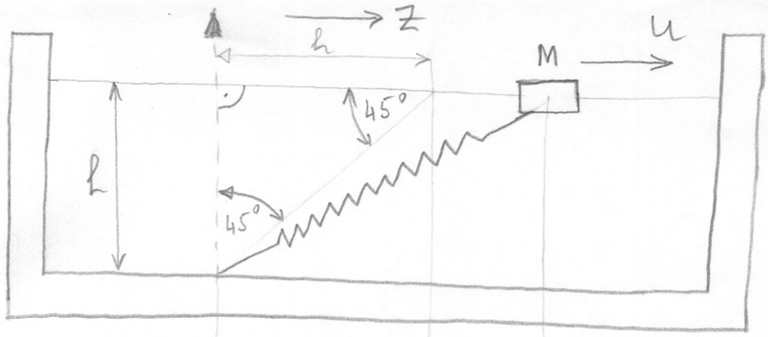
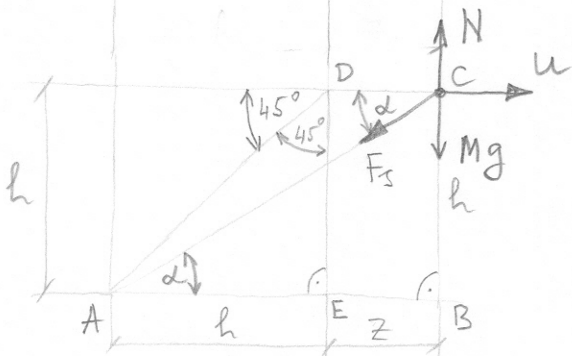
2°

[redacted]





$$M \cdot \vec{a} = \sum \vec{F_i}$$

$$Z: \; M \cdot \ddot{Z} = u - F_s \cdot \cos\alpha \quad \dots (1)$$

$$\left.\begin{aligned} F_s &= K \cdot (AC - AD) \\ AC &= \sqrt{h^2 + (h+z)^2} \\ AD &= h\sqrt{2} \end{aligned}\right\} \Rightarrow F_s = K \cdot \left(\sqrt{h^2 + (h+Z)^2} - h\sqrt{2}\right) \quad \dots (2)$$

$$\Delta ABC \Rightarrow \cos\alpha = \frac{AB}{AC} = \frac{h+Z}{\sqrt{h^2 + (h+z)^2}} \quad \dots (3)$$

$[(2),(3)] \Rightarrow (1) \Rightarrow$

$$M\ddot{Z} = u - K \cdot \left(\sqrt{h^2 + (h+z)^2} - h\sqrt{2}\right) \frac{h+Z}{\sqrt{h^2 + (h+Z)^2}} \Rightarrow$$

$$\boxed{\ddot{Z} = -\frac{K}{M} \cdot \left(\sqrt{h^2 + (h+Z)^2} - h\sqrt{2}\right) \frac{h+Z}{\sqrt{h^2 + (h+z)^2}} + \frac{1}{M} u} \quad \dots (4)$$

State variables:

$x_1(t) = Z(t) \rightarrow \dot{x}_1(t) = \dot{Z}(t)$

$x_2(t) = \dot{Z}(t) \rightarrow \dot{x}_2(t) = \ddot{Z}(t)$

$$\dot{x}_1(t) = x_2(t) \quad \ldots(5)$$

$$\dot{x}_2(t) = -\frac{k}{M}\cdot\left(\sqrt{h^2+(h+x_1)^2} - h\sqrt{2}\right)\frac{h+x_1}{\sqrt{h^2+(h+x_1)^2}} + \frac{1}{M}\cdot u \quad \ldots(6)$$

$$y(t) = x_1(t) \quad \ldots(7)$$

Scalar equations (5), (6) and (7) can be written in the vector form:

$$\underline{\dot{x}} = \underline{f}(\underline{x}, u)$$
$$y = g(\underline{x})$$

where

$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$ – state vector

$\underline{f}(\underline{x}, u): \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ – nonlinear vector function defined by

$$\underline{f}(\underline{x}, u) = \begin{pmatrix} x_2(t) \\ -\frac{k}{M}\cdot\left(\sqrt{h^2+(h+x_1(t))^2} - h\sqrt{2}\right)\frac{h+x_1(t)}{\sqrt{h^2+(h+x_1(t))^2}} + \frac{1}{M}u(t) \end{pmatrix}$$

$g(\underline{x}): \mathbb{R}^2 \to \mathbb{R}$ – scalar function defined by:

$$g(\underline{x}) = x_1(t)$$

Properties of this system:

a) System is causal since causality is built in every state-space model

b) System is time invariant since vector function $\underline{f}(\cdot)$ and scalar function $g(\cdot)$ do not depend explicitly on time.

c) System is nonlinear since the vector function $\underline{f}(\cdot)$ is nonlinear.

