EE/AEM 5231 09/04/12

"linear systems & optimal control"

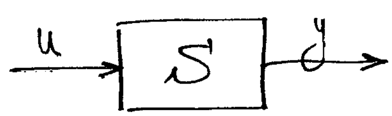
<u>Linear systems</u> (and <u>optimal control</u>)

We want to study systems!

We will take a fairly broad view of a system

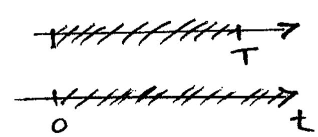
System : A mapping from input signals to output signals

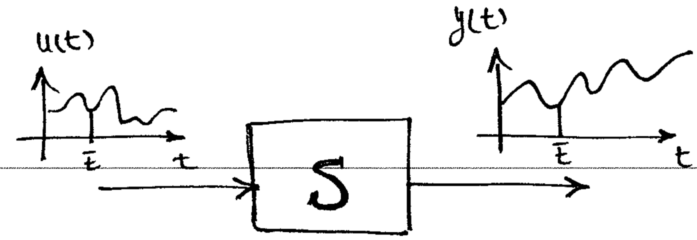
$$\text{Input signals} \xrightarrow{\;S\;} \text{Output signals}$$



Signal : a function of time

time
- continuous; $t \in [0, T]$
  - $t \in [0, \infty)$
- discrete;
  - $t = 0, 1, 2, \ldots$
  - $t \in \mathbb{N}_0$
  - $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$



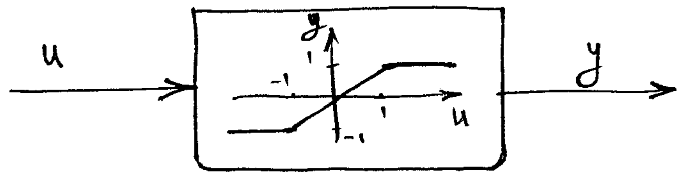


$$y = Su$$

$$y(t) = [\mathbf{S}u](t)$$

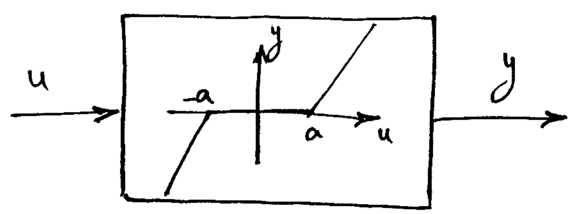
u : input signal
y : output signal

Ex 1



$$y(t) = [Su](t) = [Sat\{u\}](t) = \begin{cases} -1 & u(t) < -1 \\ u(t) & |u(t)| \leq 1 \\ +1 & u(t) > +1 \end{cases}$$

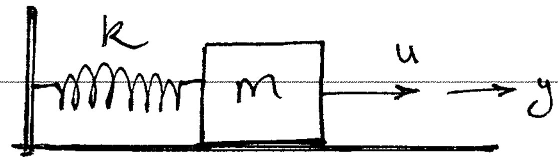
Ex 2



$$y(t) = [Su](t) = \begin{cases} u(t) - a & u(t) > a \\ 0 & |u(t)| \leq a \\ u(t) + a & u(t) < -a \end{cases}$$

dead-zone
soft-thresholding
shrinkage

Ex 3



u: external force acting on mass

y: position of mass

a good indicator of number of initial conditions needed.

$$m\frac{d^2y}{dt^2} + ky = u$$

m & k → known

but we need some additional information to solve the problem!

initial conditions $y(0) = y_0$ initial position

$\dot{y}(0) = \frac{dy}{dt}(t_0)$ initial velocity

→ We want to introduce certain properties that will characterize systems.

1) Linearity
2) Time invariance
3) causality
4) Memory [static or dynamic?]

static: ex) $V = RI$

$y(t) = Ru(t)$

dynamic: ex) diff. eq.

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Stability, more advanced discussions in a few weeks

Linearity

$$u \longrightarrow \boxed{S} \longrightarrow y$$

$$y(t) = [Su](t)$$

(1) $\bar{u}(t) = \alpha \cdot u(t)$; α scalar

$$\bar{y}(t) = [S\bar{u}](t) = [S\,\alpha \cdot u](t) = \alpha \cdot [Su](t) = \alpha \cdot y(t) \qquad \text{homogeneity}$$

(2)
$$\left.\begin{array}{l} \bar{u}(t) = u_1(t) + u_2(t) \\ y_i(t) = [S u_i](t) \\ \quad i = 1,2 \end{array}\right\} \Rightarrow \bar{y}(t) = y_1(t) + y_2(t) \qquad \text{additivity}$$

System is linear if (1) & (2) hold

$\Updownarrow$

if principle of superposition holds

$$* \quad \boxed{\bar{y}(t)} = S[\overbrace{\alpha \cdot u_1 + \beta \cdot u_2}^{\bar{u}}](t) = \alpha[Su_1](t) + \beta[Su_2](t)$$

$$\boxed{= \alpha \cdot y_1(t) + \beta \cdot y_2(t)}$$

α, β scalars

the system in example (2) is not linear while the system in example (3) is.