Dynamics of inertialess flows of viscoelastic fluids: the role of uncertainty

Mihailo Jovanović
www.umn.edu/∼mihailo

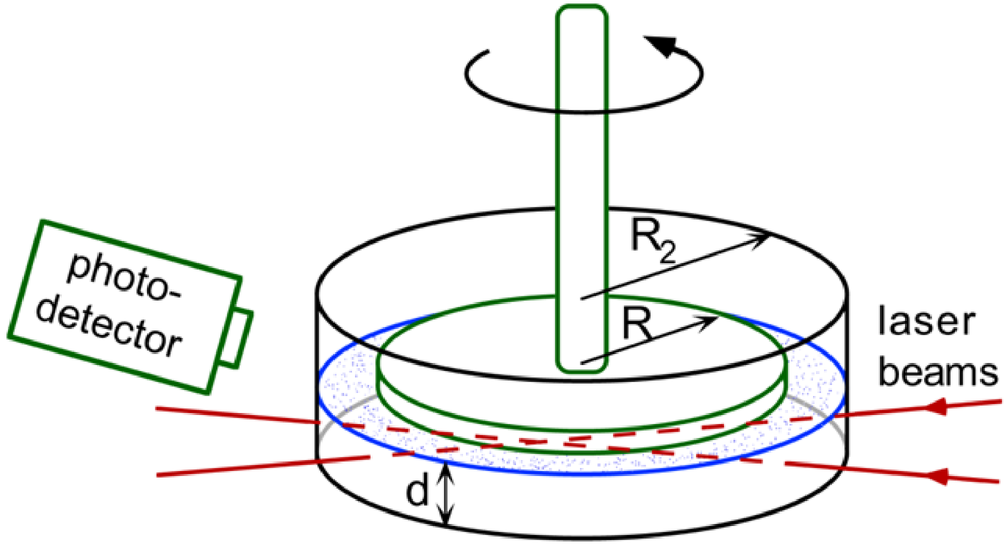
joint work with:
Satish Kumar, Binh Lieu

University of Minnesota

Workshop on Complex Fluids and Flows in Industry and Nature

Turbulence without inertia

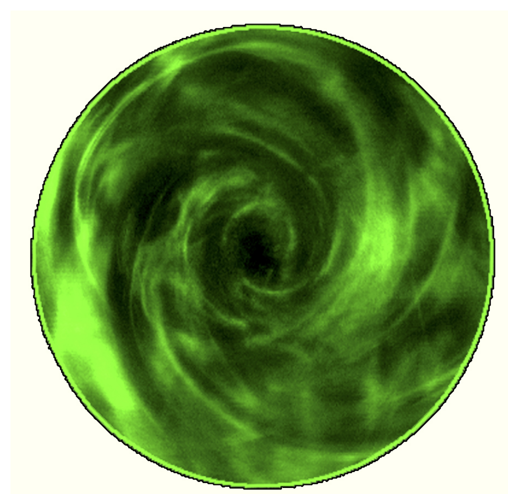
NEWTONIAN: **inertial turbulence** VISCOELASTIC: **elastic turbulence**



Groisman & Steinberg, *Nature '00*

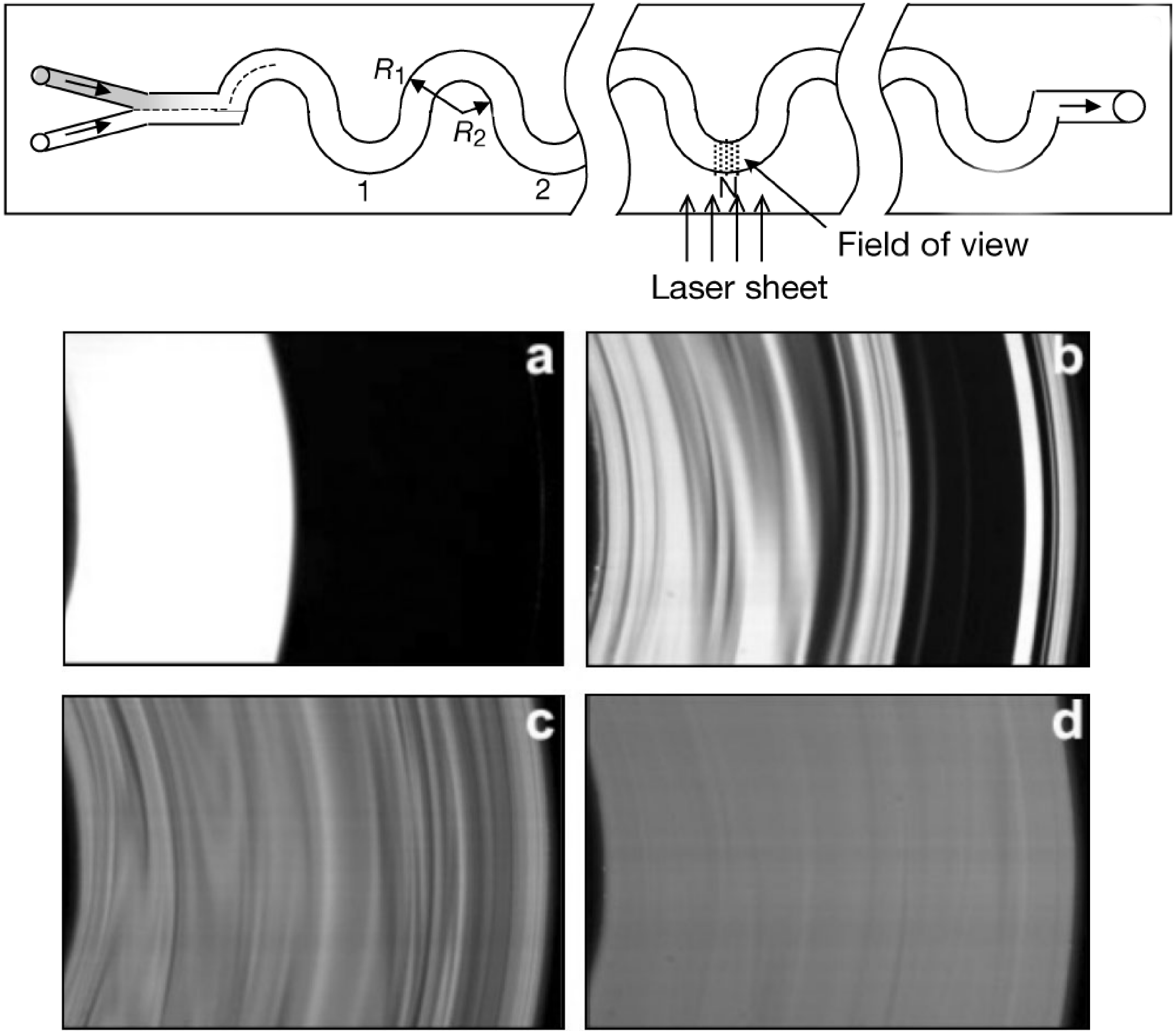
NEWTONIAN:

VISCOELASTIC:



☞ FLOW RESISTANCE: **increased 20 times!**

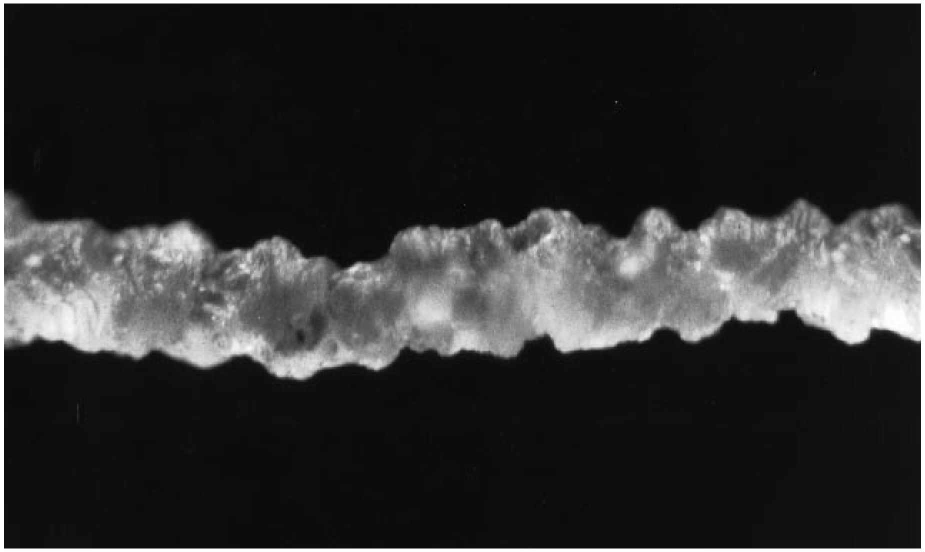
Turbulence: good for mixing . . .



