Due Wednesday 11/12/08

1. Give, if possible, an example of the following
   (a) A continuous-time signal with bounded energy and unbounded magnitude.
   (b) A continuous-time signal with bounded energy, unbounded action.
   (c) A discrete-time signal with bounded energy and unbounded power.
   (d) A discrete-time signal with bounded action and unbounded magnitude.
   (e) A linear time-invariant discrete-time system that is BIBO stable and ℓ_2 unstable.
   (f) A continuous-time linear time-invariant state space system that is asymptotically stable and BIBO unstable.
   (g) A discrete-time linear time-invariant state space system that is asymptotically stable and BIBO unstable.
   (h) A discrete-time linear time-invariant state space system where the origin is stable in the sense of Lyapunov and BIBO unstable.

2. Determine the H_∞, H_2, and L_1 norms of the following systems:
   (a) $H(s) = \frac{1}{s+a}$, with $a > 0$. How do these norms compare to each other for different values of a? What happens for $a = 0$?
   (b) $$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1 + u, \\ y &= x_1. \end{aligned}$$

3. Write a MATLAB program to compute the H_∞ norm of a SISO transfer function using the grid (in frequency) method. Test your program on the function
   $$H(s) = \frac{1}{s^2 + 10^{-6}s + 1},$$
   and compare your answer to the exact solution (computed by hand using the definition of the H_∞ norm).

4. Consider the system parameterized by k and R
   $$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -(1+k^2)/R & 0 \\ k & -(2+k^2)/R \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \\ y &= x_2. \end{aligned}$$
   (a) For what values of k and R is this system stable?
   (b) Derive the formula for the H_2 norm of this system as a function of k and R. Using this formula, plot the H_2 norm as a function of k for $R = 1$ and $R = 1000$, and as a function of R for $k = 2$.
   (c) Find the solution of the unforced system (i.e. determine operator $G(t)$ that maps initial conditions to the output $y(t)$, $y(t) = G(t)x_0$).
   (d) Plot the maximal singular value of $G(t)$ as a function of time (on time interval $t \in (0, 1000)$) for two different cases: a) $R = 1000$, $k = 0$; b) $R = 1000$, $k = 2$. How do these two cases compare to each other. Explain the obtained results.