1°

State - space represantion:

$$\dot{x}_1(t) = x_2(t) \qquad \dots (1)$$

$$\dot{x}_2(t) = -\frac{K}{M}\left(\sqrt{h^2+(h+x_1)^2} - h\sqrt{2}\right)\frac{h+x_1}{\sqrt{h^2+(h+x_1)^2}} + \frac{1}{M}\cdot u \qquad \dots (2)$$

$$y(t) = x_1(t) \qquad \dots (3)$$

State variables:

$$x_1(t) = z(t)$$
$$x_2(t) = \dot{z}(t)$$

Let's translate coordinate system:

$$z_1 = z + h \rightarrow z = z_1 - h \Rightarrow \begin{aligned} x_1 &= z_1 \\ x_2 &= \dot{z}_1 \\ y &= z_1 \end{aligned}$$

x_1 and x_2 are not the same as x_1 & x_2 in eq. (1) & (2)
$x_{1N}^{NEW} = x_{1O}^{OLD} + h$
$x_{2N} = (\dot{z} + \dot{h}) = \dot{z} = x_{2O}$

New state-space represantion:

$$\dot{x}_1(t) = x_2(t) \qquad \dots (4)$$

$$\dot{x}_2(t) = -\frac{K}{M}\cdot\left\{\sqrt{h^2+x_1^2} - h\sqrt{2}\right\}\frac{x_1}{\sqrt{h^2+x_1^2}} + \frac{1}{M}\cdot u \qquad \dots (5)$$

$$y(t) = x_1(t) \qquad \dots (6)$$

Equilibrium points: $\underline{\dot{x}} = \underline{f}(\underline{x}; u)$

$$\underline{f}(\underline{x}_e, 0) = \underline{0}$$

$$\underline{f}(\underline{x}, u) = \begin{pmatrix} x_2(t) \\ -\frac{K}{M}\left\{\sqrt{h^2+x_1^2} - h\sqrt{2}\right\}\frac{x_1}{\sqrt{h^2+x_1^2}} + \frac{1}{M}u \end{pmatrix} \Rightarrow$$

$$\underline{f}(\underline{x},0)=\begin{pmatrix} x_2(t) \\ -\frac{K}{M}\left\{\sqrt{h^2+x_1^2}-h\sqrt{2}\right\}\frac{x_1}{\sqrt{x_1^2+h^2}}\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Longleftrightarrow$$

$$\left.\begin{aligned} & x_2=0 \\ & -\frac{K}{M}\left\{\sqrt{h^2+x_1^2}-h\sqrt{2}\right\}\frac{x_1}{\sqrt{x_1^2+h^2}}=0\Rightarrow \\ & \Rightarrow x_1=0 \vee \sqrt{h^2+x_1^2}-h\sqrt{2}=0\Rightarrow h^2+x_1^2=2h^2\Rightarrow \\ & \Rightarrow x_1^2-h^2=0\Rightarrow (x_1-h)(x_1+h)=0\Rightarrow x_1=h \vee x_1=-h \end{aligned}\right\}\Rightarrow$$

$\Rightarrow$ Candidates for equilibrium points are:

$$\underline{x}=\begin{pmatrix}-h\\0\end{pmatrix} \vee \underline{x}=\begin{pmatrix}0\\0\end{pmatrix} \vee \underline{x}=\begin{pmatrix}h\\0\end{pmatrix}$$