Groisman & Steinberg, *Nature '01*

. . . bad for processing

POLYMER MELT EMERGING FROM A CAPILLARY TUBE

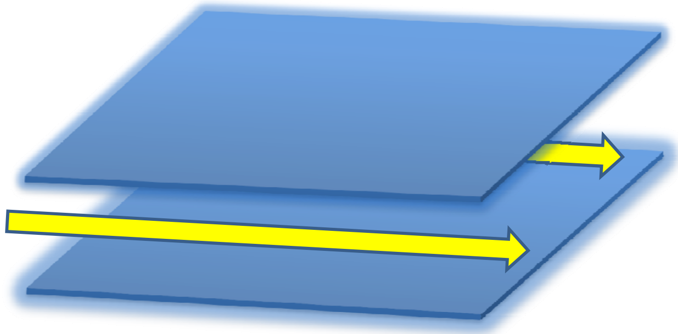


Kalika & Denn, *J. Rheol. '87*

CURVILINEAR FLOWS: **purely elastic instabilities**

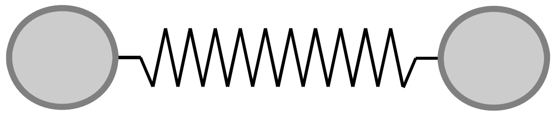
Larson, Shaqfeh, Muller, *J. Fluid Mech. '90*

RECTILINEAR FLOWS: **no modal instabilities**



Oldroyd-B fluids

HOOKEAN SPRING:



$$
\begin{aligned}
(Re/We)\,\partial_t \mathbf{v} &= -Re\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} - \nabla p + \beta\,\Delta\mathbf{v} + (1-\beta)\,\nabla\cdot\boldsymbol{\tau} + \mathbf{d} \\
0 &= \nabla\cdot\mathbf{v} \\
\partial_t\,\boldsymbol{\tau} &= -\boldsymbol{\tau} + \nabla\mathbf{v} + (\nabla\mathbf{v})^T + We\left(\boldsymbol{\tau}\cdot\nabla\mathbf{v} + (\nabla\mathbf{v})^T\cdot\boldsymbol{\tau} - (\mathbf{v}\cdot\nabla)\boldsymbol{\tau}\right)
\end{aligned}
$$

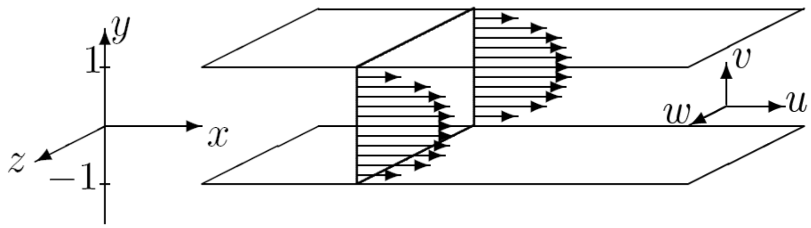
VISCOSITY RATIO: $\beta := \dfrac{\text{solvent viscosity}}{\text{total viscosity}}$

WEISSENBERG NUMBER: $We := \dfrac{\text{fluid relaxation time}}{\text{characteristic flow time}}$

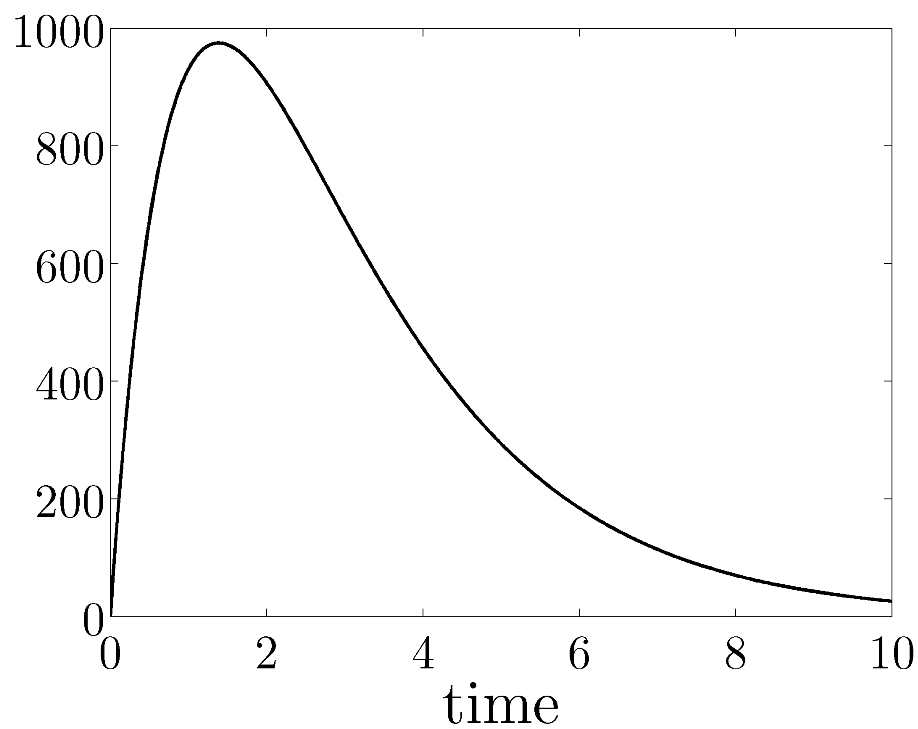
REYNOLDS NUMBER: $Re := \dfrac{\text{inertial forces}}{\text{viscous forces}}$

Transient growth analysis

- STUDY TRANSIENT BEHAVIOR OF FLUCTUATIONS' ENERGY



kinetic energy:



☞ SPECTRUM: **misleading measure of transient response**

Sureshkumar *et al.*, *JNNFM '99*; Atalik & Keunings, *JNNFM '02*;
Kupferman, *JNNFM '05*; Doering *et al.*, *JNNFM '06*;
Renardy, *JNNFM '09*; Jovanović & Kumar, *Phys. Fluids '10*

- **HIGH FLOW SENSITIVITY**
  even in the absence of inertia
  - ⋆ large transient responses
  - ⋆ large noise amplification
  - ⋆ small stability margins

