Due Friday 11/16/12

1. Many systems can be put in the form of Lienard's equation

$$\ddot{x}(t) + f(x(t))\,\dot{x}(t) + g(x(t)) = 0$$

where

- f is an even function of $x(t)$, and $f(\cdot) \geq 0$
- g is a monotonically increasing function and $g(0) = 0$.

For such systems a Lyapunov function can be constructed as,

$$V(x) = \int_0^{x_1} g(\tau)\,\mathrm{d}\tau + \frac{1}{2}\,x_2^2.$$

Based on this, investigate the stability of Van der Pol's equation,

$$\ddot{x}(t) + \epsilon\,(x^2(t) - 1)\,\dot{x}(t) + x(t) = 0.$$

2. The Morse oscillator is a model that is frequently used in chemistry to study reaction dynamics. The equations for an unforced Morse oscillator are given by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\mu(e^{-x_1} - e^{-2x_1}). \end{aligned}$$

(a) Find the equilibrium points of the system.

(b) Investigate their stability properties.

3. Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= -\frac{x_2}{1+x_1^2} - 2x_1, \\ \dot{x}_2 &= \frac{x_1}{1+x_1^2}. \end{aligned}$$

(a) Show that the origin is an equilibrium point.

(b) Using the candidate Lyapunov function

$$V(x) = x_1^2 + x_2^2,$$

what are the stability properties of the equilibrium point?

(c) Linearize the nonlinear system around the equilibrium point.

(d) What can you deduce about the stability properties of the origin using Lyapunov indirect method?

(e) Obtain a suitable Lyapunov function by solving the Lyapunov equation

$$A^T P + PA = -Q,$$

where

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

4. Problem 8.6 from the book (page 78).

5. Determine the H_∞, H_2, and L_1 norms of the following systems:

(a) $H(s) = \frac{1}{s+a}$, with $a > 0$. How do these norms compare to each other for different values of a? What happens for $a = 0$?

(b)
$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1 + u, \\ y &= x_1. \end{aligned}$$