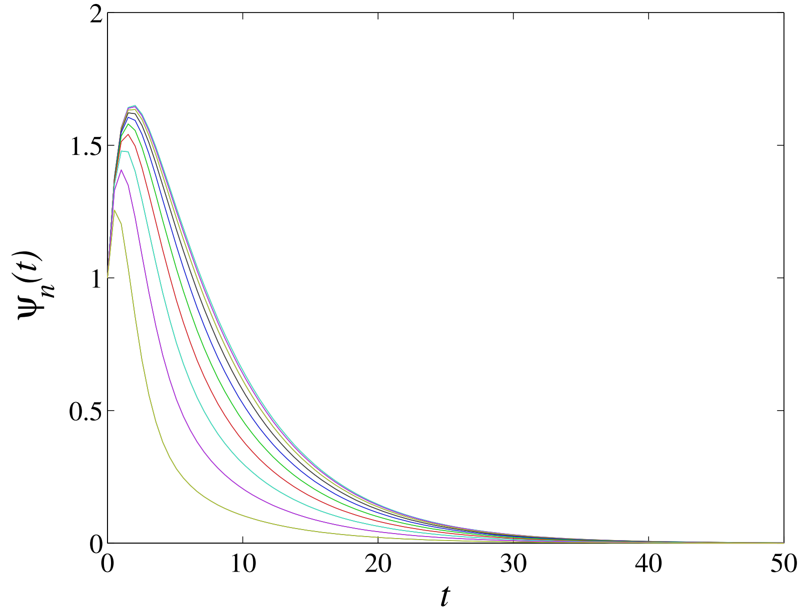
$q = 0$:



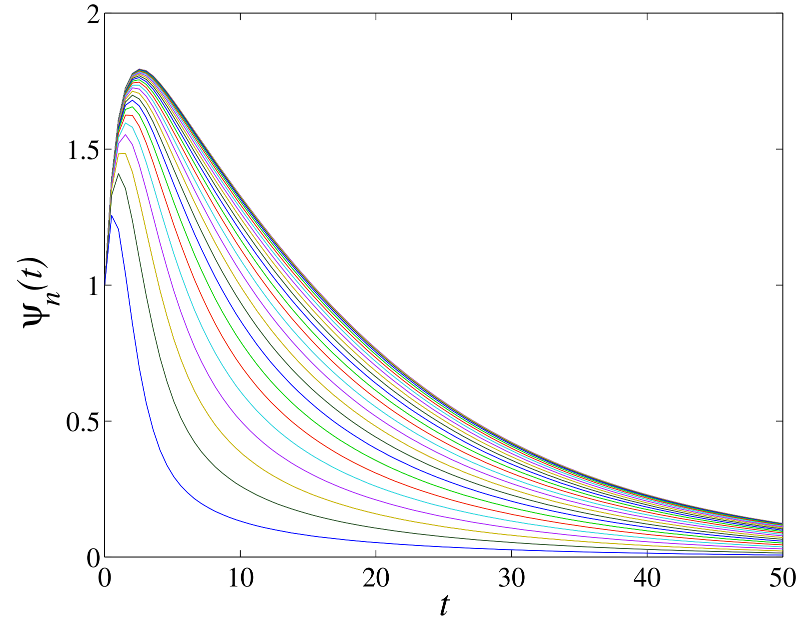
$q = 1$:



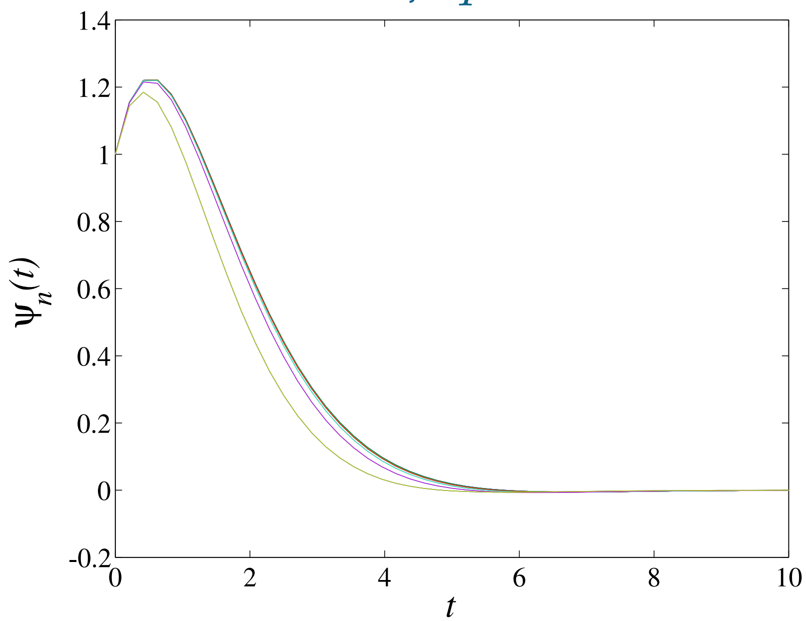
$M = 20,\ q = 0$:



