Nonmodal amplification of disturbances in inertialess flows of viscoelastic fluids

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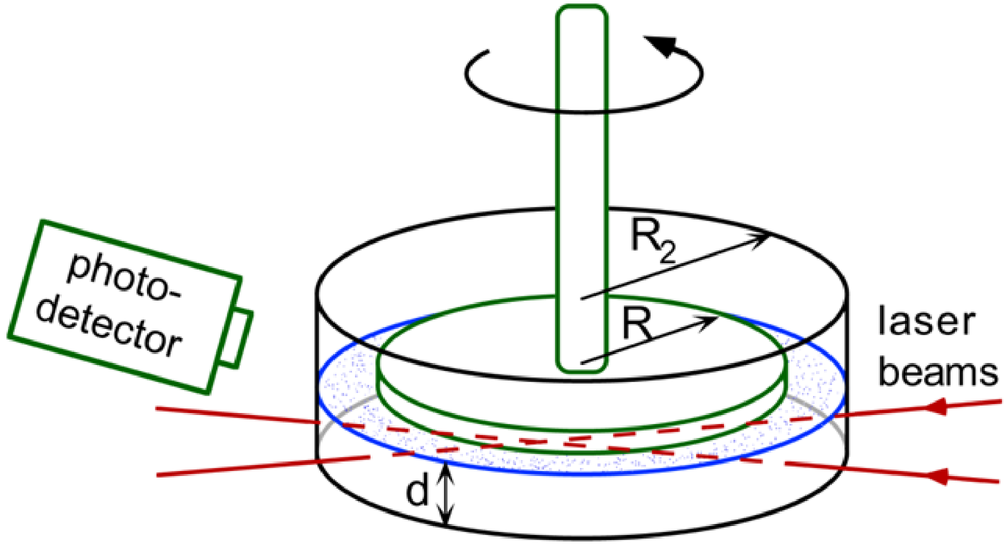
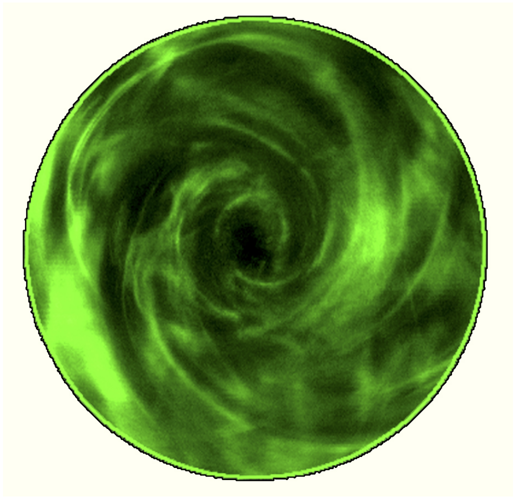
Turbulence without inertia

