Collaborative Research: Algorithms for design of structured distributed controllers with application to large-scale vehicular formations

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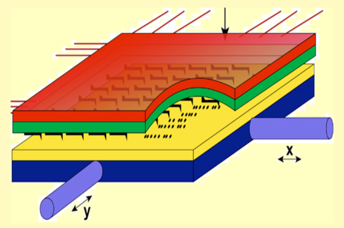
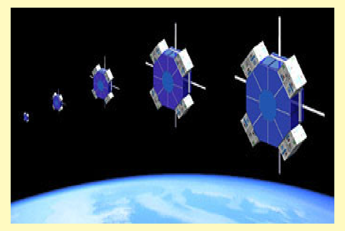
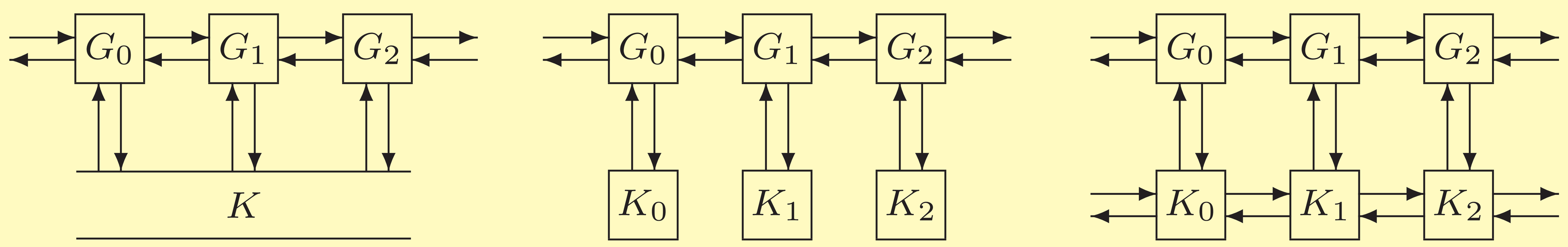
Distributed systems

- **Of increasing importance in modern technology**

wind farms	micro-cantilevers	satellite constellations
		

- **Controller architectures**

CENTRALIZED FULLY DECENTRALIZED LOCALIZED



best performance, excessive communication | worst performance, no communication | many possible architectures

Structured optimal control problem

$$\begin{aligned} \dot{x} &= A\,x + B_1\,d + B_2\,u \\ z &= \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} Q^{1/2}x \\ R^{1/2}u \end{bmatrix} \\ u &= -K\,x \qquad K \in \mathcal{S} \end{aligned}$$

CENTRALIZED FULLY DECENTRALIZED LOCALIZED

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \qquad \begin{bmatrix} \diamond & & & \\ & \star & & \\ & & \triangle & \\ & & & \divideontimes \end{bmatrix} \qquad \begin{bmatrix} * & * & & \\ * & * & * & \\ & * & * & * \\ & & * & * \end{bmatrix}$$

OBJECTIVE: **find stabilizing** $K \in \mathcal{S}$ **that minimizes** $\|\, d \to z \,\|_2^2$

- **Structured minimum variance problem**

$$\begin{aligned} &\text{minimize} \quad \operatorname{trace}\left(P(K) B_1 B_1^T\right) \\ &\text{subject to} \quad (A - B_2K)^T P + P\,(A - B_2K) = -\left(Q + K^T R K\right), \quad K \in \mathcal{S} \end{aligned}$$

- **Necessary conditions for optimality**

$$\begin{aligned} (A - B_2K)^T P + P\,(A - B_2K) &= -\left(Q + K^T R K\right) \\ (A - B_2K)\,L + L\,(A - B_2K)^T &= -B_1 B_1^T \\ \left[(RK - B_2^T P)\,L\right] \circ I_{\mathcal{S}} &= 0 \end{aligned}$$

no constraints $\quad \begin{cases} K_c = R^{-1} B_2^T P \\ A^T P + PA - PB_2R^{-1}B_2^T P + Q = 0 \end{cases}$

Structured optimal design

- **Expensive control of stable open-loop systems**

$$R = (1/\varepsilon)\,I, \quad 0 < \varepsilon \ll 1$$

Perturbation analysis

$$P = \sum_{n=0}^{\infty} \varepsilon^n P_n, \quad L = \sum_{n=0}^{\infty} \varepsilon^n L_n, \quad K = \sum_{n=0}^{\infty} \varepsilon^n K_n$$

$$O(1): \quad K_0 = 0$$

$$O(\varepsilon): \begin{cases} A^T P_0 + P_0 A = -Q \\ A L_0 + L_0 A^T = -B_1 B_1^T \\ [K_1\, L_0] \circ I_{\mathcal{S}} = [B_2^T P_0 L_0] \circ I_{\mathcal{S}} \end{cases}$$

followed by homotopy

- **Augmented Lagrangian method**

$$K = \begin{bmatrix} \diamond & \\ & \star \end{bmatrix} \Rightarrow \Lambda = \begin{bmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{bmatrix}, \quad I_{\mathcal{S}}^c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