TO COUNTER THIS SENSITIVITY: **must account for modeling imperfections**

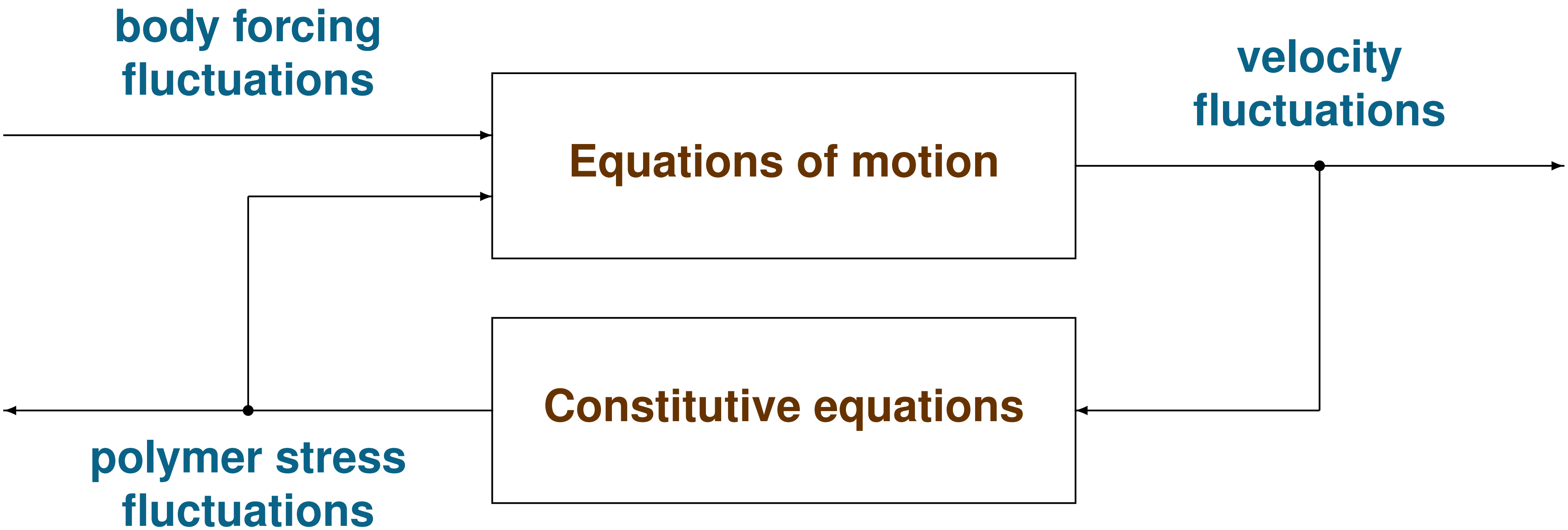
STABILITY + RECEPTIVITY + ROBUSTNESS

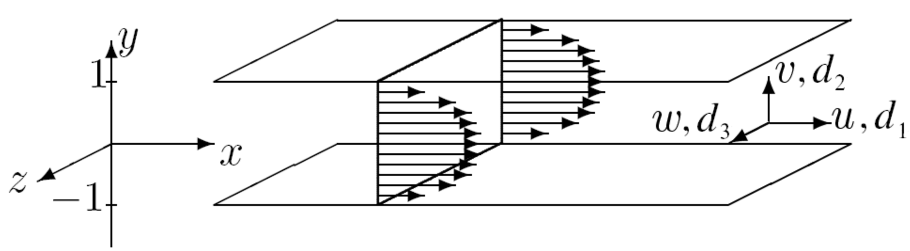
RECEPTIVITY → **flow disturbances**

ROBUSTNESS → **unmodeled dynamics**

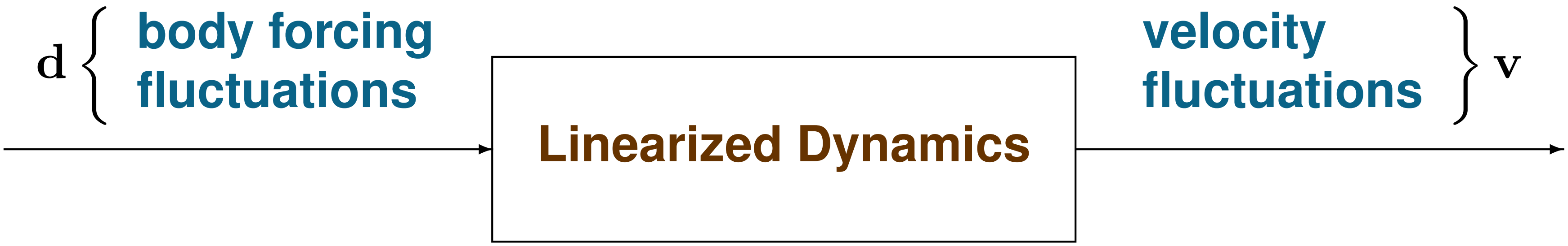
Tools for quantifying sensitivity

- INPUT-OUTPUT ANALYSIS: **spatio-temporal frequency responses**



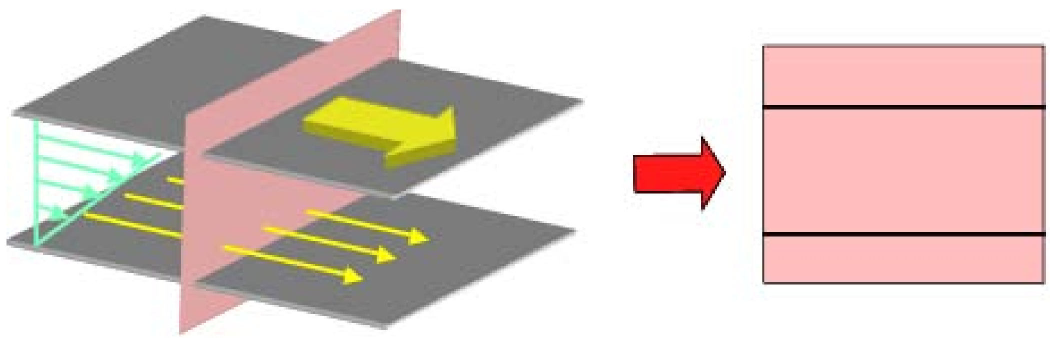


$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$



- INSIGHT INTO AMPLIFICATION MECHANISMS

  importance of streamwise elongated structures



Hoda, Jovanović, Kumar, *J. Fluid Mech. '08, '09*

Jovanović & Kumar, *JNNFM '11*

Non-modal amplification of disturbances

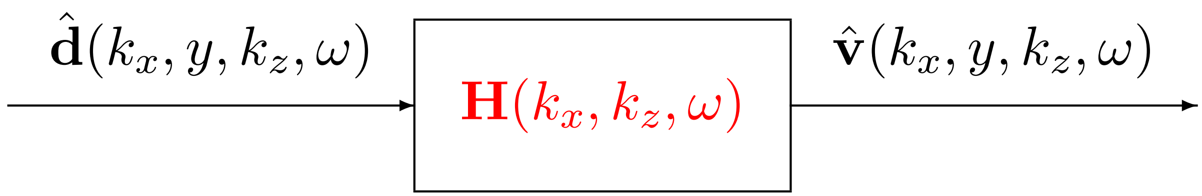
harmonic forcing:

$$\mathbf{d}(x,y,z,t) \;=\; \hat{\mathbf{d}}(k_x,y,k_z,\omega)\, \mathrm{e}^{\mathrm{i}(k_x x \,+\, k_z z \,+\, \omega t)}$$

$\downarrow$ **steady-state response**

$$\mathbf{v}(x,y,z,t) \;=\; \hat{\mathbf{v}}(k_x,y,k_z,\omega)\, \mathrm{e}^{\mathrm{i}(k_x x \,+\, k_z z \,+\, \omega t)}$$

- FREQUENCY RESPONSE



$\mathbf{H}(k_x,k_z,\omega)$: **operator in y**

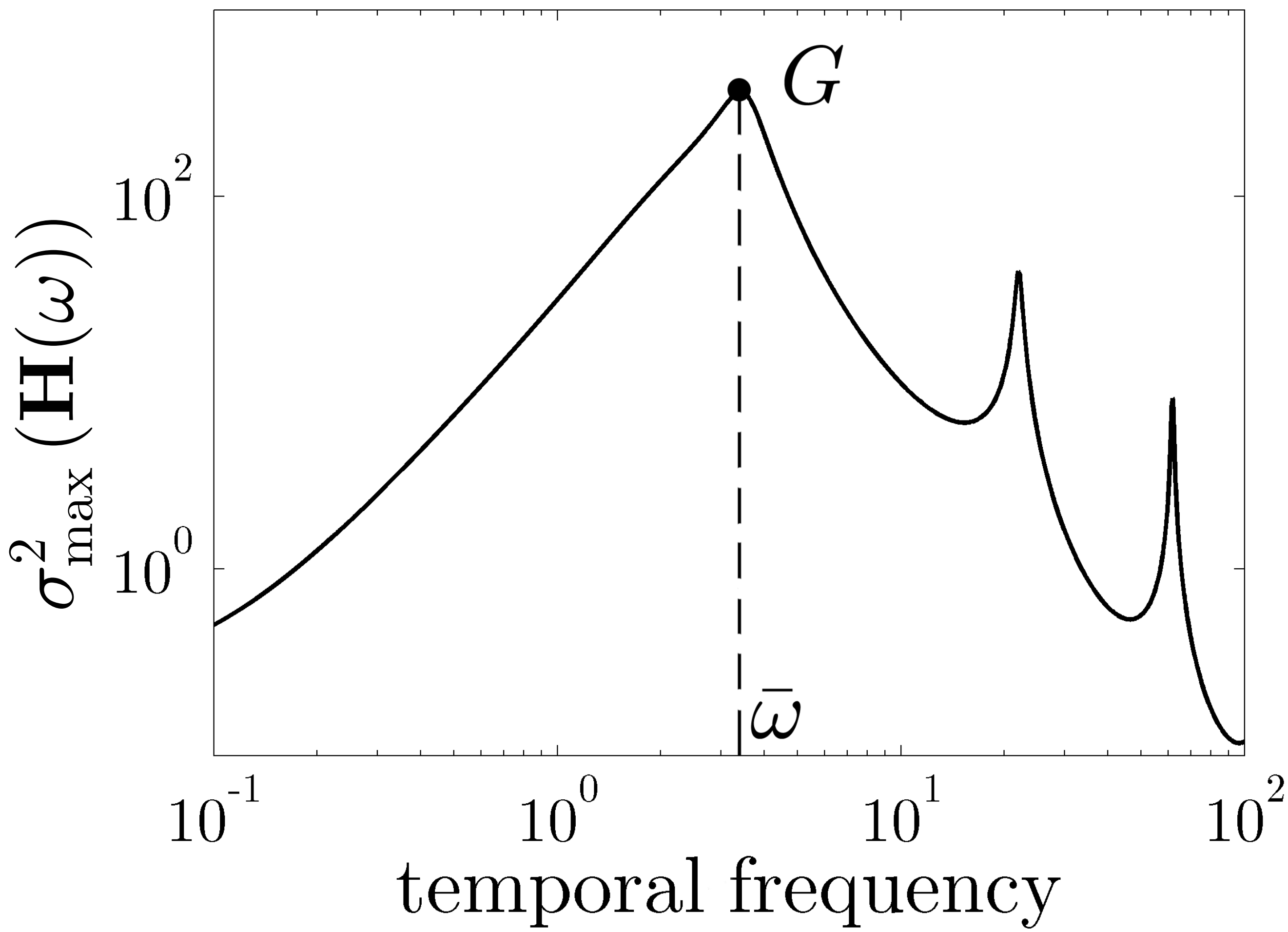
$$\hat{\mathbf{v}}(k_x,y,k_z,\omega) \;=\; \int_{-1}^{1} \mathbf{H}_{\mathrm{ker}}(k_x,k_z,\omega;y,\eta)\, \hat{\mathbf{d}}(k_x,\eta,k_z,\omega)\, \mathrm{d}\eta$$