NEWTONIAN: **inertial turbulence** VISCOELASTIC: **elastic turbulence**



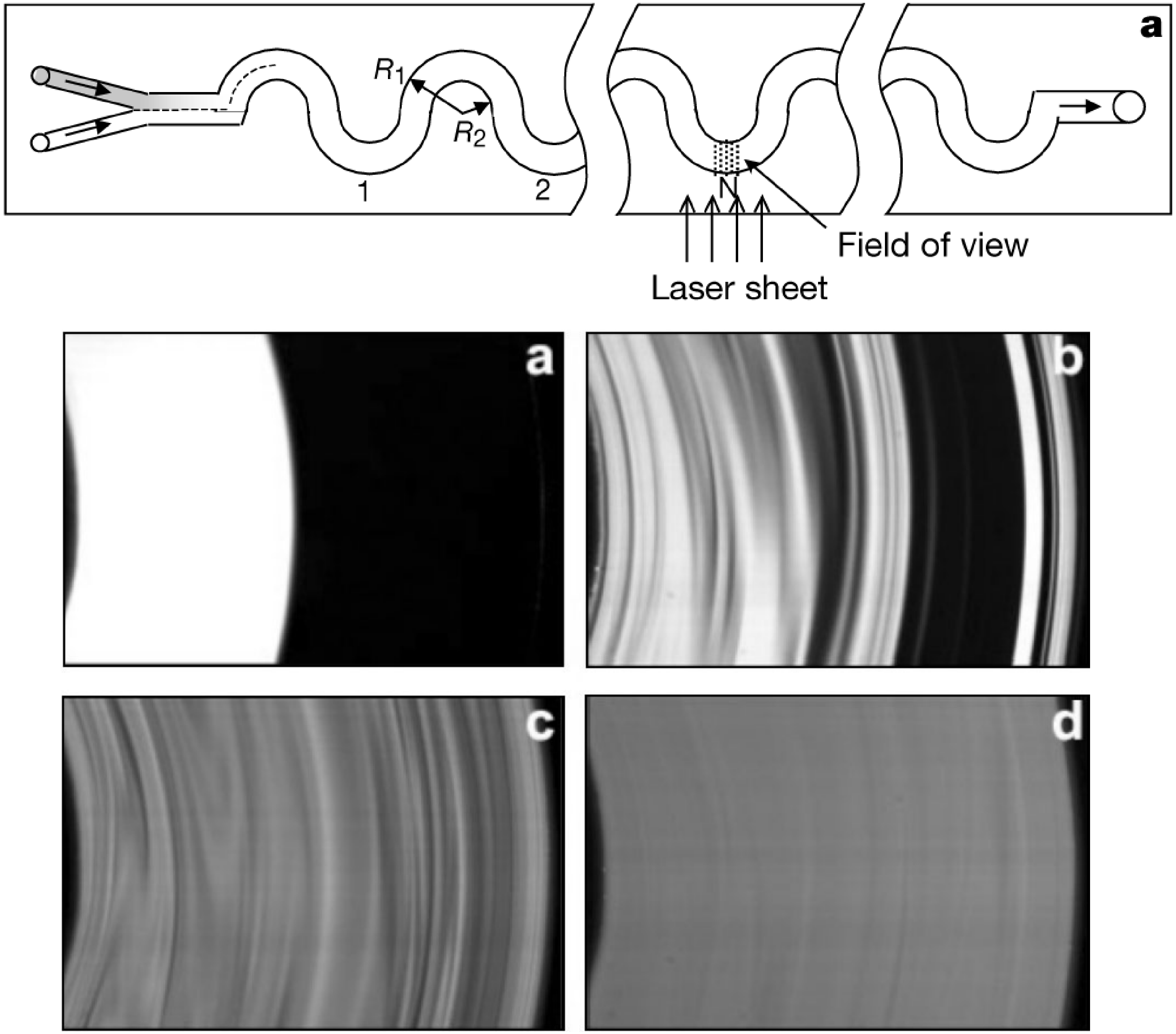
Groisman & Steinberg, *Nature '00*

NEWTONIAN: VISCOELASTIC:



☞ FLOW RESISTANCE: **increased 20 times!**

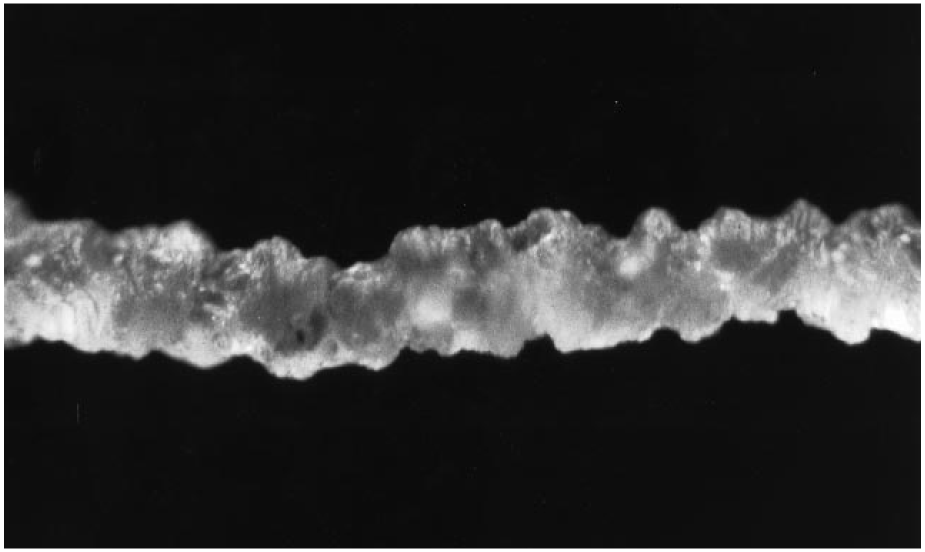
Good for mixing . . .



Groisman & Steinberg, *Nature '01*

. . . bad for processing

DISTORTION OF A POLYMER MELT EMERGING FROM A CAPILLARY TUBE

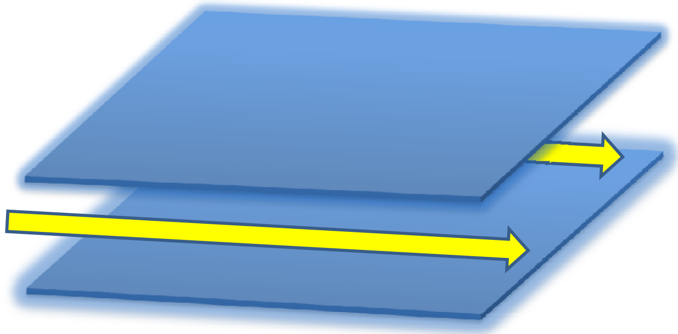


Kalika & Denn, *J. Rheol. '87*

CURVILINEAR FLOWS: **purely elastic instabilities**

Larson, Shaqfeh, Muller, *J. Fluid Mech. '90*

RECTILINEAR FLOWS: **no modal instabilities**



Transition in Newtonian fluids

- LINEAR HYDRODYNAMIC STABILITY: **unstable normal modes**
  - ⋆ **successful in:** Benard Convection, Taylor-Couette flow, etc.
  - ⋆ **fails in:** wall-bounded shear flows (channels, pipes, boundary layers)

- DIFFICULTY #1

Inability to predict: **Reynolds number for the onset of turbulence** (Re_c)

Experimental onset of turbulence: $\left\{ \begin{array}{l} \textbf{much before instability} \\ \textbf{no sharp value for } Re_c \end{array} \right.$