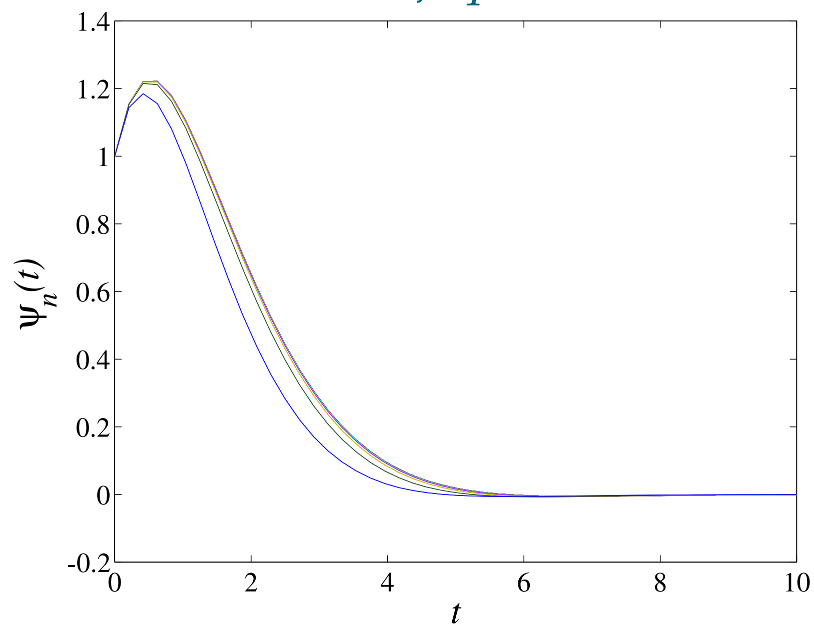
$M = 50,\ q = 0$:



$M = 20,\ q = 1$:



$M = 50,\ q = 1$:

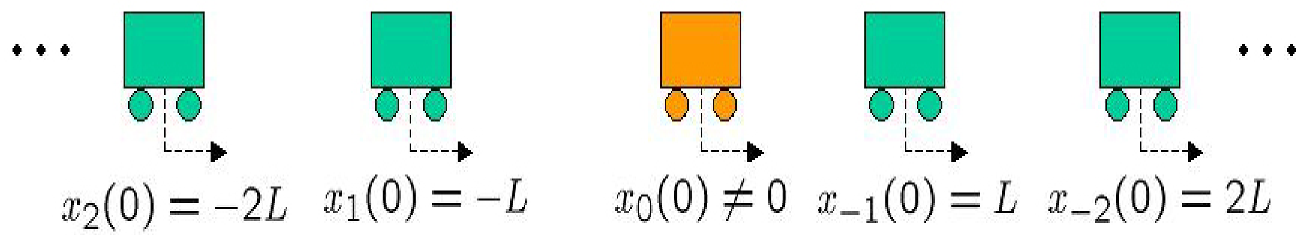


‘Problematic’ initial conditions for LA & MK

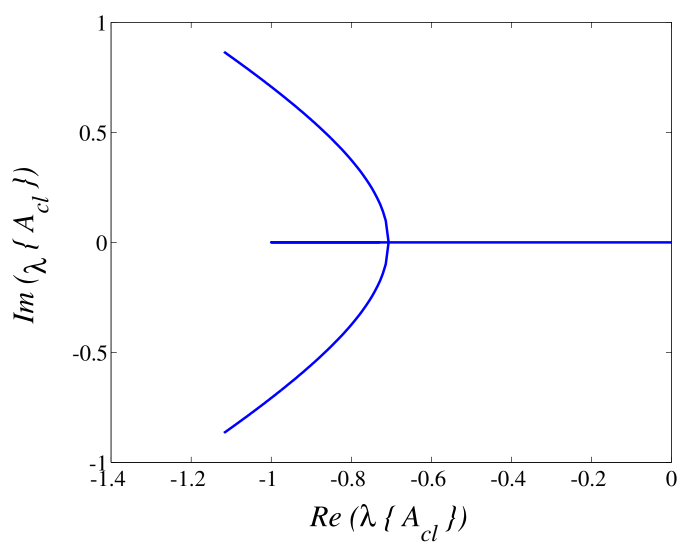
- LA & MK FOR INFINITE PLATOONS:
  non-zero mean initial conditions cannot be driven to zero!