Input-output gains

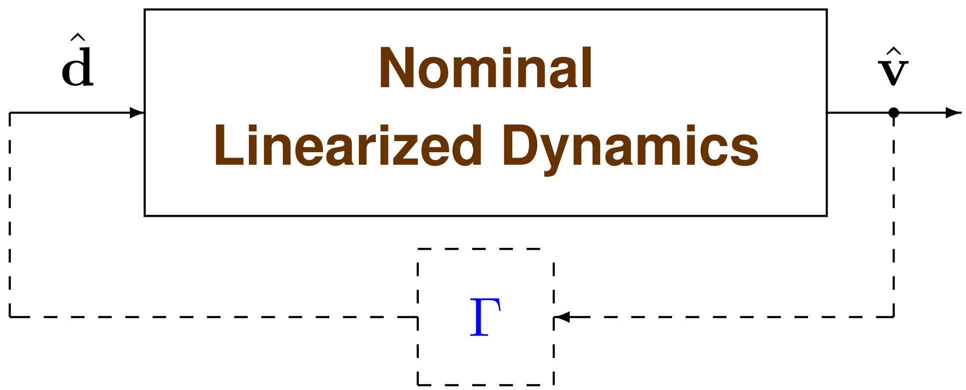
- **Determined by singular values of** $\mathbf{H}(k_x, k_z, \omega)$

worst case amplification:

$$G(k_x, k_z) \; = \; \max \frac{\text{output energy}}{\text{input energy}} \; = \; \max_{\omega} \sigma^2_{\max} \left(\mathbf{H}(k_x, k_z, \omega)\right)$$



Robustness interpretation



modeling uncertainty
(can be nonlinear or time-varying)

small-gain theorem:

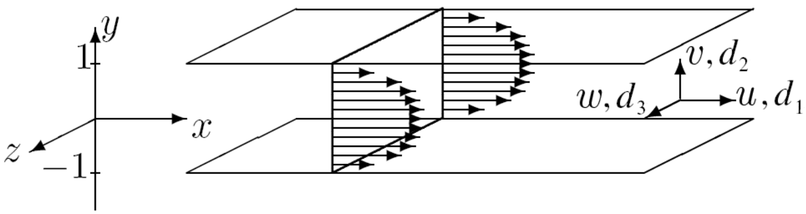
stability for all Γ with

$$\max_{\omega} \sigma_{\max}^2(\Gamma(\omega)) \le \gamma \quad \Leftrightarrow \quad \gamma < \frac{1}{G}$$

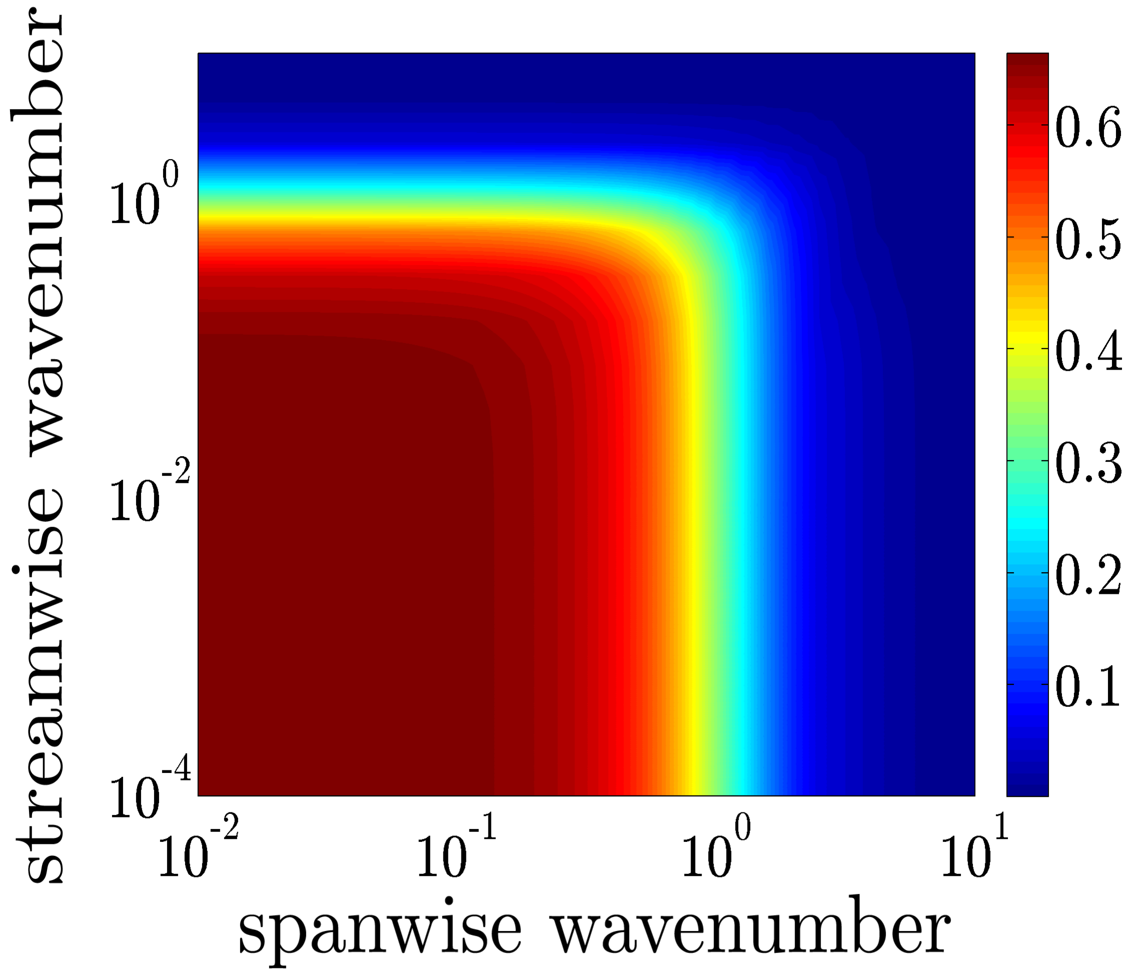
LARGE
worst case amplification $\Rightarrow$ **small stability margins**