- DIFFICULTY #2

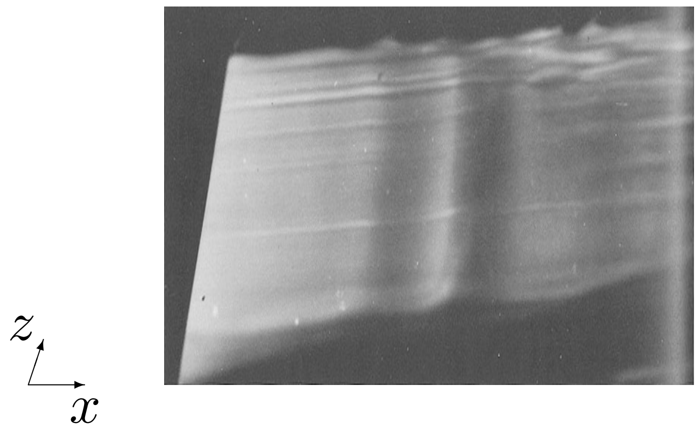
Inability to predict: **flow structures observed at transition**

(except in carefully controlled experiments)

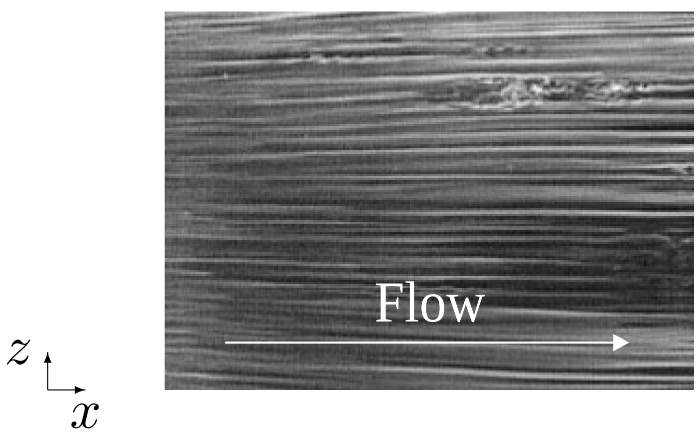
LINEAR STABILITY:

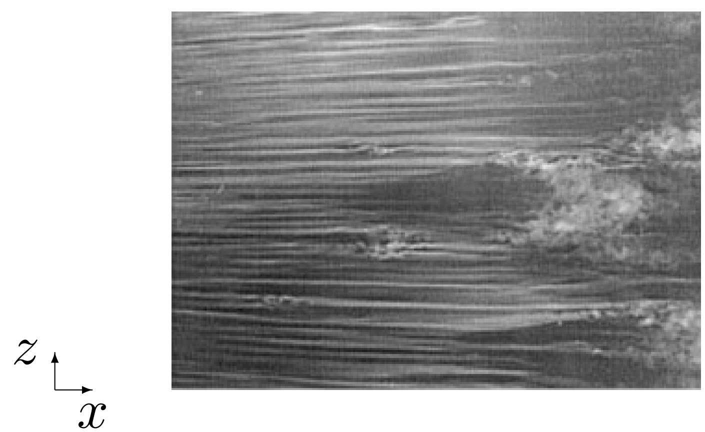
⋆ For $Re \geq Re_c \;\Rightarrow\;$ exp. growing normal modes

corresponding e-functions (TS-waves) $\Big\} :=$ exp. growing flow structures



EXPERIMENTS: **streaky boundary layers and turbulent spots**





Matsubara & Alfredsson, *J. Fluid Mech. ’01*

- FAILURE OF LINEAR HYDRODYNAMIC STABILITY
  caused by high flow sensitivity
  - ⋆ large transient responses
  - ⋆ large noise amplification
  - ⋆ small stability margins

TO COUNTER THIS SENSITIVITY: **must account for modeling imperfections**

TRANSITION ≈ STABILITY + RECEPTIVITY + ROBUSTNESS

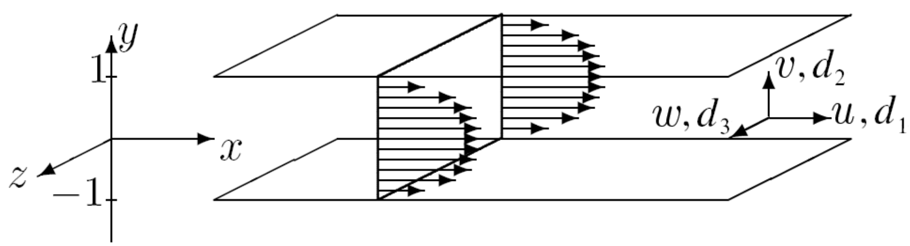
RECEPTIVITY ↓ **flow disturbances**

ROBUSTNESS ↓ **unmodeled dynamics**

Tools for quantifying sensitivity

- INPUT-OUTPUT ANALYSIS: **spatio-temporal frequency responses**

$\mathbf{d}$ { Free-stream turbulence; Surface roughness; Neglected nonlinearities } → **Linearized Dynamics** → Fluctuating velocity field } $\mathbf{v}$