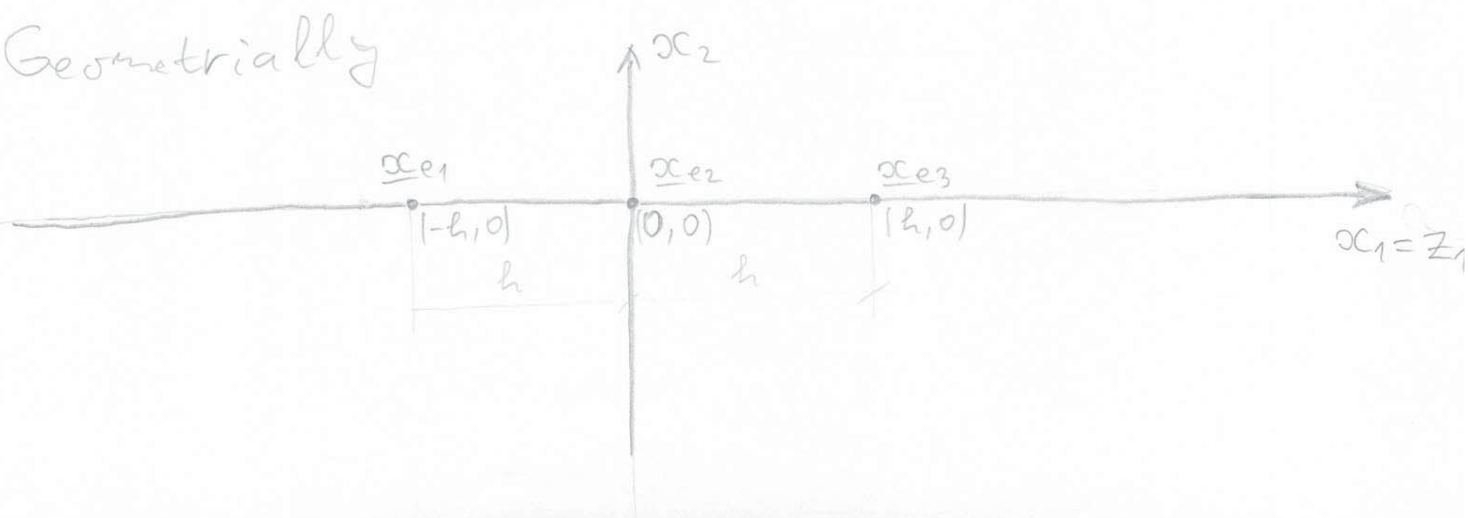
We should determine if our system has unique solut
through these points.

Since function $\underline{f}(\underline{x},0)\in C^{(1)}(\mathbb{R}^2)\Rightarrow$ System has unique solution through every point in $\mathbb{R}^2\Rightarrow$

$\Rightarrow$ There are 3 e.p.:

$$\underline{x}_{e1}=\begin{pmatrix}-h\\0\end{pmatrix};\ \underline{x}_{e2}=\begin{pmatrix}0\\0\end{pmatrix};\ \underline{x}_{e3}=\begin{pmatrix}h\\0\end{pmatrix}$$

Geometrially



b) Linearization:

$$x_1(t) = x_{10}(t) + \delta x_1(t)$$

$$x_2(t) = x_{20}(t) + \delta x_2(t)$$

$$u(t) = u_0(t) + \delta u(t)$$

$$\dot{x}_1(t) = f_1(x_1(t), x_2(t), u(t)) = x_2(t) \text{ - linear function}$$

$$\dot{x}_2(t) = f_2(x_1(t), x_2(t), u(t)) = \underbrace{-\frac{K}{M}\left\{\sqrt{h^2+x_1^2} - h\sqrt{2}\right\}\frac{x_1}{\sqrt{h^2+x_1^2}}}_{\text{only this term needs linearization!}}$$

$$\frac{\partial f_2}{\partial x_1} = ?$$

$$f_2(\cdot) = -\frac{K}{M}\cdot\left\{x_1 - h\sqrt{2}\,\frac{x_1}{\sqrt{h^2+x_1^2}}\right\}$$

$$\frac{\partial f_2}{\partial x_1} = -\frac{K}{M}\left\{1 - h\sqrt{2}\,\frac{\sqrt{h^2+x_1^2} - x_1\cdot\frac{2x_1}{2\sqrt{h^2+x_1^2}}}{h^2+x_1^2}\right\} =$$

$$= -\frac{K}{M}\cdot\left\{1 - h\sqrt{2}\,\frac{h^2+\cancel{x_1^2}-\cancel{x_1^2}}{(h^2+x_1^2)\sqrt{h^2+x_1^2}}\right\} =$$

$$= -\frac{K}{M}\cdot\left\{1 - \frac{h^3\sqrt{2}}{(h^2+x_1^2)^{\frac{3}{2}}}\right\}$$

$$\boxed{\left.\frac{\partial f_2}{\partial x_1}\right|_{x_{1e}=-h} = \left.\frac{\partial f_2}{\partial x_1}\right|_{x_{1e}=h}} = -\frac{K}{M}\cdot\left\{1 - \frac{h^3\sqrt{2}}{(h^2+h^2)^{\frac{3}{2}}}\right\} =$$

$$= -\frac{K}{M}\cdot\left\{1 - \frac{h^3\sqrt{2}}{\sqrt{2h^2}\cdot 2h^2}\right\} = -\frac{K}{M}\cdot\left\{1 - \frac{\cancel{h^3}\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}\cdot\cancel{h^3}}\right\} = -\frac{K}{M}\left\{1-\frac{1}{2}\right\}$$

$$= \boxed{-\frac{K}{2M}}$$

$$\boxed{\left.\frac{\partial f_2}{\partial x_1}\right|_{x_{1e}=0}} = -\frac{K}{M}\left\{1 - \frac{h^3\sqrt{2}}{h^2\cdot\sqrt{h^2}}\right\} = \boxed{-\frac{K}{M}\cdot(1-\sqrt{2})}$$