Inertialess channel flow: worst case amplification

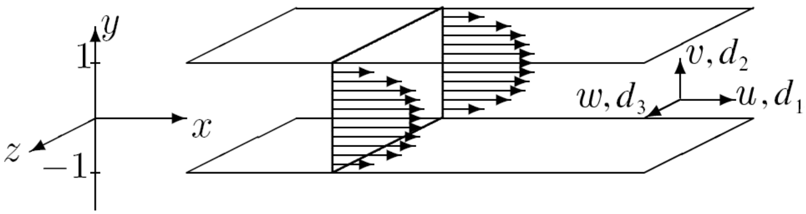
$We = 10$, $\beta = 0.5$, $Re = 0$

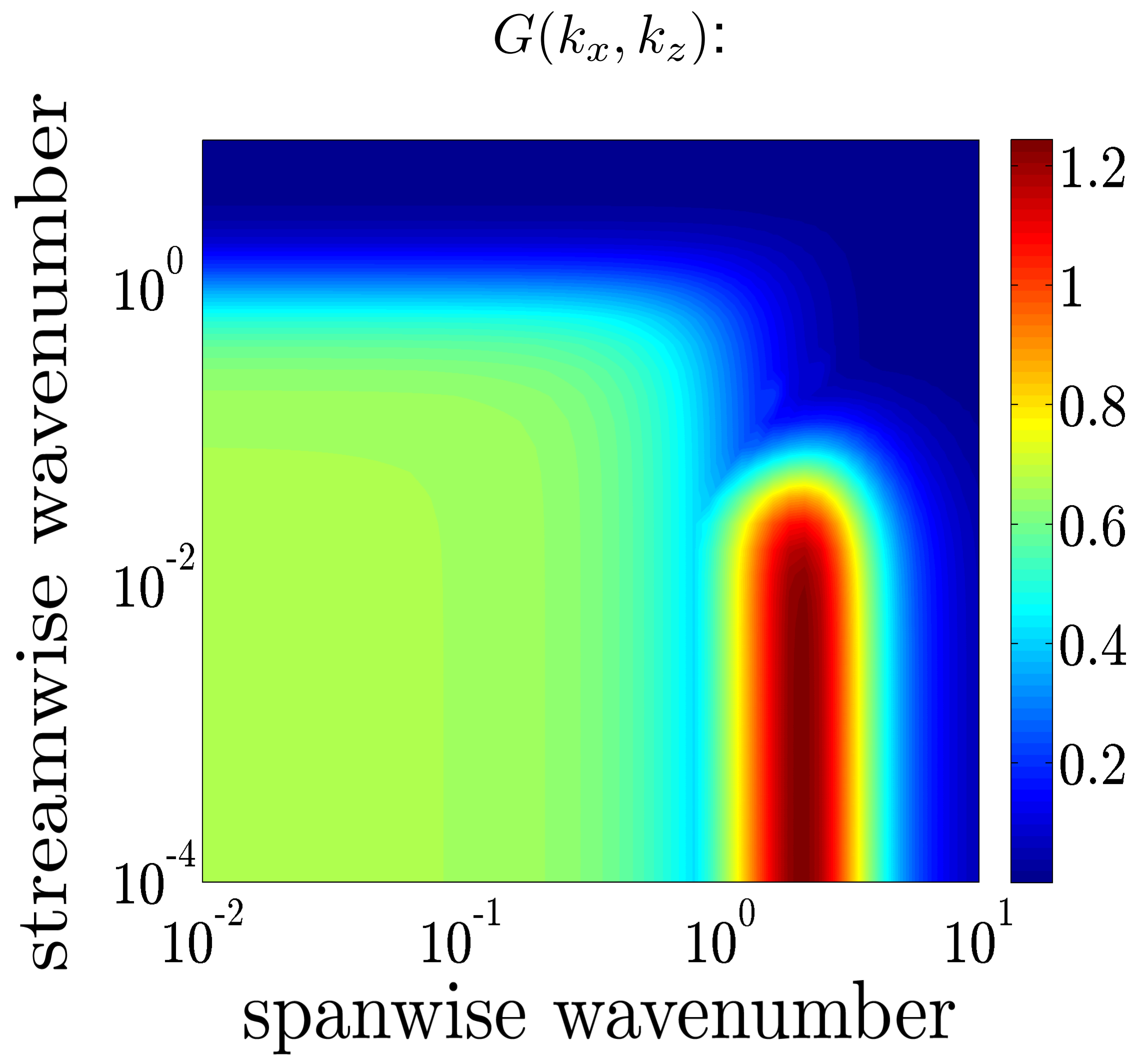


$G(k_x, k_z)$:



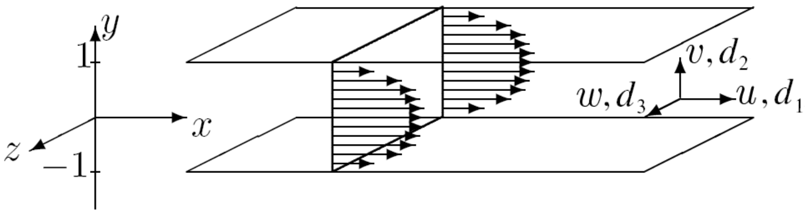
$We = 50$, $\beta = 0.5$, $Re = 0$

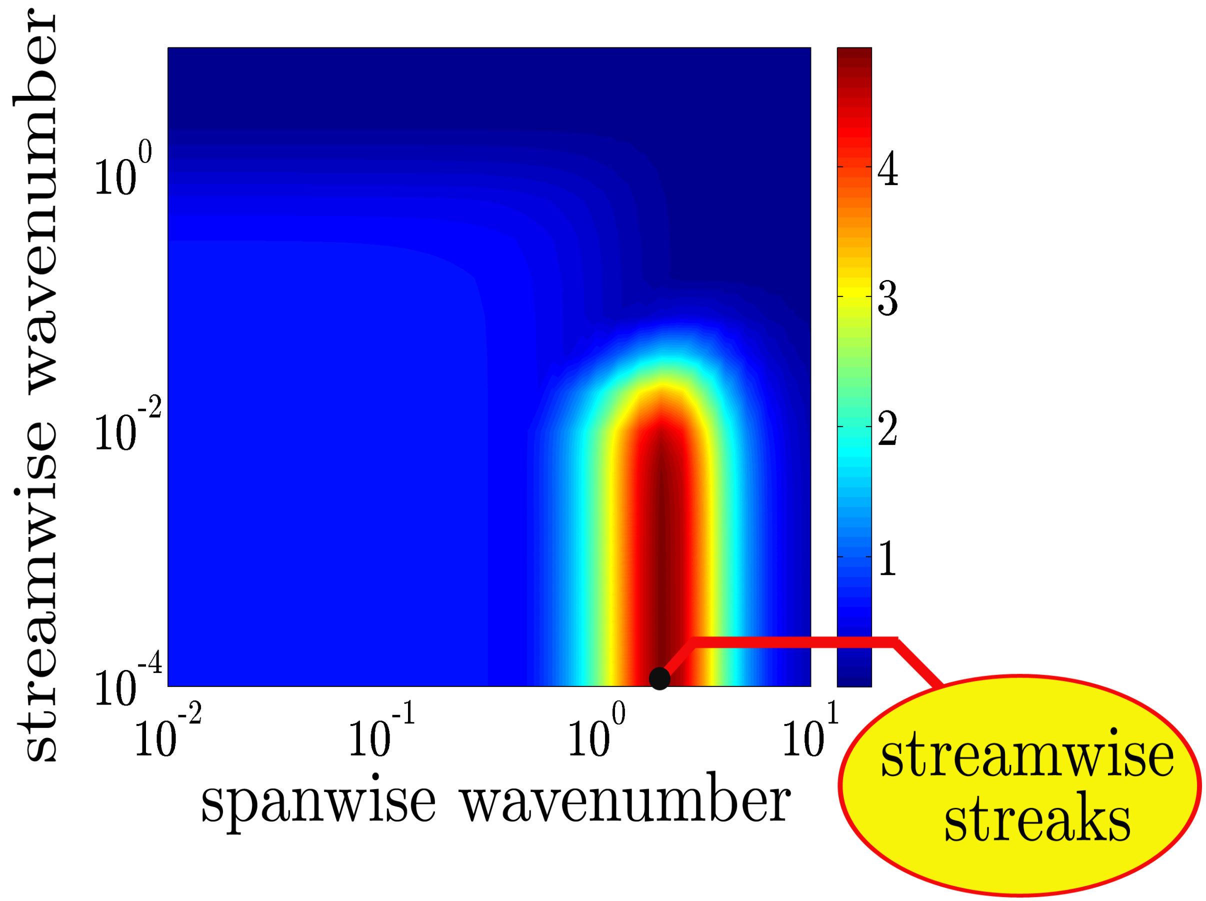




$G(k_x, k_z)$:

$We = 100$, $\beta = 0.5$, $Re = 0$





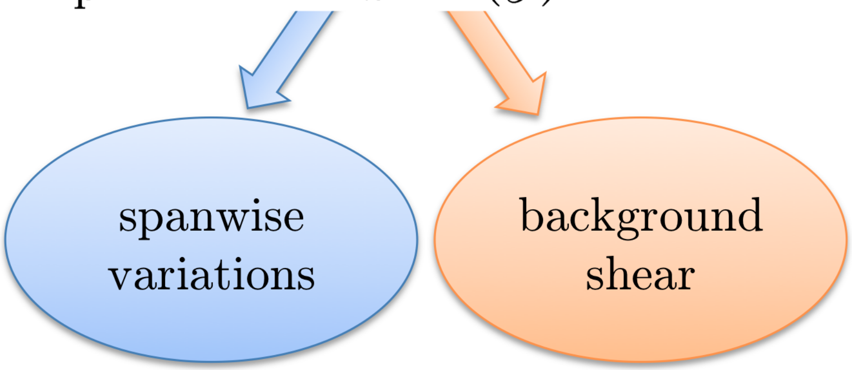
- **Dominance of streamwise elongated structures**
  streamwise streaks!

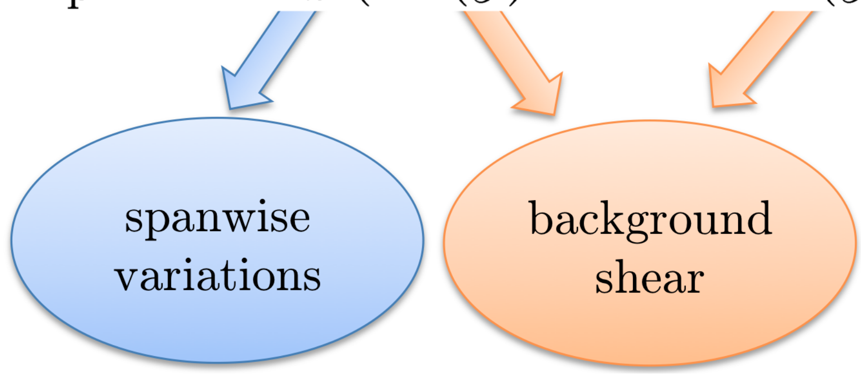
Streamwise-constant flows

LINEARIZED DYNAMICS OF NORMAL VORTICITY η

Inertial Newtonian	Inertialess viscoelastic
$\partial_t \eta \;=\; \Delta\eta \;+\; Re\,\mathbf{C}_{\rm p1}\,v$	$\Delta\,\partial_t \eta \;=\; -(1/\beta)\Delta\eta \;+\; We\,(1 - 1/\beta)\,\mathbf{C}_{\rm p2}\,\vartheta$
$\downarrow$	$\partial_t \vartheta \;=\; -\vartheta + v$ $\qquad\downarrow$
source	**source**

$$\mathbf{C}_{\rm p1} \;=\; -\mathrm{i}k_z\,U'(y) \qquad\qquad \mathbf{C}_{\rm p2} \;=\; \mathrm{i}k_z\,(U'(y)\Delta \;+\; 2U''(y)\partial_y)$$



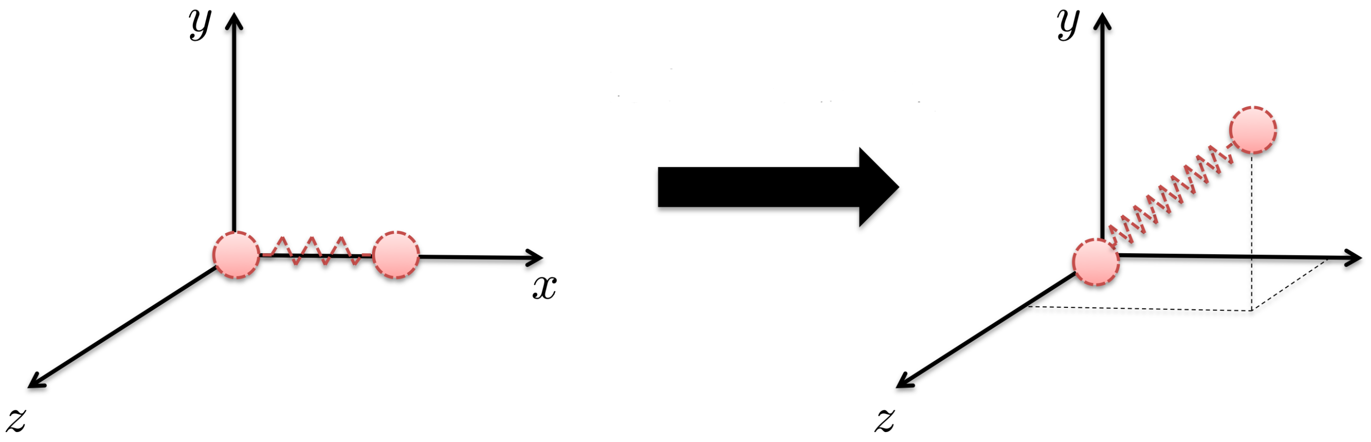


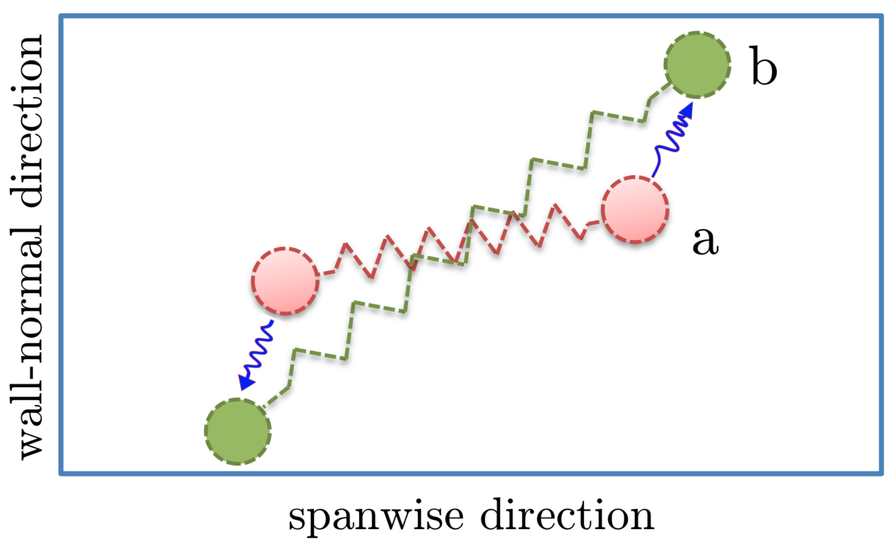
- SOURCE TERM IN INERTIALESS VISCOELASTIC FLOWS

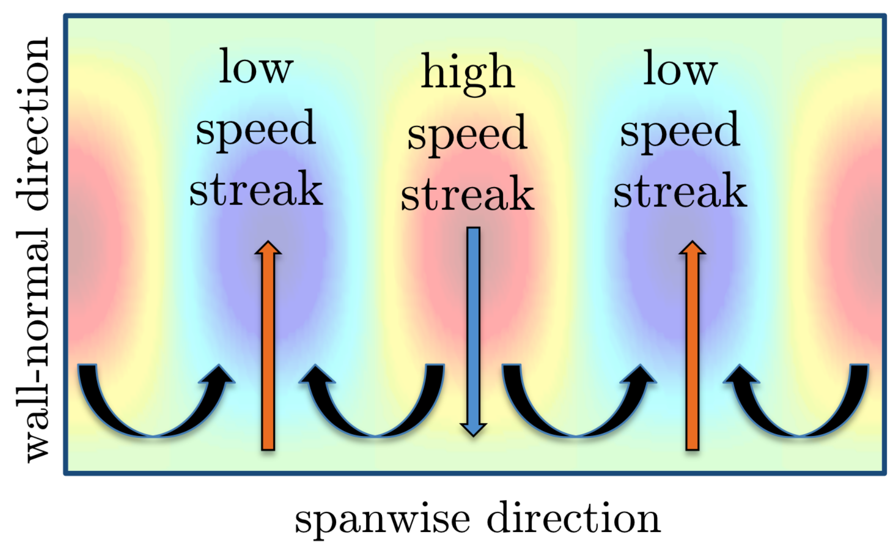
$$\mathbf{C}_{\rm p2}\,\vartheta \;=\; \partial_{yz}\,(U'(y)\tau_{22}) \;+\; \partial_{zz}\,(U'(y)\tau_{23})$$