$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

IMPLICATIONS FOR:

transition: insight into mechanisms

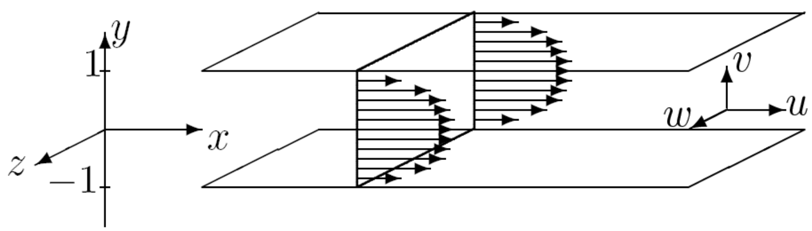
control: control-oriented modeling

Transient growth analysis

- STUDY TRANSIENT BEHAVIOR OF FLUCTUATIONS' ENERGY

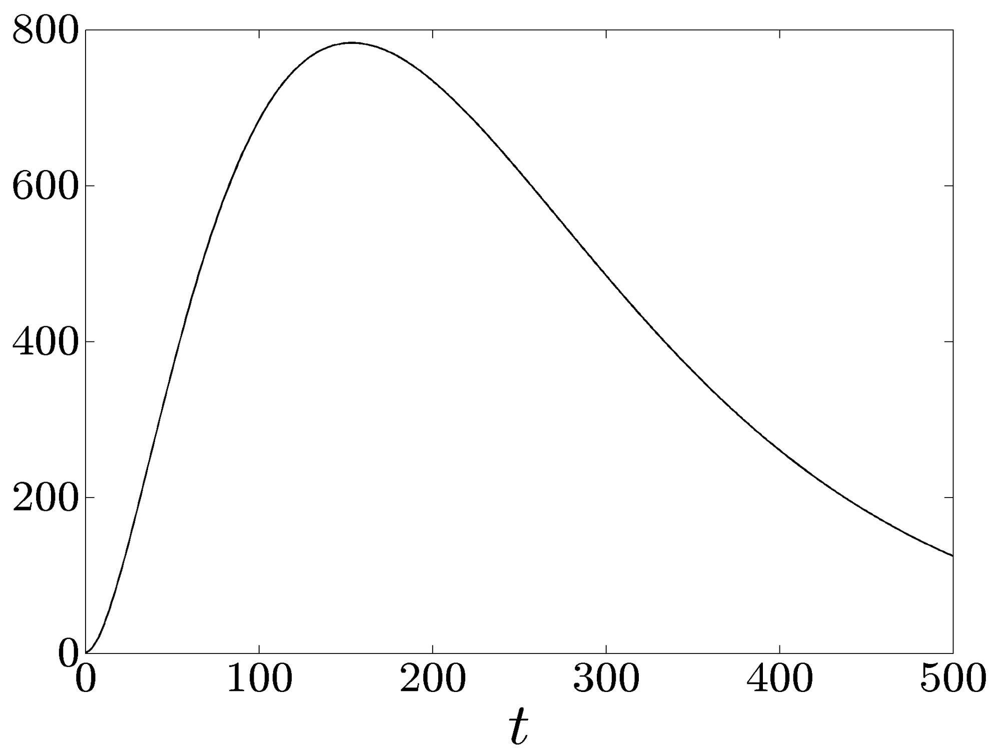
  Farrell, Butler, Gustavsson, Henningson, Reddy, Trefethen, etc.

$Re < 5772$



$$\psi_t = A\,\psi$$

kinetic energy:

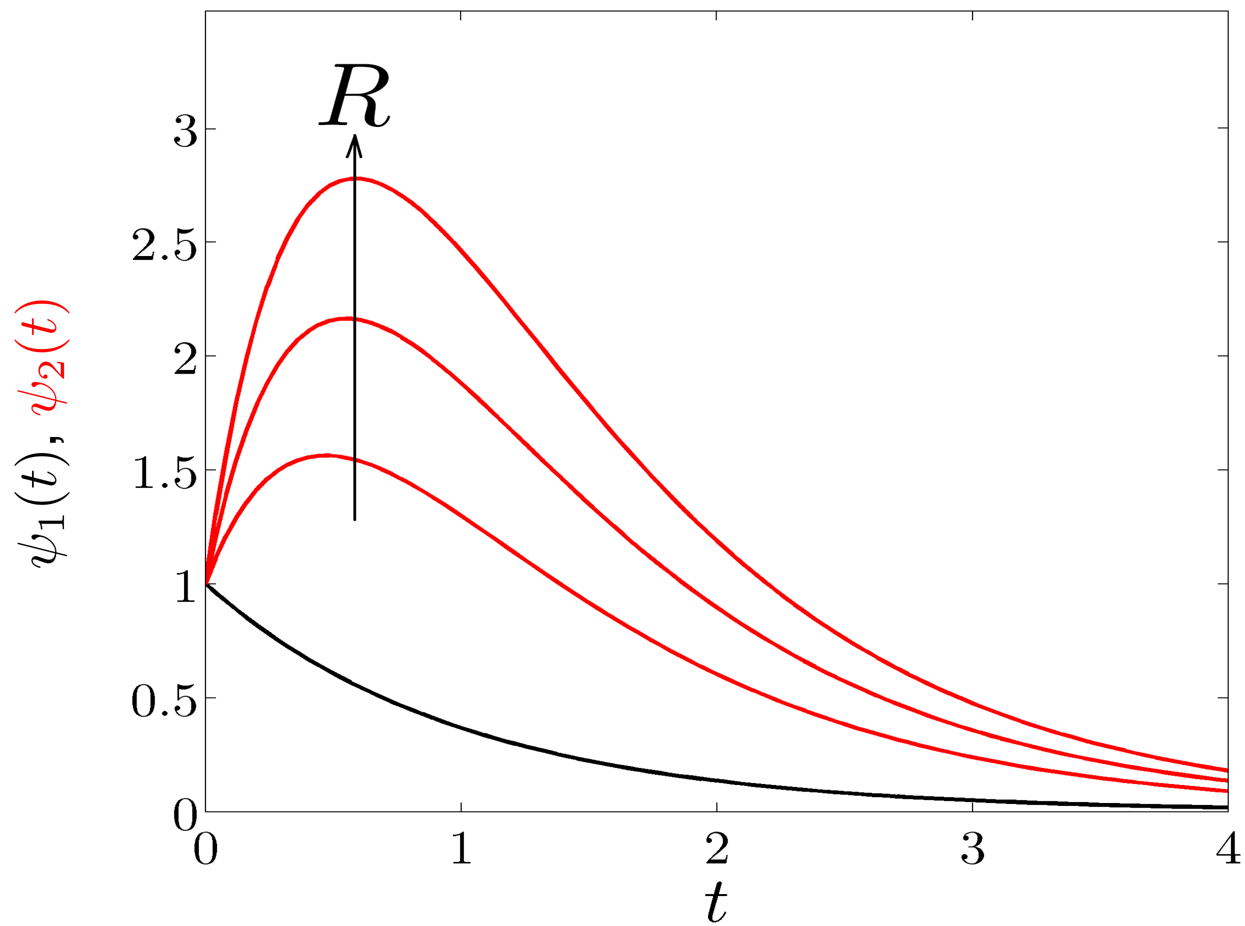


☞ E-VALUES: **misleading measure of transient response**

A toy example

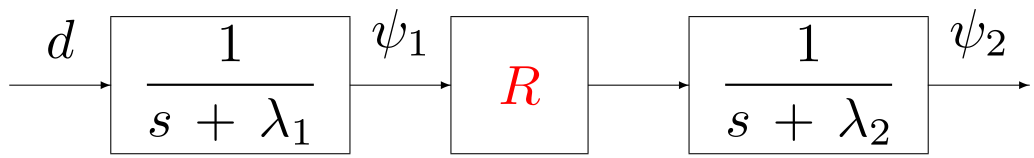
$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 \\ R & -\lambda_2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \quad \lambda_1, \lambda_2 > 0$$

$\lambda_1 = 1, \lambda_2 = 2$:



Non-modal amplification of disturbances

$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 \\ R & -\lambda_2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d$$



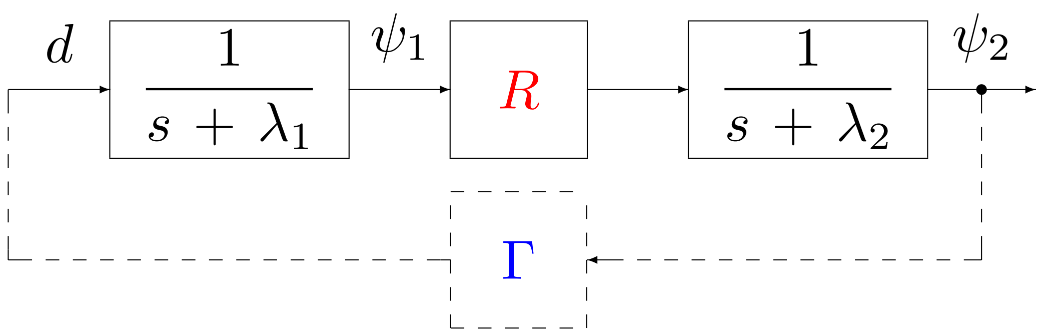
Worst case amplification

$$\max \frac{\text{energy of } \psi_2}{\text{energy of } d} = \max_{\omega} |H(\mathrm{i}\omega)|^2 = \frac{R^2}{(\lambda_1 \lambda_2)^2}$$

Variance amplification

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |H(\mathrm{i}\omega)|^2 \, \mathrm{d}\omega = \frac{R^2}{\lambda_1 \lambda_2 (\lambda_1 + \lambda_2)}$$

Robustness



modeling uncertainty

(can be nonlinear or time-varying)

small-gain theorem:

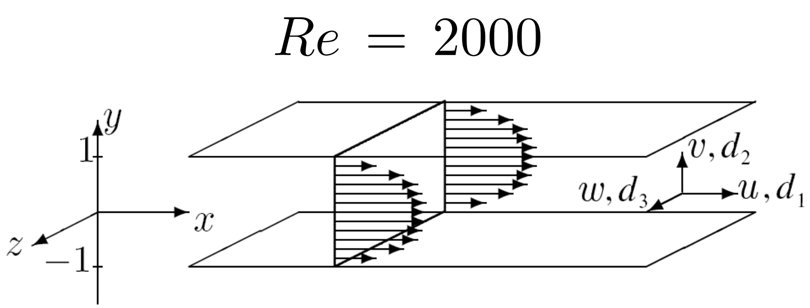
stability for all Γ with

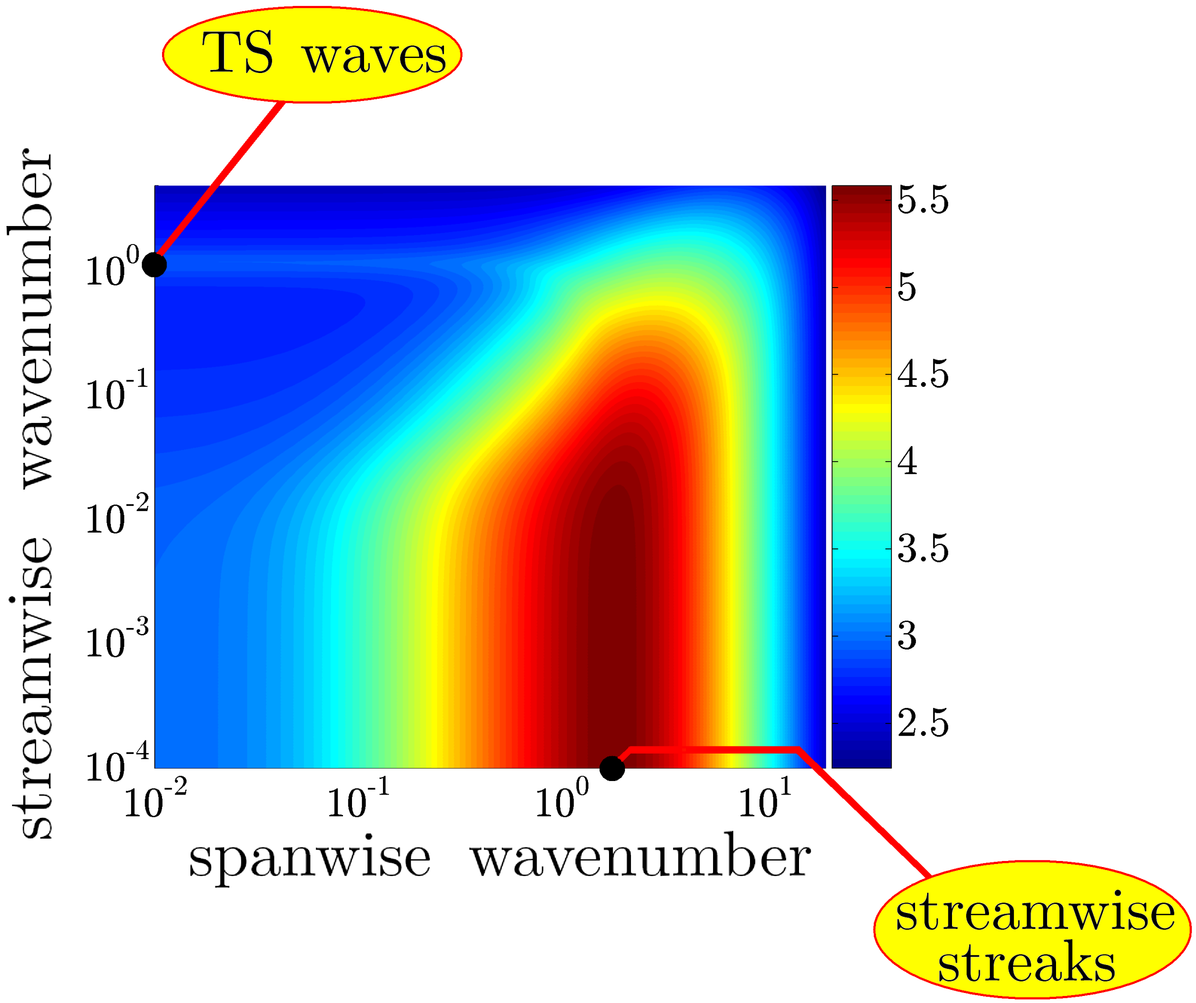
$$\max_{\omega} |\Gamma(\mathrm{i}\omega)| \le \gamma$$