Linearized State-Space Represantion

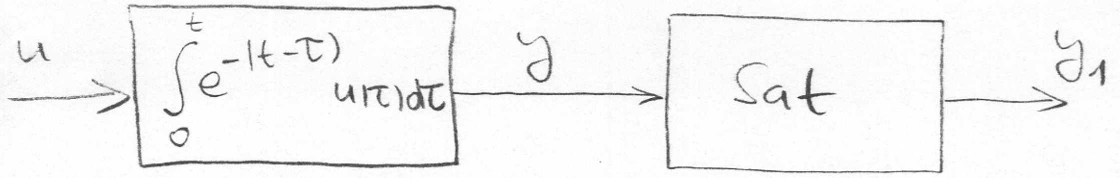
a) Around $\underline{x}_{e2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\left.\begin{aligned} \frac{d\,\delta x_1}{dt} &= \delta x_2 \\ \frac{d\,\delta x_2}{dt} &= \frac{(\sqrt{2}-1)\cdot K}{M}\cdot \delta x_1 + \frac{1}{M}\delta u \\ \delta y &= \delta x_1 \end{aligned}\right\} \Rightarrow \quad A = \begin{pmatrix} 0 & 1 \\ \frac{(\sqrt{2}-1)K}{M} & 0 \end{pmatrix};\ B = \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}\ D = (0)$$

b) Around $\underline{x}_{e1} = \begin{pmatrix} -h \\ 0 \end{pmatrix}$ and $\underline{x}_{e3} = \begin{pmatrix} h \\ 0 \end{pmatrix}$

$$\left.\begin{aligned} \frac{d\,\delta x_1}{dt} &= \delta x_2 \\ \frac{d\,\delta x_2}{\partial t} &= -\frac{K}{2M}\,\delta x_1 + \frac{1}{M}\delta u \\ \delta y &= \delta x_1 \end{aligned}\right\} \Rightarrow \quad A = \begin{pmatrix} 0 & 1 \\ -\frac{K}{2M} & 0 \end{pmatrix};\ B = \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix};\ D = (0)$$

1.



[redacted]

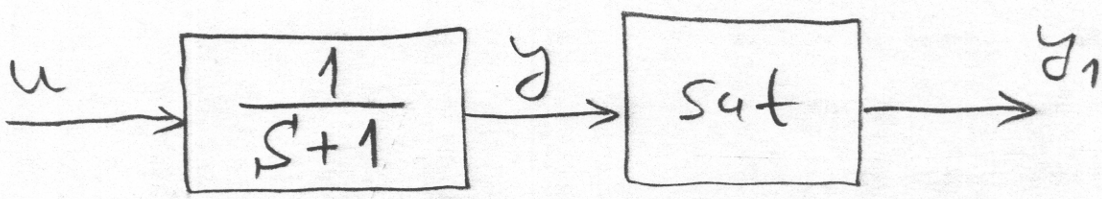
$$\Rightarrow y(t) = \int_0^t e^{-(t-\tau)} u(\tau) d\tau$$ - Convolution form

$$\Updownarrow$$

$$y(t) = \int_0^t h(t-\tau) u(\tau) d\tau,$$ where $h(\cdot)$ is the impulse response of our system

$$\therefore h(t-\tau) = e^{-(t-\tau)} \Rightarrow h(t) = e^{-t} \,/\, \mathcal{L} \Rightarrow$$

$$\Rightarrow \boxed{H(s) = \frac{1}{s+1}}$$ $\Rightarrow$ Our system can be represented as:



Properties: causal, time invariant, dynamical (non-memoryless), nonlinear (due sat element

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$u(t)$ is a periodic signal, with period $T = 8$. $\Rightarrow$ It can be represented in the form of Fourier series:

$$u(t) = \sum_{n=-\infty}^{+\infty} \hat{u}_n \cdot e^{jn\omega_0 t},$$ where

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\hat{u}_n = \frac{1}{T}\int_0^T u(t)\cdot e^{-jn\omega_0 t}\,dt$$

On the interval $[0, 8]$

$$u(t) = \begin{cases} t, & 0 \le t \le 2 \\ 4-t, & 2 \le t \le 6 \\ t-8, & 6 \le t \le 8 \end{cases}$$

Using the help of Mathematica one gets: that for $n \neq 0$:

$$\hat{u}_n = -\frac{2}{n^2\pi^2} e^{-j2n\pi}\cdot\left(-1+e^{j\frac{n\pi}{2}}\right)^3\cdot\left(1+e^{j\frac{n\pi}{2}}\right)$$