Inertialess lift-up mechanism

$$\Delta\,\partial_t\,\eta \;=\; -(1/\beta)\Delta\eta \;+\; \color{red}{We}\,(1\,-\,1/\beta)\,\mathbf{C}_{\mathrm{p}2}\,\vartheta$$
$$\;=\; -(1/\beta)\Delta\eta \;+\; \color{red}{We}\,(1\,-\,1/\beta)\,(\partial_{yz}\,(U'(y)\tau_{22})\,+\,\partial_{zz}\,(U'(y)\tau_{23}))$$



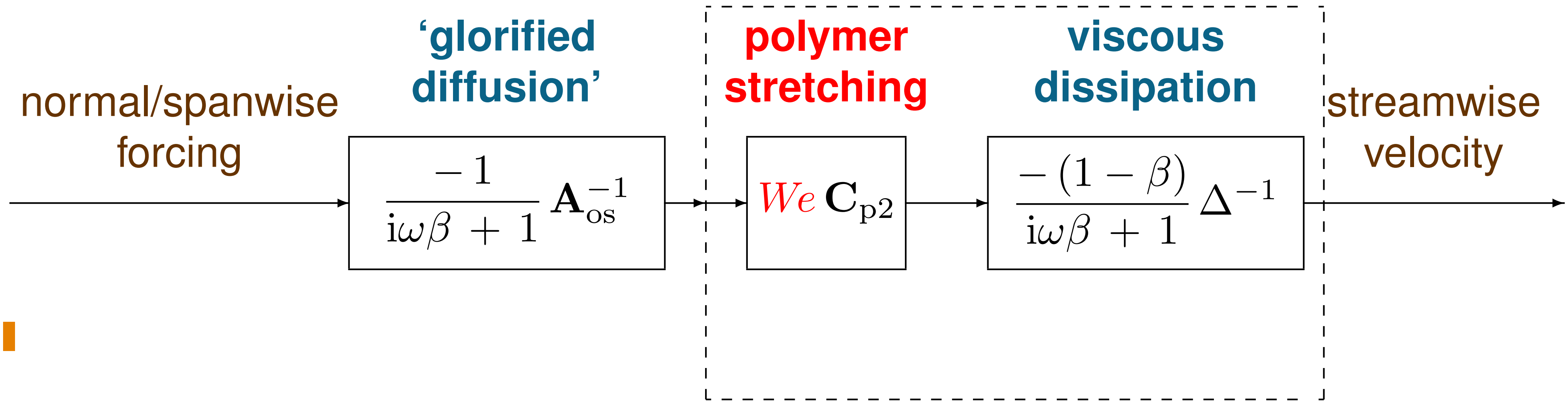




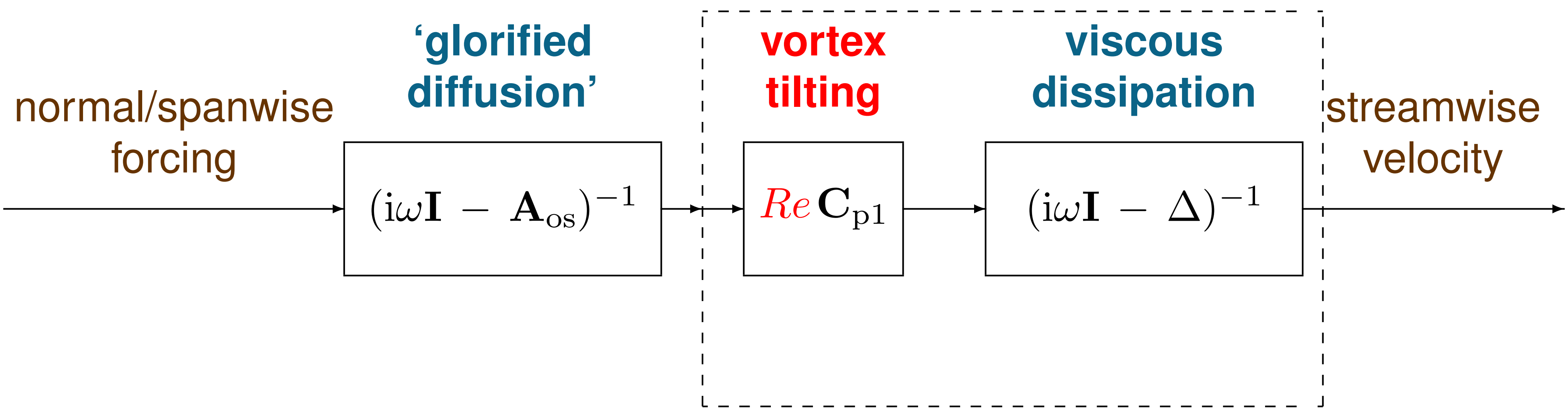
Amplification mechanism

- **Highest amplification:** $(d_2, d_3) \to u$

INERTIALESS VISCOELASTIC:



INERTIAL NEWTONIAN:

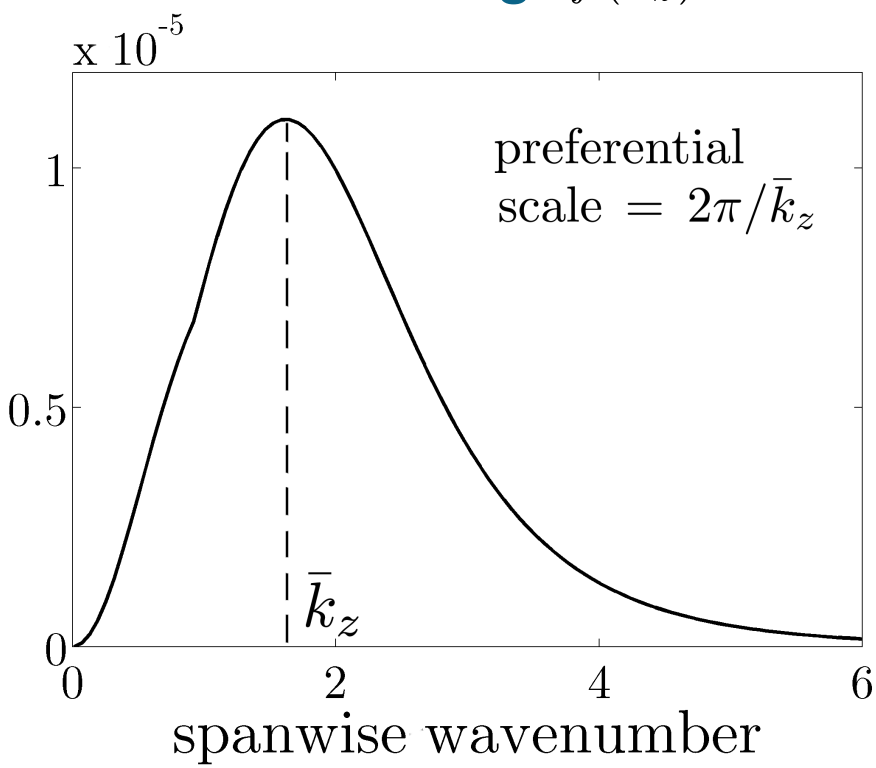


Spatial frequency responses

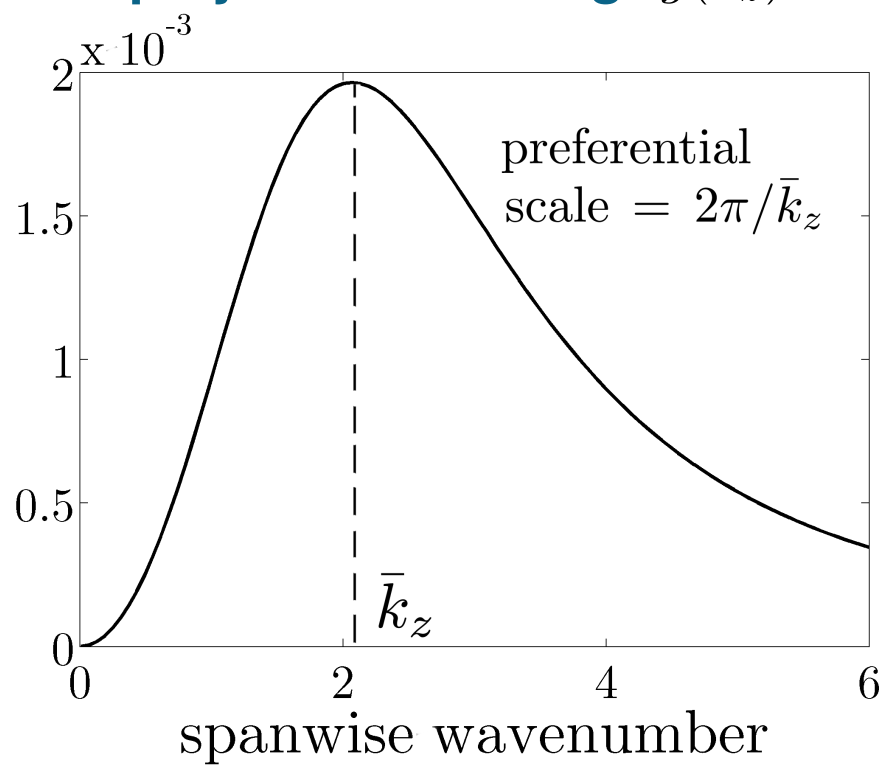
$$(d_2, d_3) \xrightarrow{\textbf{amplification}} u$$

INERTIAL NEWTONIAN:	$G(k_z; Re) \;=\; Re^2\, f(k_z)$
INERTIALESS VISCOELASTIC:	$G(k_z; We, \beta) \;=\; We^2\, g(k_z)\,(1-\beta)^2$

vortex tilting: $f(k_z)$



polymer stretching: $g(k_z)$

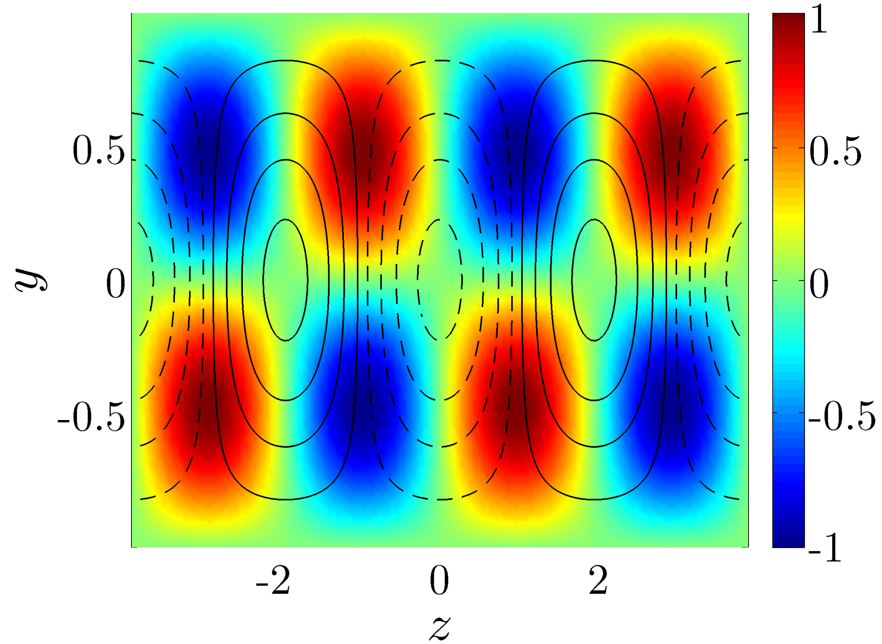


Dominant flow patterns

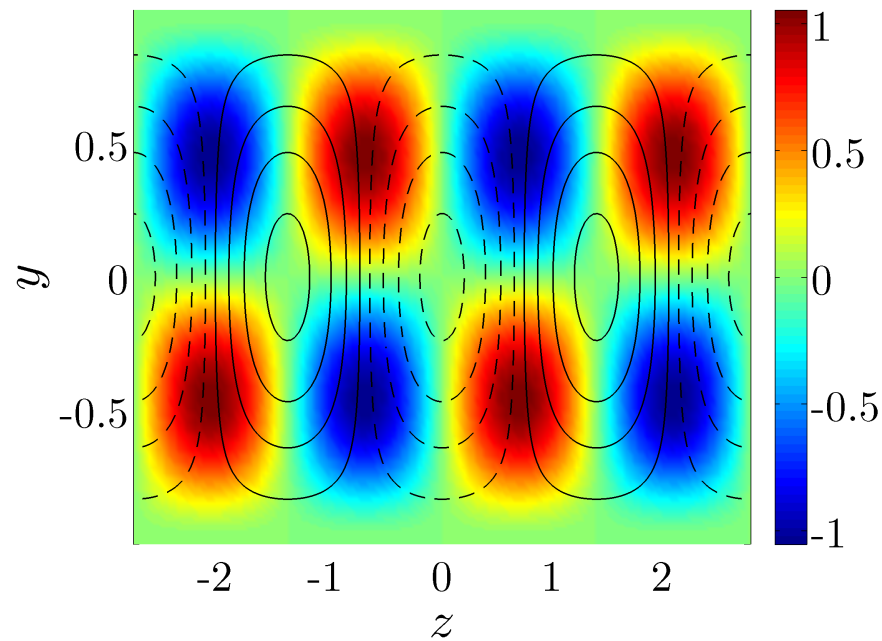
- FREQUENCY RESPONSE PEAKS

☞ **streamwise vortices and streaks**

Inertial Newtonian:



Inertialess viscoelastic:

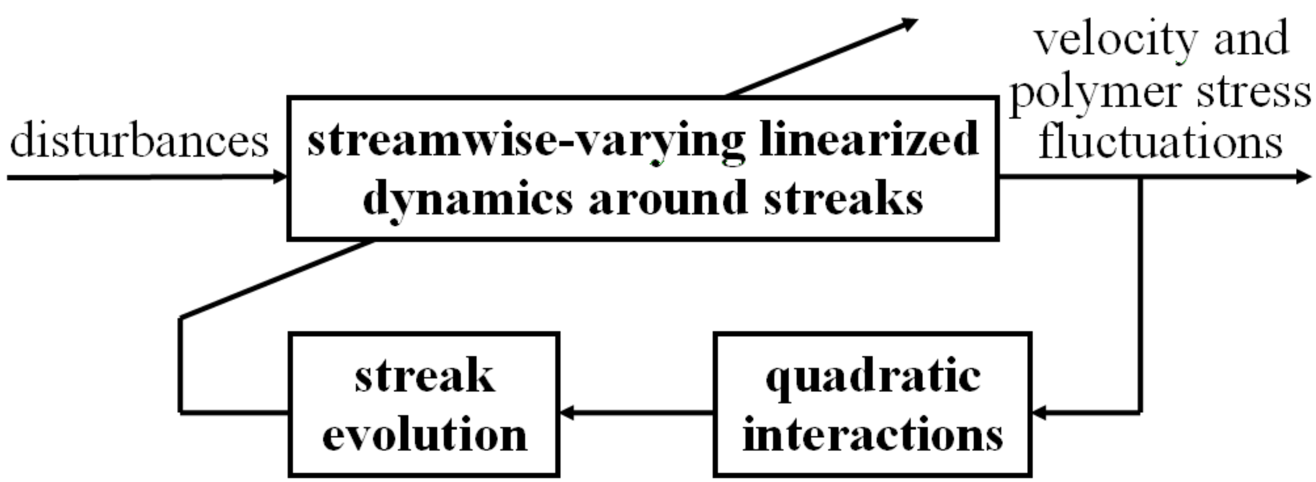


- CHANNEL CROSS-SECTION VIEW: { color plots: streamwise velocity; contour lines: stream-function }

Outlook

- ONGOING EFFORT
  - ⋆ **direct numerical simulations of inertialess flows**
    track 'linear' and 'nonlinear' stages of disturbance development
  - ⋆ **secondary stability/receptivity analysis**
    study influence of streamwise-varying disturbances on streaks

- **Challenge: relative roles of flow sensitivity and nonlinearity**
  - ⋆ **self-sustaining process for transition to elastic turbulence?**



Acknowledgments

TEAM:

Satish

Binh

SUPPORT:

NSF CAREER Award CMMI-06-44793

U of M Digital Technology Center Seed Grant

COMPUTING RESOURCES:

Minnesota Supercomputing Institute