MK:

$$\sum_{n \in \mathbb{Z}} \psi_n(0) \neq 0 \quad \Rightarrow \quad \lim_{t \to \infty} \sum_{n \in \mathbb{Z}} \psi_n(t) \neq 0$$



☞ MANY MODES HAVE VERY SLOW RATES OF CONVERGENCE



Remarks

- CONTROL OF VEHICULAR PLATOONS

Jovanović & Bamieh, IEEE TAC'05:

- ⋆ analytically showed ill-posedness of several widely cited results
- ⋆ formulated well-posed control problems

SMALL SCALE $\xrightarrow{\text{CAREFUL}}$ LARGE SCALE

SMALL SCALE $\neq$ LARGE SCALE $\approx$ INFINITE DIMENSIONAL

large-but-finite systems	$\Rightarrow$	large scale computations
infinite dimensional abstractions with spatial invariance	$\Rightarrow$	almost analytical results

Outline

❶ ILL-POSEDNESS OF SEVERAL WIDELY CITED RESULTS

⋆ small scale $\xrightarrow{\text{GAP}}$ large scale $\approx$ infinite

FORMULATION OF WELL-POSED CONTROL PROBLEMS

⋆ spatially invariant theory

❷ PEAKING IN VEHICULAR PLATOONS

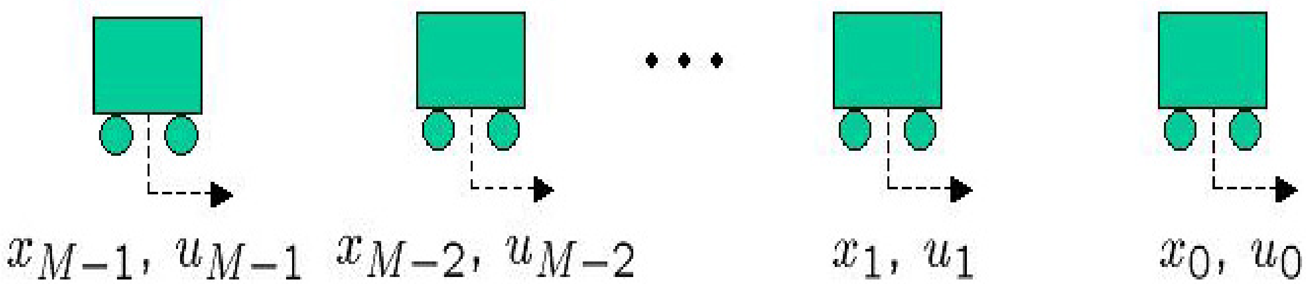
large platoons uniform convergence rates	$\Rightarrow$	large transient peaks

CONTROL IN THE PRESENCE OF SATURATION

⋆ explicit constraints on feedback gains to avoid magnitude and rate saturation

❸ CONCLUDING REMARKS

LQR design



$$
\left.
\begin{array}{l}
\text{absolute position error:} \quad \psi_n(t) \;:=\; x_n(t) \,-\, v_d t \,+\, nL \\
\text{absolute velocity error:} \quad \phi_n(t) \;:=\; \dot{x}_n(t) \,-\, v_d \\
\qquad\qquad\qquad\qquad \Downarrow \\
\begin{bmatrix} \dot{\psi}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n
\end{array}
\right\} \; n \in \{0, \ldots, M-1\}
$$