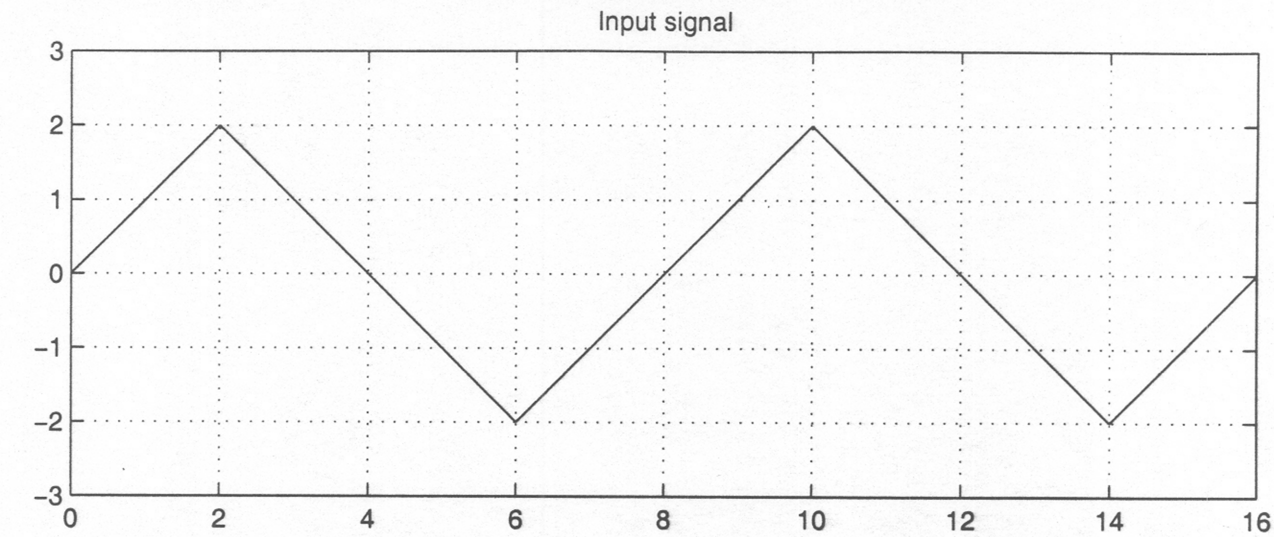
and $\hat{a}_0 = 0$. $\Rightarrow$ For $n \neq 0 \Rightarrow$

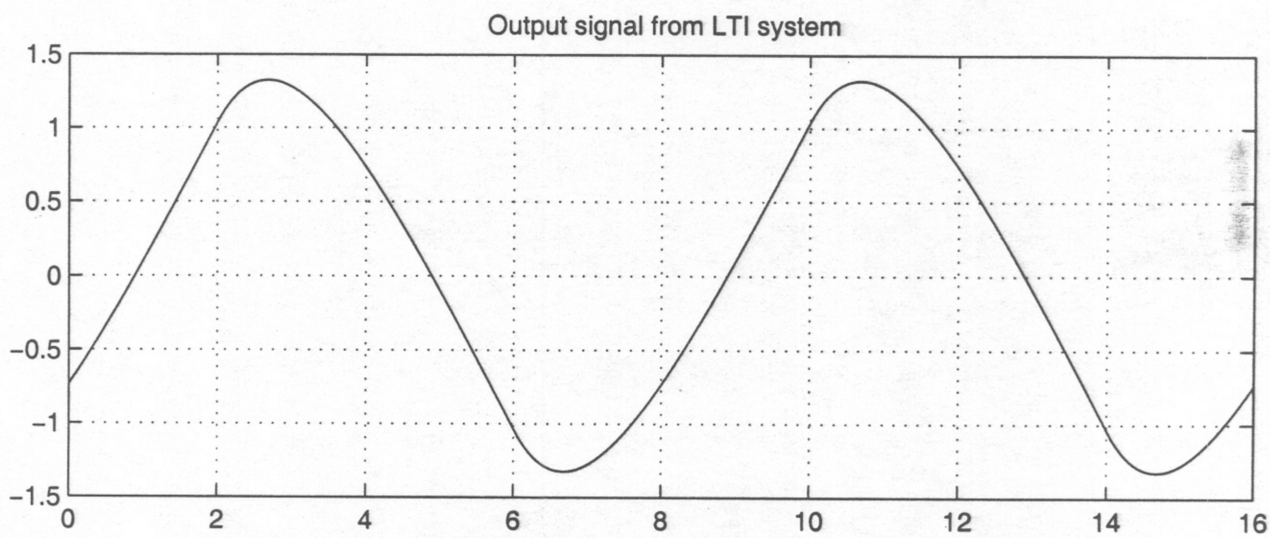
$$\hat{u}_n = -\frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]\cdot\left\{1+(-1)^n - 2\cos\frac{n\pi}{2} - j2\sin\frac{n\pi}{2}\right\}$$