$$\Updownarrow$$

$$\gamma < \lambda_1 \lambda_2 / R$$

Ensemble average energy density



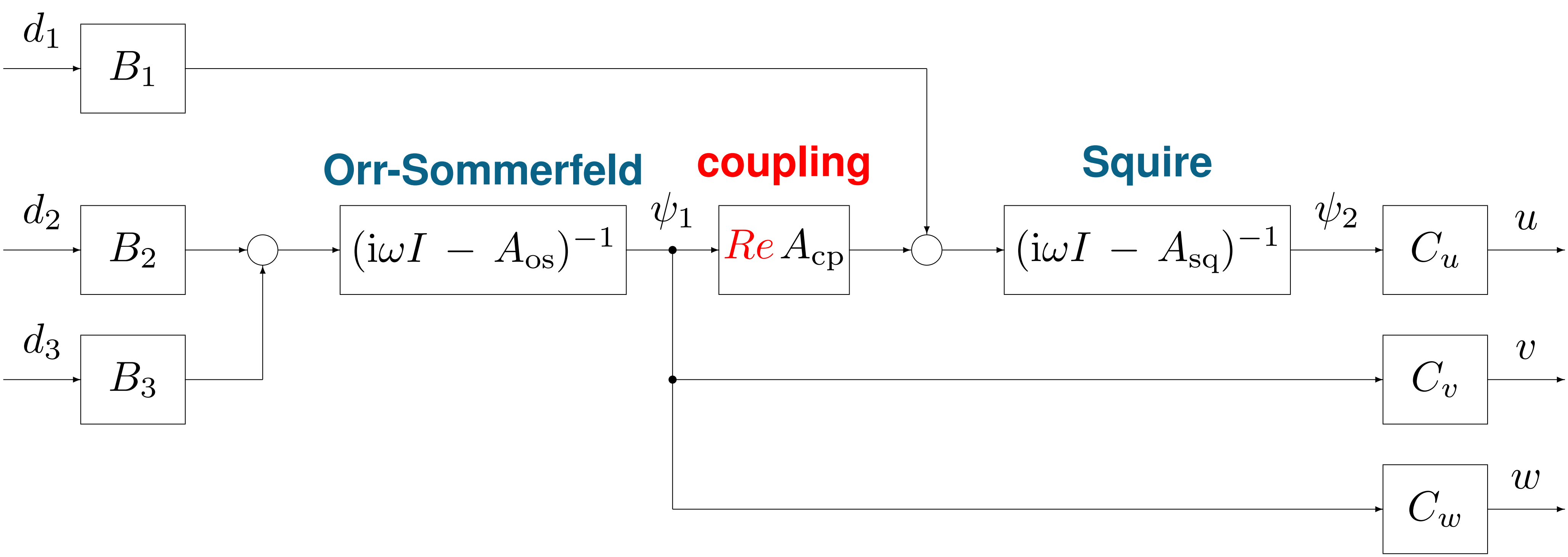


- **Dominance of streamwise elongated structures**
  streamwise streaks!

Influence of Re: streamwise-constant model

$$\begin{bmatrix} \psi_{1t} \\ \psi_{2t} \end{bmatrix} = \begin{bmatrix} A_{\mathrm{os}} & 0 \\ Re\, A_{\mathrm{cp}} & A_{\mathrm{sq}} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \begin{bmatrix} 0 & B_2 & B_3 \\ B_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

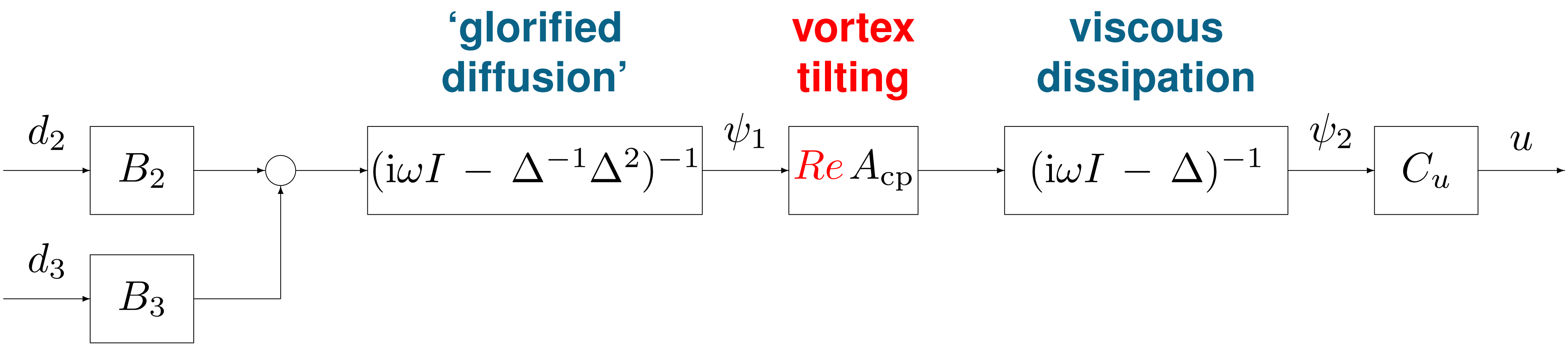
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 & C_u \\ C_v & 0 \\ C_w & 0 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$



Jovanović & Bamieh, *J. Fluid Mech.* '05

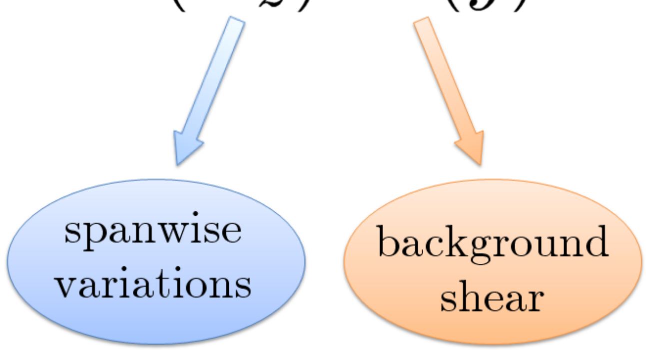
Amplification mechanism in flows with high Re

