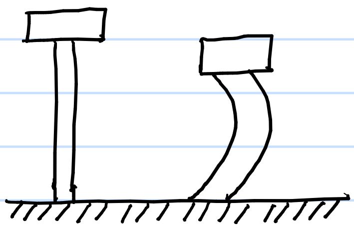
EE 8215 S16 : Nonlinear Systems

Lecture 3 – Jan 26th 2016, Tuesday

Bifurcations

➤ Bifurcations : sudden change in trends in equ^m points or stability as parameters are changed.

Eg ①: A buckling beam : weight on top of a beam



Types of Bifurcations:

1) Fold : $\dot{x} = \alpha \pm x^2$ $\alpha \in \mathbb{R}$: real parameter

2) Transcritical : $\dot{x} = \alpha x \mp x^2$ $x(t) \in \mathbb{R}$

3) Pitchfork : $\dot{x} = \alpha x \mp x^3$

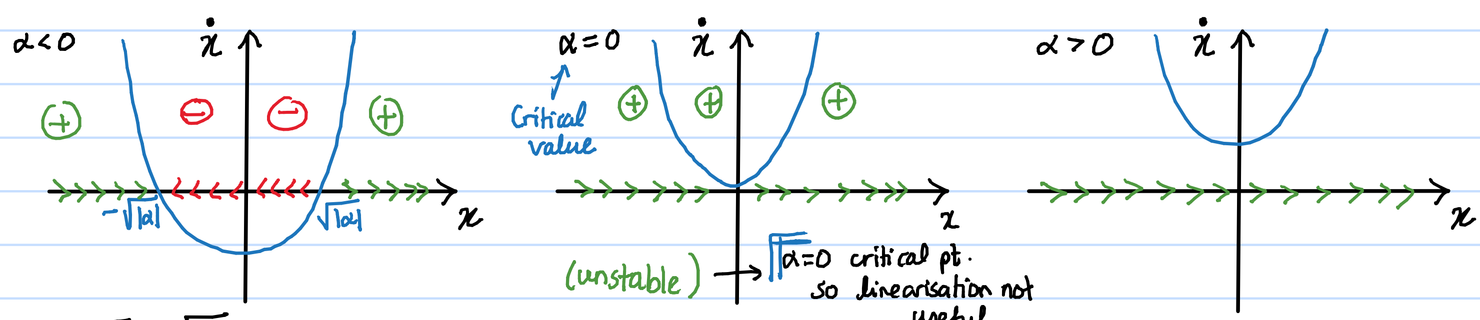
1) Fold :

$$\dot{x} = \alpha + x^2$$

Equ^m pts: $\alpha + \bar{x}^2 = 0$

$$\bar{x}^2 = -\alpha$$

$$\bar{x} = \begin{cases} \pm\sqrt{|\alpha|} ; & \alpha \leq 0 \\ \text{none} ; & \alpha > 0 \end{cases}$$



• $\bar{x} = -\sqrt{|\alpha|}$; $\alpha \leq 0$

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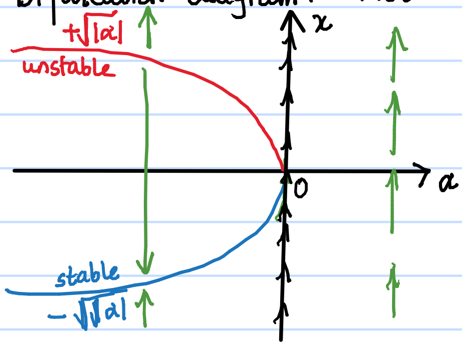
• Linearisation: $\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}} = 2\bar{x} = \begin{cases} -2\sqrt{|\alpha|} & \text{(stable)} \\ +2\sqrt{|\alpha|} & \text{(unstable)} \end{cases}$ ($\alpha < 0$)

- Bifurcation diagram: Plot of x vs. α (state vs. parameter)



2) Transcritical

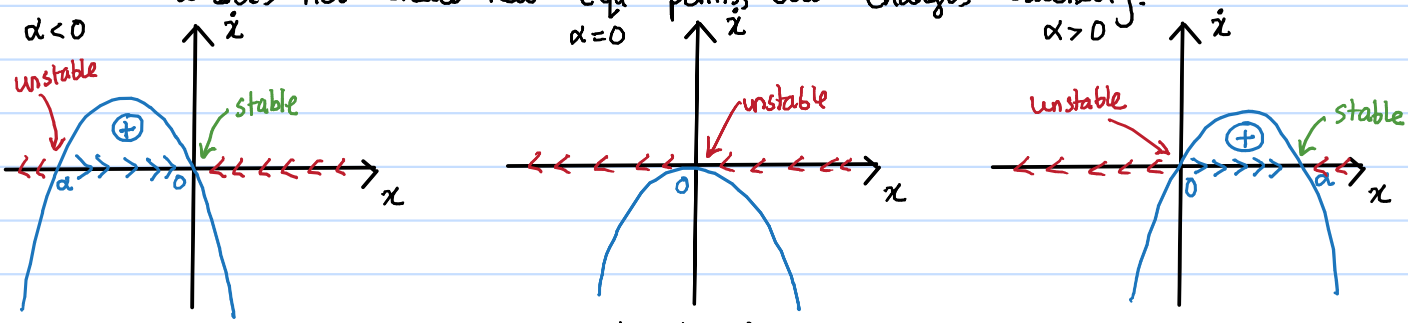
$$\dot{x} = \alpha x - x^2 \quad \Rightarrow \quad f(x) = x(\alpha - x)$$

Equ^m pts. : $\bar{x}(\alpha - \bar{x}) = 0$

$$\bar{x}_1 = 0 \qquad \bar{x}_2 = \alpha \quad \rightarrow \quad \forall\, \alpha \in \mathbb{R}$$

Linearisation: $\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}} = \alpha - 2\bar{x} = \begin{cases} \alpha \;;\; \bar{x} = 0 \\ -\alpha \;;\; \bar{x} = \alpha \end{cases}$

α does not create new equ^m points, but changes stability.



$x(0) > 0 \Rightarrow$ always back to zero

departure from origin

⟶ shrinking region of attraction

Bifurcation diagram