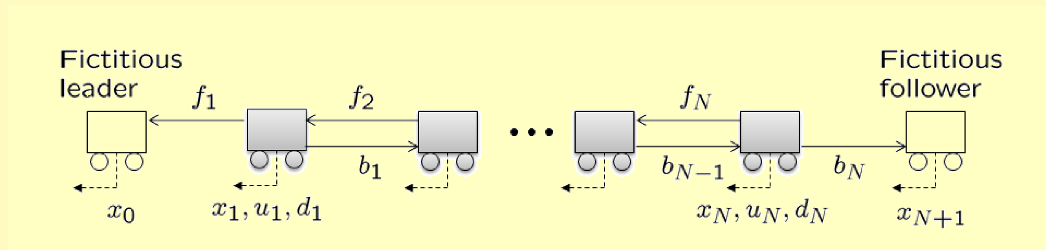
$$\text{minimize} \quad \mathcal{L}_\gamma(K, \Lambda) = J(K) + \operatorname{trace}\left(\Lambda^T K\right) + \gamma\, \|K \circ I_{\mathcal{S}}^c\|_F^2$$

update: $\Lambda_{i+1} = \Lambda_i + \gamma_i\,(K_i \circ I_{\mathcal{S}}^c), \quad \gamma_{i+1} = c\,\gamma_i, \quad c > 1$

stopping criterion: $\|K_i \circ I_{\mathcal{S}}^c\|_F <$ tolerance

Vehicular formations

- **Local feedback design for tight spacing at highway speeds**



RELATIVE POSITION FEEDBACK:

$$\begin{aligned} \dot{\tilde{x}}_n &= \tilde{u}_n + d_n \\ \tilde{u}_n &= -f_n\,(\tilde{x}_n - \tilde{x}_{n-1}) - b_n\,(\tilde{x}_n - \tilde{x}_{n+1}) \\ \tilde{u} &= -K\tilde{x} = -\begin{bmatrix} F_f & F_b \end{bmatrix} \begin{bmatrix} C_f \\ C_b \end{bmatrix} \tilde{x} \end{aligned}$$

$$K \sim \underbrace{\begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix}}_{F_f} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{C_f} + \underbrace{\begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}}_{F_b} \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}_{C_b}$$

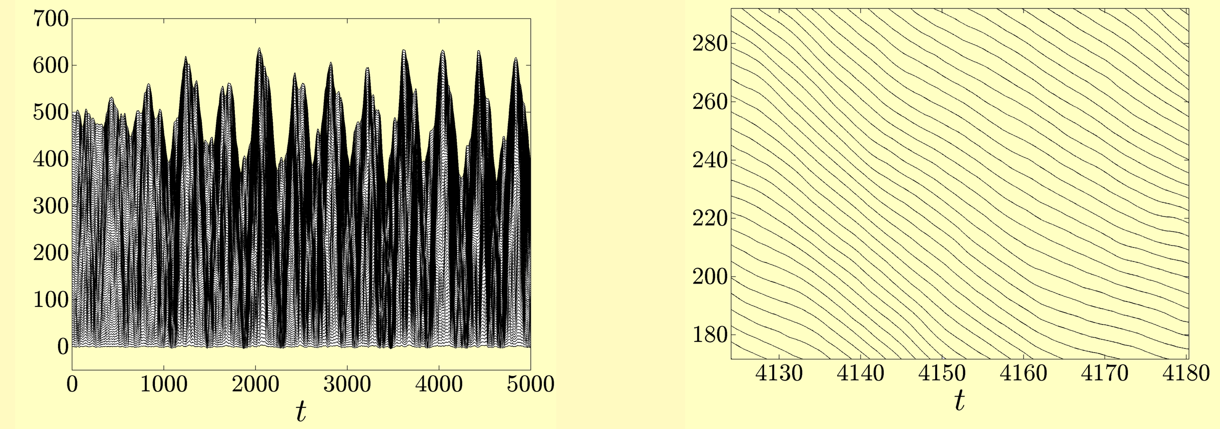
- **Performance measures** $\quad (1/N)\,\|\, d \to z_1 \,\|_2^2$

microscopic: local position deviation $\quad \tilde{x}_n - \tilde{x}_{n-1}, \quad Q_l = C_f + C_b$