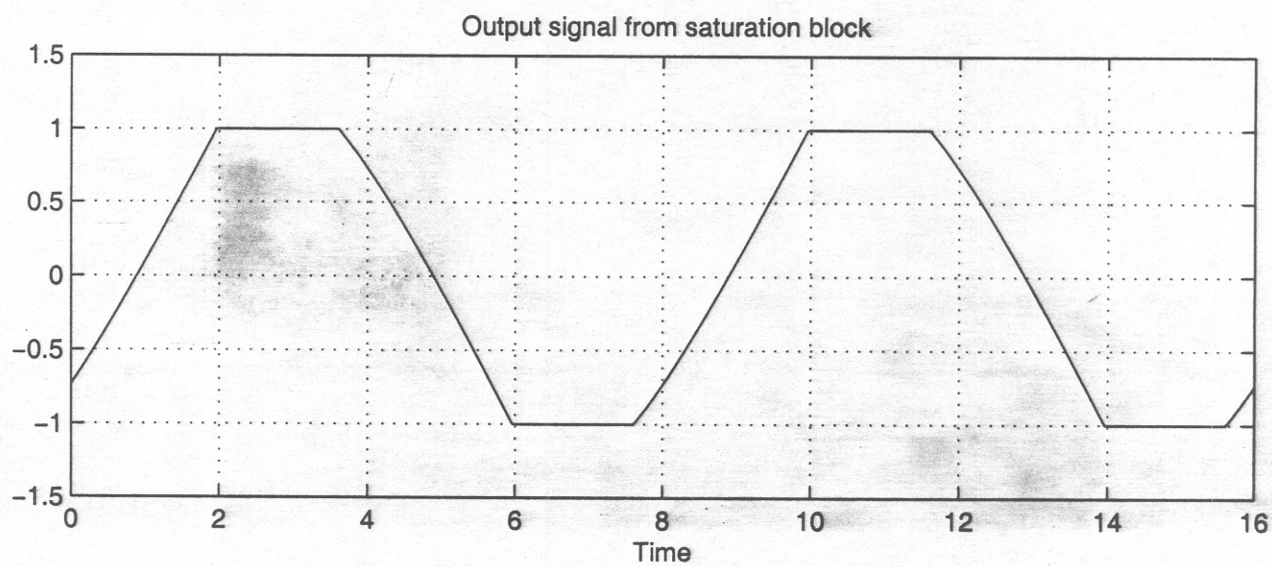
Obviously $\hat{u}_n = 0$, $\forall n = 2K$, $K = 0, \pm 1, \pm 2, \ldots$

$$\hat{u}_{2K+1} = (-1)^{K+1}\cdot\frac{8j}{(2K+1)^2\pi^2}, \quad \forall K = 0, \pm 1, \pm 2, \ldots$$

$\hookrightarrow$ A bit of algebra will convince you that this is true.


Input signal
3
2
1
0
−1
−2
−3
0
2
4
6
8
10
12
14
16


Output signal from LTI system
1.5
1
0.5
0
−0.5
−1
−1.5
0
2
4
6
8
10
12
14
16


Output signal from saturation block
1.5
1
0.5
0
−0.5
−1
−1.5
0
2
4
6
8
10
12
14
16
Time

1° $\underline{x}(K+1) = A(K)\underline{x}(K) + B(K)\underline{u}(K)$

$A(K+N) = A(K)$; $B(K+N) = B(K)$

$\underline{z}(K) = \underline{x}(KN)$

$$\underline{v}(K) = \begin{pmatrix} \underline{u}(KN) \\ \underline{u}(KN+1) \\ \vdots \\ \underline{u}(KN+N-1) \end{pmatrix}$$

Show that $\underline{z}(K+1) = F \cdot \underline{z}(K) + G \cdot \underline{v}(K)$, where F & G are constant matrices.