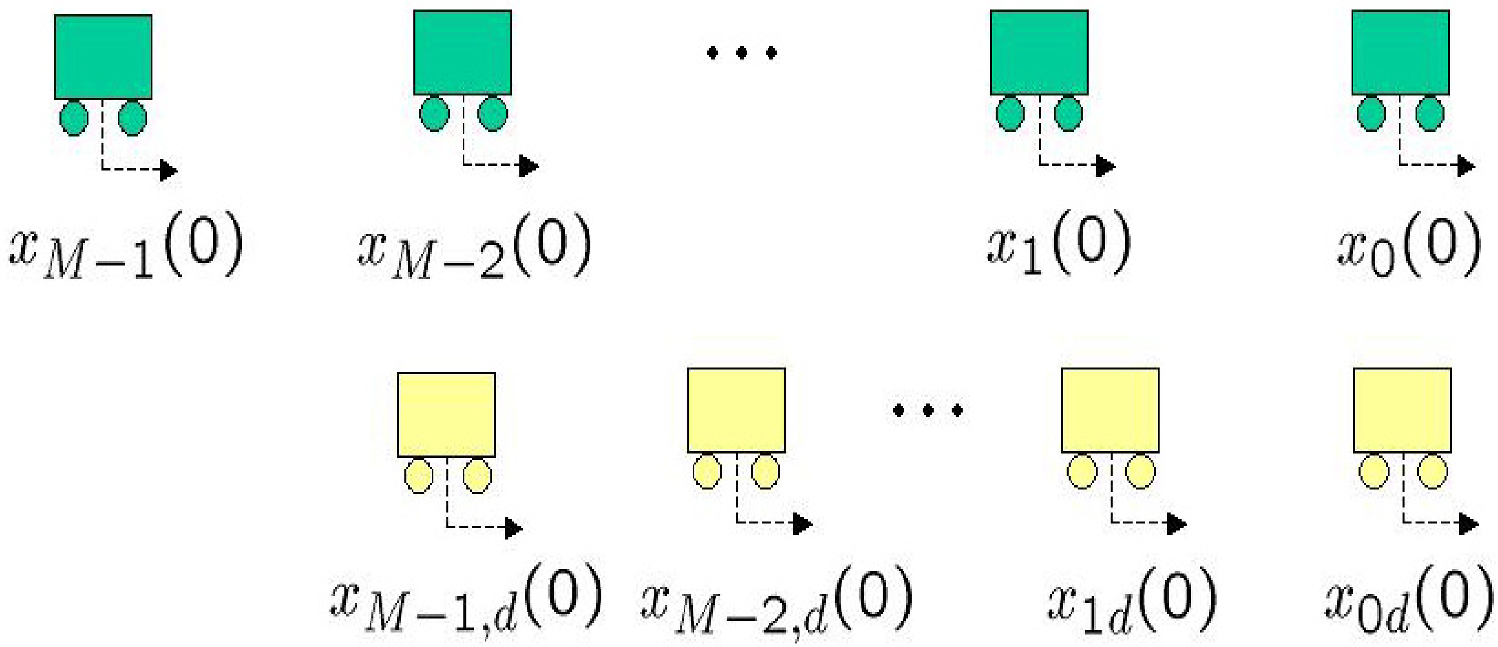
PERFORMANCE INDEX:

$$
J \;:=\; \frac{1}{2} \int_0^\infty \left(\sum_{n=1}^{M-1} (\psi_n(t) \,-\, \psi_{n-1}(t))^2 \,+\, \sum_{n=0}^{M-1} (\psi_n^2(t) \,+\, \phi_n^2(t) \,+\, u_n^2(t)) \right) \mathrm{d}t
$$

