Due Tuesday 10/23/12

1. Consider the following system

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -\lambda_1 & 0 \\ R & -\lambda_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \end{aligned} \tag{S}$$

where x_1 and x_2 are the states, u is the input, and y is the output.

(a) Determine conditions for stability of this system.

(b) Determine if the matrix A is normal. Find the right and the the left eigenvectors of the matrix A (once you determine the right eigenvectors, you should normalize the left eigenvector to ensure bi-orthogonality).

(c) Determine the expression for the system's output arising from the initial conditions in x_1 and x_2. You should use the results obtained in part (b) here.

(d) Sketch output components determined in part (c) for $\lambda_1 = 1$ and $\lambda_2 = 2$. How do your results change if R is increased? Explain your observations.

(e) Determine times at which components of $y(t)$ caused by $x_1(0)$ and $x_2(0)$ achieve their respective peaks.

(f) Assuming stability of system (S) and positivity of R, determine values of uncertain parameters a and b for which stability is preserved in the presence of modeling uncertainty, $u(s) = (a + b/s)y(s) + d(s)$, where d denotes a process disturbance.

2. Consider the second-order system

$$\ddot{y} + g(y)\,\dot{y} + y = 0,$$

with equilibrium point $y = \dot{y} = 0$. Determine for which values of $g(0)$ local linearization around this equilibrium point will be unstable, stable, or marginal stable.

3. Problem 8.4 from the book, parts (a), (b) and (c) (page 78). What can be said about the stability of this system?

4. Problem 9.4 from the book, parts (a), (b) (page 86).