$\underline{z}(K) = \underline{x}(KN) \Rightarrow \underline{z}(K+1) = \underline{x}[(K+1)N] \Rightarrow$

$\Rightarrow \underline{z}(K+1) = \underline{x}[KN+N] = \underline{x}[(KN+N-1)+1] =$

$= A(KN+N-1) \cdot \underline{x}(KN+N-1) + B(KN+N-1)\underline{u}(KN+N-1) =$

$= A(KN+N-1) \cdot \{A(KN+N-2)\underline{x}(KN+N-2) + B(KN+N-2)\underline{u}(KN+N-2)$

$+ B(KN+N-1) \cdot \underline{u}(KN+N-1) =$

$= A(KN+N-1)A(KN+N-2)\underline{x}(KN+N-2) +$

$+ A(KN+N-1) \cdot B(KN+N-2)\underline{u}(KN+N-2) +$

$+ B(KN+N-1)\underline{u}(KN+N-1) =$

$= A(KN+N-1)A(KN+N-2)\{A(KN+N-3) \cdot \underline{x}(KN+N-3) + B(KN+N-3)\underline{u}(KN+N-$

$+ A(KN+N-1)B(KN+N-2)\underline{u}(KN+N-2) +$

$+ B(KN+N-1)\underline{u}(KN+N-1) =$

$= A(KN+N-1)A(KN+N-2)A(KN+N-3)\underline{x}(KN+N-3) +$

$+ A(KN+N-1)A(KN+N-2)B(KN+N-3)\underline{u}(KN+N-3) +$

$+ A(KN+N-1)B(KN+N-2)\underline{u}(KN+N-2) +$

$+ B(KN+N-1)\underline{u}(KN+N-1)$

If we keep on doing this, we can get:

$$\underline{z}(k+1) = A(kN+N-1)\,A(kN+N-2)\cdots A(kN+N-N)\,\underline{x}(kN+N-N) +$$

$$+\,A(kN+N-1)\,A(kN+N-2)\cdots A(kN+N-(N-1))\,B(kN+N-N)\,\underline{u}(kN+N-N) +$$

$$\cdots$$

Equation (1) can be now written in the following way:

$$\underline{z}(k+1) = A(N-1)\,A(N-2)\cdots A(0)\,\underline{x}(kN) +$$

$$+\,A(N-1)\,A(N-2)\cdots A(1)\,B(0)\cdot\underline{u}(kN) +$$

$$+\,A(N-1)\,A(N-2)\cdots A(2)\,B(1)\,\underline{u}(kN+1) +$$

$$\vdots$$

$$+\,A(N-1)\,B(N-2)\,\underline{u}(kN+N-2) +$$

$$+\,B(N-1)\cdot\underline{u}(kN+N-1) \qquad \ldots (2)$$

Taking into acount eq. (2), the fact that:

$$\left.\begin{aligned} &\underline{x}(kN) = \underline{z}(k), \\ &\underline{v}(k) = \begin{pmatrix} \underline{u}(kN) \\ \underline{u}(kN+1) \\ \vdots \\ \underline{u}(kN+N-2) \\ \underline{u}(kN+N-1) \end{pmatrix} \end{aligned}\right\} \qquad (3)$$

(2-1)

and defining matrices F and G as:

$$F = A(N-1)A(N-2)\cdots A(0) \quad \cdots (4)$$

$$G = [A(N-1)\cdots A(1)B(0);\ A(N-1)\cdots A(2)B(1);\ \ldots;\ A(N-1)B(N-2);\ B(N-1)] \quad \cdots (5)$$

one can get: $\{[(3),(4),(5)] \rightarrow (2)\}$

$$\boxed{\underline{z}(k+1) = F\cdot \underline{z}(k) + G\cdot \underline{v}(k)}$$

It is obvious that matrices F and G do not depend on k, i.e. that they are constant matrices.

Problem 5: see Brogan section 9.8

4° $\dot{\underline{x}}(t) = e^{-tA}\cdot B\cdot e^{tA}\cdot \underline{x}(t)$ ... (1) [redacted]

Solution of (1) is given by

$\underline{x}(t) = e^{-tA}\cdot e^{(t-t_0)(A+B)} e^{t_0 A}\cdot \underline{x}(t_0)$ ... (2) ?

(2) is a solution of (1) iff:

1° $\underline{x}(t_0) = \underline{x}(t_0)$

2° (2) satisfies (1) for every t

1° $\boxed{\underline{x}(t_0) \overset{?}{=}}\ e^{-t_0 A}\cdot e^{(t_0-t_0)(A+B)}\cdot e^{t_0 A}\cdot \underline{x}(t_0) =$

$$= e^{-t_0 A}\cdot e^{0\cdot(A+B)}\cdot e^{t_0 A}\cdot \underline{x}(t_0) =$$

$$= e^{-t_0 A}\cdot I\cdot e^{t_0 A}\cdot \underline{x}(t_0) =$$

$$= e^{-t_0 A}\cdot e^{t_0 A}\cdot \underline{x}(t_0) = e^{(-t_0+t_0)\cdot A}\cdot \underline{x}(t_0) = e^{0\cdot A}\cdot \underline{x}(t_0)$$

$$= I\cdot \underline{x}(t_0) \boxed{\overset{?}{=} \underline{x}(t_0)} \checkmark$$