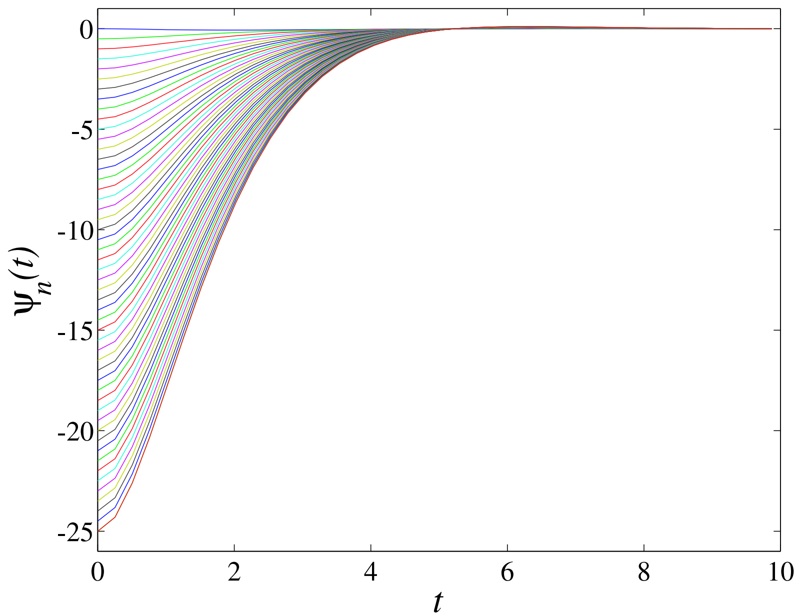
Example

☞ PEAKING: CERTAIN INITIAL CONDITIONS CAN LEAD TO LARGE CONTROLS

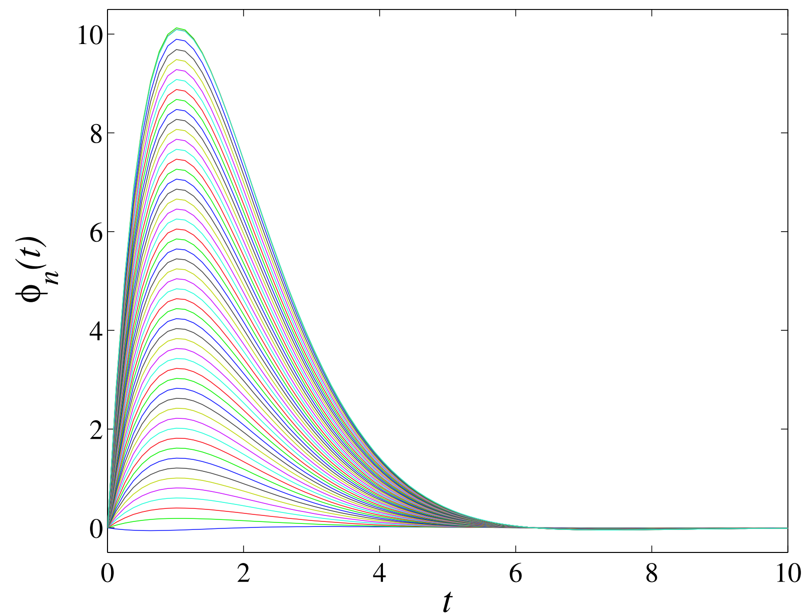
$$\begin{aligned} x_n(0) &= -n(L + S) & \Rightarrow \quad \psi_n(0) &= -nS \\ \dot{x}_n(0) &= v_d & \Rightarrow \quad \phi_n(0) &= 0 \end{aligned}$$



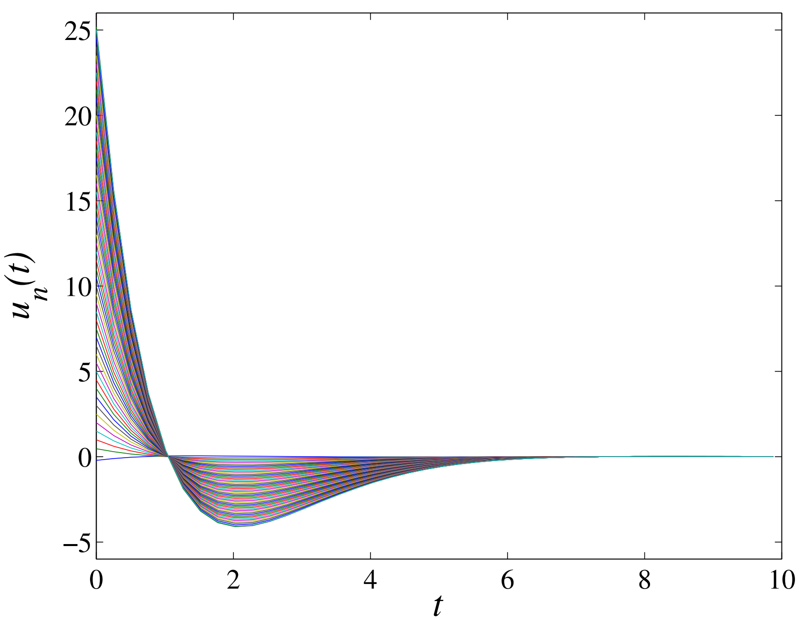
POSITION:



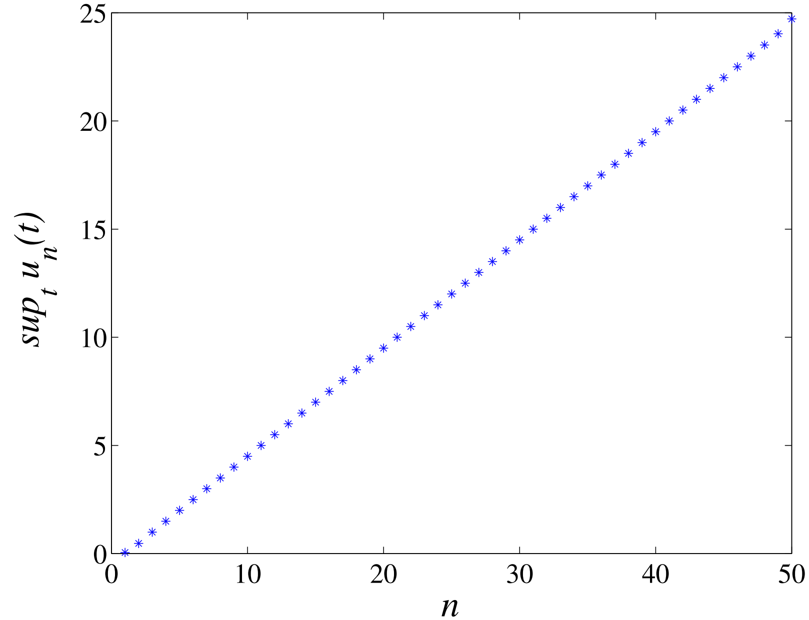
VELOCITY:



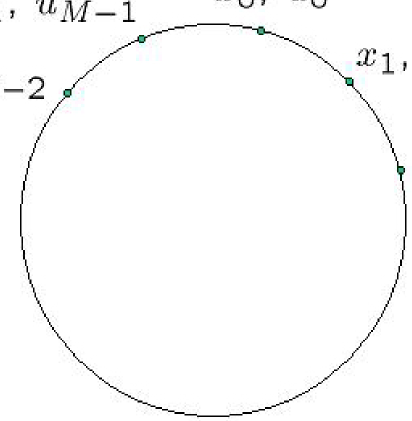
CONTROL:



$\sup_t u_n(t)$:



LQR design for a platoon on a circle



$$\begin{bmatrix} \dot{\psi}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n$$
$$=: A_n \boldsymbol{\xi}_n + B_n u_n$$

SPATIALLY INVARIANT PERFORMANCE INDEX:

$$\boxed{J := \frac{1}{2} \int_0^\infty \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} \left(\boldsymbol{\xi}_n^*(t) Q_{n-m} \boldsymbol{\xi}_m(t) + u_n^*(t) R_{n-m} u_m(t) \right) \mathrm{d}t}$$

MAIN IDEA: EXPLOIT SPATIAL INVARIANCE

LARGE SCALE PROBLEM:

$$\begin{bmatrix} \dot{\psi}_n \\ \dot{\phi}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_n \\ \phi_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_n =: A_n \boldsymbol{\xi}_n + B_n u_n$$

$$J := \frac{1}{2} \int_0^\infty \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} (\boldsymbol{\xi}_n^*(t) Q_{n-m} \boldsymbol{\xi}_m(t) + u_n^*(t) R_{n-m} u_m(t))\, \mathrm{d}t$$

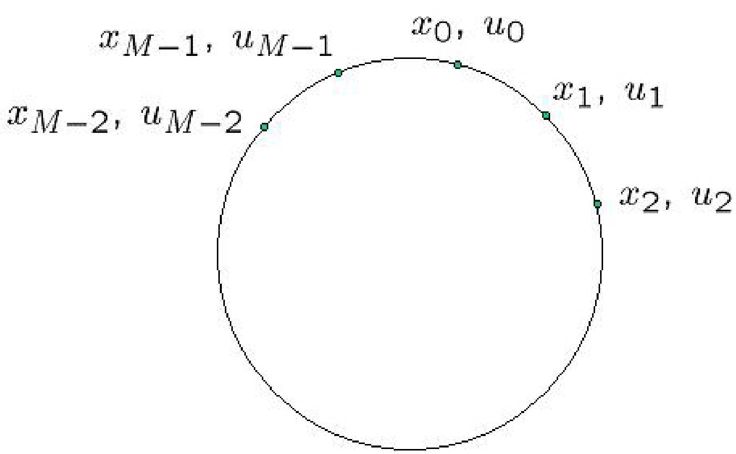
$$\big\downarrow \text{SPATIAL DFT}$$

BLOCK DIAGONAL PROBLEM:

$$\begin{bmatrix} \dot{\hat{\psi}}_k \\ \dot{\hat{\phi}}_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\psi}_k \\ \hat{\phi}_k \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u}_k =: \hat{A}_k \hat{\boldsymbol{\xi}}_k + \hat{B}_k \hat{u}_k$$

$$J = \frac{\sqrt{M}}{2} \int_0^\infty \sum_{k=0}^{M-1} \left(\hat{\boldsymbol{\xi}}_k^*(t) \hat{Q}_k \hat{\boldsymbol{\xi}}_k(t) + \hat{u}_k^*(t) \hat{R}_k \hat{u}_k(t) \right) \mathrm{d}t$$

$$\hat{R}_k > 0, \quad \hat{Q}_k := \begin{bmatrix} \hat{q}_{11k} & \hat{q}_{21k}^* \\ \hat{q}_{21k} & \hat{q}_{22k} \end{bmatrix} \geq 0, \quad k \in \{0, \ldots, M-1\}$$