macroscopic: global position deviation $\quad \tilde{x}_n, \quad Q_g = I$

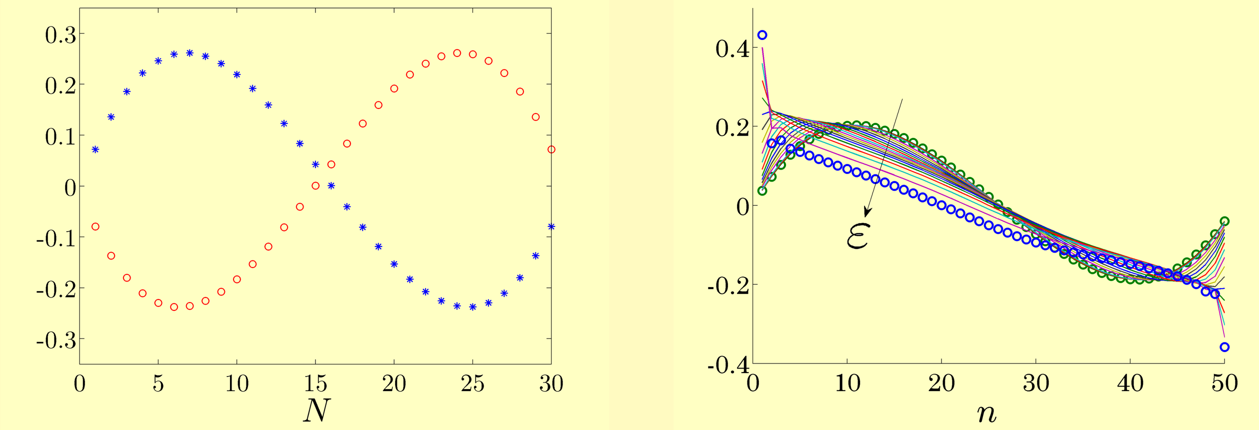
Performance vs. size

- **Incoherence phenomenon**
  double integrators: relative position/velocity fdbk



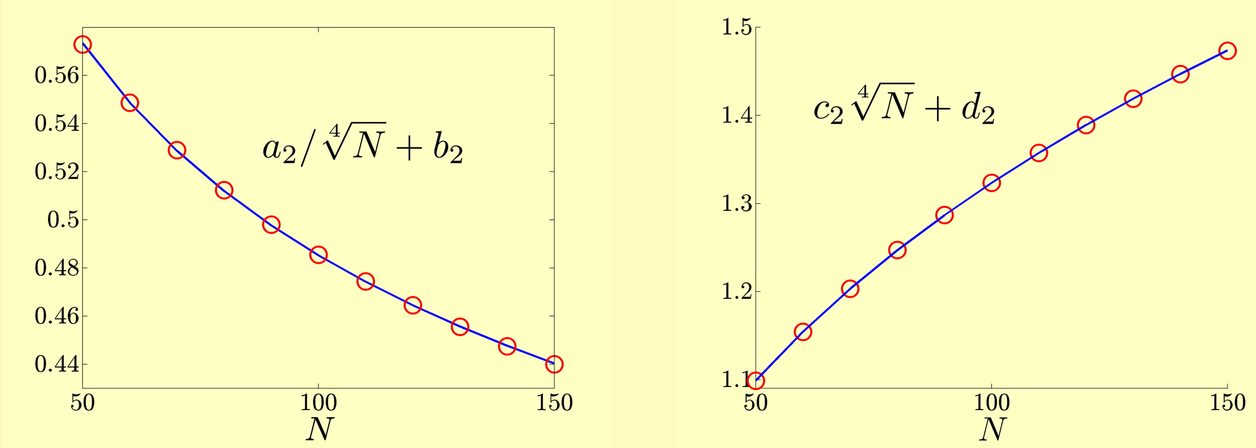
- **Structured optimal design**

perturbation analysis: forward/backward gains homotopy: forward gains

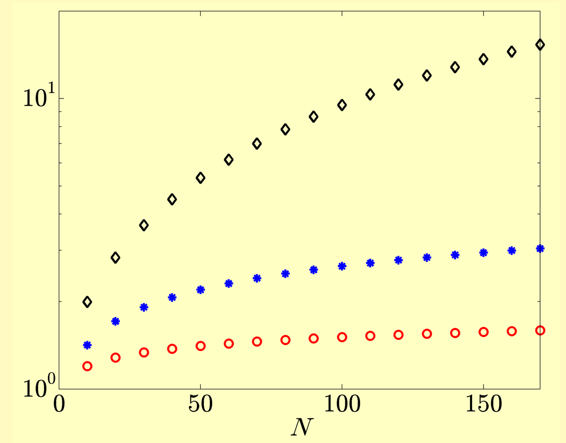


- **Performance vs. size**

microscopic: macroscopic:



- **Macroscopic performance:** spatially uniform vs. structured optimal vs. centralized optimal



Publications

[1] F. Lin, M. Fardad, and M. R. Jovanović, "Augmented Lagrangian approach to design of structured optimal state feedback gains", *IEEE Trans. Automat. Control*, submitted (2010)

[2] F. Lin, M. Fardad, and M. R. Jovanović, "Optimal control of vehicular formations with nearest neighbor interactions", *IEEE Trans. Automat. Control*, submitted (2010)