2° $\dot{\underline{x}}(t) = \left\{ \frac{d}{dt}(e^{-tA})\cdot e^{(t-t_0)(A+B)} + e^{-tA}\cdot \frac{d}{dt}\left(e^{(t-t_0)(A+B)}\right)\right\} e^{t_0 A}\underline{x}(t_0)$ ...(3)

$$\boxed{\frac{d}{dt}\left(e^{-tA}\right) \overset{?}{=}} \frac{d}{dt}\left\{\sum_{n=0}^{\infty}\frac{1}{n!}(-tA)^n\right\} =$$

$$= \frac{d}{dt}\left\{ I - tA + \frac{1}{2!}t^2A^2 - \frac{1}{3!}t^3A^3 + \frac{1}{4!}t^4A^4 - \dots \right\} =$$

$$= \left\{ 0 - A + tA^2 - \frac{1}{2!}t^2A^3 + \frac{1}{3!}t^3\cdot A^4 - \dots \right\} =$$

$$= -A\cdot\left\{ I - tA + \frac{1}{2!}t^2A^2 - \frac{1}{3!}t^3A^3 + \dots \right\} =$$

$$\boxed{\overset{?}{=} -Ae^{-tA}} \ \dots(4)$$

$$\boxed{\frac{d}{dt}\left\{e^{(t-t_0)(A+B)}\right\} \overset{?}{=}} \frac{d}{dt}\left\{\sum_{n=0}^{\infty}\frac{1}{n!}\cdot(t-t_0)^n\cdot(A+B)^n\right\} =$$

$$= \frac{d}{dt}\left\{ I + (t-t_0)(A+B) + \frac{1}{2!}(t-t_0)^2(A+B)^2 + \frac{1}{3!}(t-t_0)^3(A+B)^3 + \dots \right\}$$

$$= \left\{ 0 + (A+B) + (t-t_0)(A+B)^2 + \frac{1}{2!}(t-t_0)^2(A+B)^3 + \dots \right\} =$$

$$= (A+B)\cdot\left\{ I + (t-t_0)(A+B) + \frac{1}{2!}(t-t_0)^2(A+B)^2 + \cdots \right\} =$$

$$= \boxed{(A+B)\cdot e^{(t-t_0)(A+B)}} \quad \ldots (5)$$

$[(4),(5)] \to (3) \Rightarrow$

$$\boxed{\dot{\underline{x}}(t)} = \left\{ -A e^{-tA}\cdot e^{(t-t_0)(A+B)} + e^{-tA}\cdot(A+B)\cdot e^{(t-t_0)(A+B)} \right\} e^{t_0 A}\underline{x}(t$$

$$= \left\{ -A\cdot e^{-tA} + e^{-tA}(A+B) \right\}\cdot e^{(t-t_0)(A+B)}\cdot e^{t_0 A}\underline{x}(t_0) =$$

$$= \left\{ -A e^{-tA} + e^{-tA}\cdot A + e^{-tA}\cdot B \right\}\cdot e^{(t-t_0)(A+B)}\cdot e^{t_0 A}\underline{x}(t_0) =$$

IF
$$\left.\begin{aligned} A\cdot e^{-tA} &= A\cdot\sum_{n=0}^{\infty}\frac{1}{n!}(-t\cdot A)^n = \sum_{n=0}^{\infty}\frac{1}{n!}(-t)^n A\cdot A^n = \sum_{n=0}^{\infty}\frac{1}{n!}(-t)^n\cdot A^{n+1} \\ e^{-tA}\cdot A &= \left\{\sum_{n=0}^{\infty}\frac{1}{n!}(-tA)^n\right\}\cdot A = \sum_{n=0}^{\infty}\frac{1}{n!}(-t)^n\cdot A^n\cdot A = \sum_{n=0}^{\infty}\frac{1}{n!}(-t)^n\cdot A^{n+1} \end{aligned}\right\} =$$

$$\Rightarrow -A\cdot e^{-tA} + e^{-tA}\cdot A = 0$$

$$= \boxed{e^{-tA}\cdot B\cdot e^{(t-t_0)(A+B)}\cdot e^{t_0 A}\cdot\underline{x}(t_0)} \quad \ldots (6)$$

$(2) \to (1) \Rightarrow$

$$\boxed{\dot{\underline{x}}(t)} = e^{-tA} B \underbrace{e^{tA}\cdot e^{-tA}}_{I}\cdot e^{(t-t_0)(A+B)}\cdot e^{t_0 A}\underline{x}(t_0) =$$

$$= \boxed{e^{-tA}\cdot B\cdot e^{(t-t_0)(A+B)}\cdot e^{t_0 A}\cdot\underline{x}(t_0)} \quad \ldots (7)$$

$(6) \equiv (7) \Rightarrow$ (2) is a solution for (1)