$$\begin{aligned}\begin{bmatrix}\dot{\psi}_n\\ \dot{\phi}_n\end{bmatrix} &= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\psi_n\\ \phi_n\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u_n\\ &=: A_n\boldsymbol{\xi}_n + B_n u_n\end{aligned}$$

$$\{\psi_n(0) = -nS,\ 0 < S < L;\quad \dot{x}_n(0) \equiv v_d\}$$

$\downarrow$ magnitude saturation unless

$$\boxed{u_{\max} \geq \frac{S^2}{6}(M-1)(2M-1)\inf_k \frac{\hat{q}_{11k}}{\hat{R}_k}}$$

IMPORTANT:

$$\boxed{\text{DETECTABILITY} \Leftrightarrow \hat{q}_{11k} > 0}$$

Trajectory generation

$$\left.\begin{array}{rcl} \ddot{\bar{x}}_n & = & \bar{u}_n \\ r_n(t) & := & \bar{x}_n(t) \; - \; v_d t \; + \; nL \end{array}\right\} \quad \Rightarrow \quad \ddot{r}_n \; = \; \bar{u}_n$$

$$\bar{u}_n \; = \; - \, p_n^2 r_n \; - \; 2 p_n \dot{r}_n \quad \Rightarrow \quad \begin{array}{rcl} r_n(t) & = & (c_n \; + \; d_n t) \mathrm{e}^{-p_n t} \\ \dot{r}_n(t) & = & (d_n \; - \; c_n p_n \; - \; d_n p_n t) \mathrm{e}^{-p_n t} \\ \bar{u}_n(t) & = & (c_n p_n^2 \; - \; 2 d_n p_n \; + \; d_n p_n^2 t) \mathrm{e}^{-p_n t} \end{array}$$

☞ DERIVED EXPLICIT CONSTRAINTS ON p_n TO GUARANTEE:

$$\begin{array}{rcl} |r_n(t)| & \leq & r_{\max} \\ |\dot{r}_n(t)| & \leq & v_{\max} \\ |\bar{u}_n(t)| & \leq & u_{\max} \end{array}$$

FUNDAMENTAL TRADEOFFS:

$$\text{large } p_n\text{'s} \quad \Rightarrow \quad \left\{ \begin{array}{l} \text{fast stabilization} \\ \text{small position overshoots} \end{array} \right.$$

$$\text{small } p_n\text{'s} \quad \Rightarrow \quad \left\{ \begin{array}{l} \text{slow stabilization} \\ \text{small velocity overshoots} \\ \text{small control effort} \end{array} \right.$$

CONTROL AROUND GENERATED TRAJECTORIES:

$$\eta_n(t) \;:=\; x_n(t) \;-\; v_d t \;+\; nL \;-\; r_n(t)$$

$$\Downarrow$$

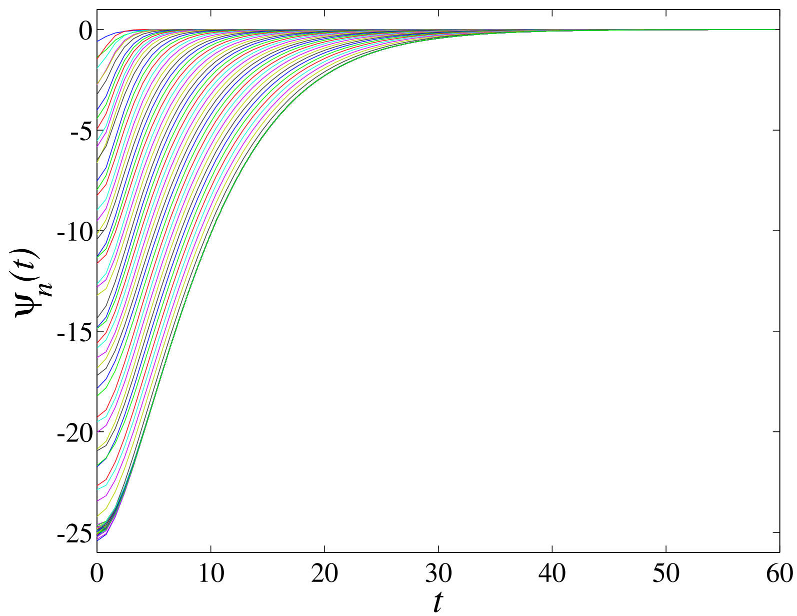
$$\ddot{\eta}_n \;=\; u_n \;-\; \bar{u}_n \;=:\; \tilde{u}_n$$

