Due Tu 09/18/12 (at the beginning of the class)

1. Problem 2.1 from the book (page 19).

2. Problem 2.2 from the book (page 20).

3. Consider the unforced mass-spring system

$$m \ddot{y} \, + \, g(y) \, = \, 0$$

with three different models for the spring force

- **hardening spring:** $g(y) \, = \, k \left(1 \, + \, y^2\right) y$;
- **softening spring:** $g(y) \, = \, k \left(1 \, - \, y^2\right) y$;
- **linear spring:** $g(y) \, = \, k \, y$,

and $k > 0$.

(a) Determine a state-space representation of this system.

(b) Find equilibrium points of the above systems. Discuss your observations for three different spring force models.

(c) Is this system

- causal,
- time-varying,
- linear,
- memoryless,
- finite-dimensional?

Explain.

(d) For three different spring force models with $m = k = 1$, use Matlab to simulate systems' responses from different initial conditions. Plot corresponding results in the phase plane (horizontal axis determined by position $y(t)$, vertical axis determined by velocity $\dot{y}(t)$) and discuss your observations.