$$\Downarrow$$

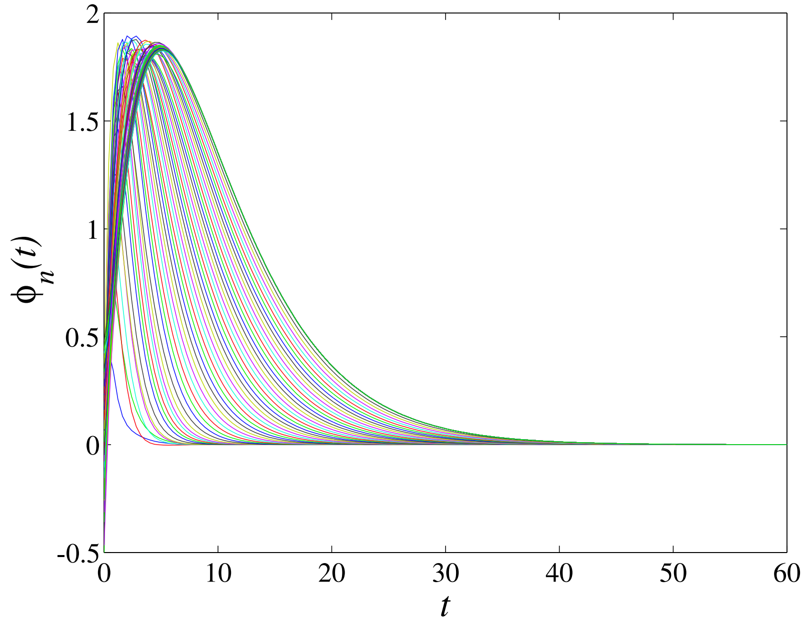
$$\boxed{u_n \;=\; \underset{\text{from trajectory generation}}{\underset{\downarrow}{\bar{u}_n}} \;+\; \underset{\text{from e.g. LQR design}}{\underset{\downarrow}{\tilde{u}_n}}}$$

PERFECT KNOWLEDGE OF INITIAL CONDITIONS: $u_n \;=\; \bar{u}_n$

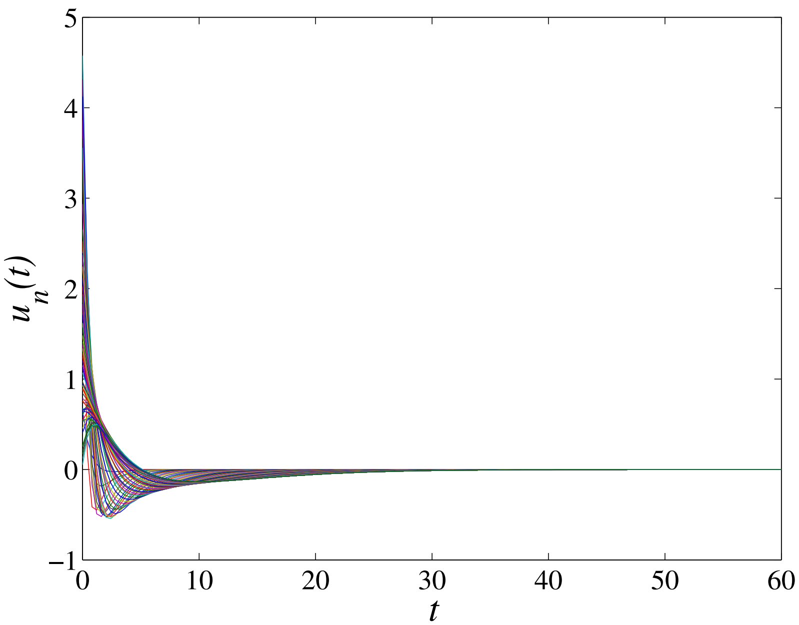
POSITION:



VELOCITY:



CONTROL:



Remarks

Jovanović, Fowler, Bamieh, & D'Andrea, Automatica'05 (submitted):

⋆ trajectory planing to avoid magnitude and rate peaking

large platoons uniform convergence rates	$\Rightarrow$	large transient peaks
fast convergence rates	$\Rightarrow$	small position overshoots
slow convergence rates	$\Rightarrow$	small velocity overshoots small control efforts

- ONGOING/FUTURE RESEARCH
  - ⋆ design of distributed controllers with favorable architectures
  - ⋆ control under communication constraints
  - ⋆ limitations & tradeoffs in the control of 2D & 3D formations
    flocking & swarming

Additional info

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