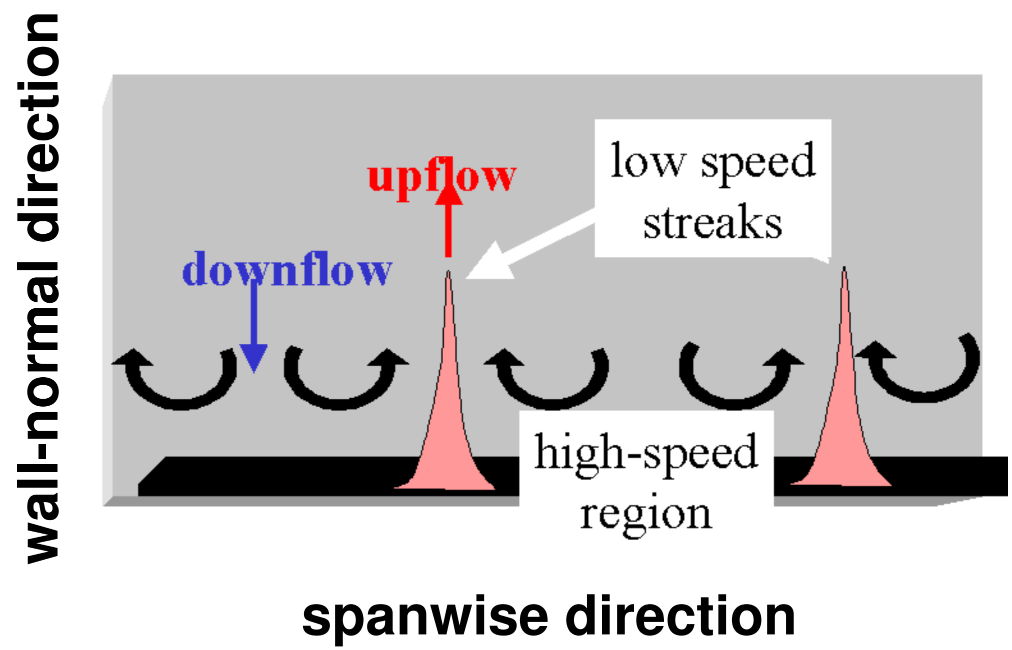
- HIGHEST AMPLIFICATION: $(d_2, d_3) \rightarrow u$



☞ AMPLIFICATION MECHANISM: **vortex tilting** or **lift-up**

$$A_{\mathrm{cp}} = -(\mathrm{i}k_z)\, U'(y)$$



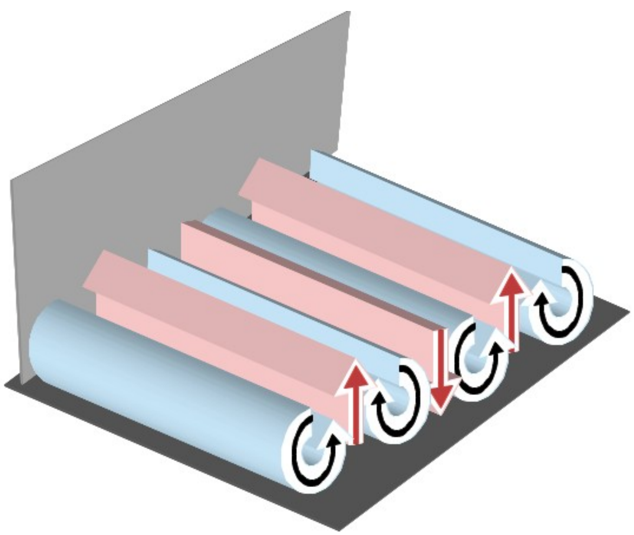


Linear analyses: Input-output vs. Stability

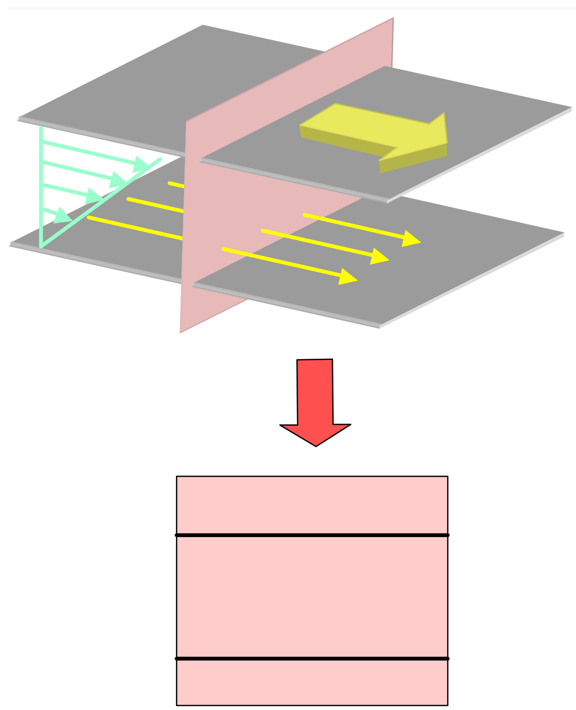
AMPLIFICATION:

$$\mathbf{v} \;=\; H\,\mathbf{d}$$

singular values of H



typical structures

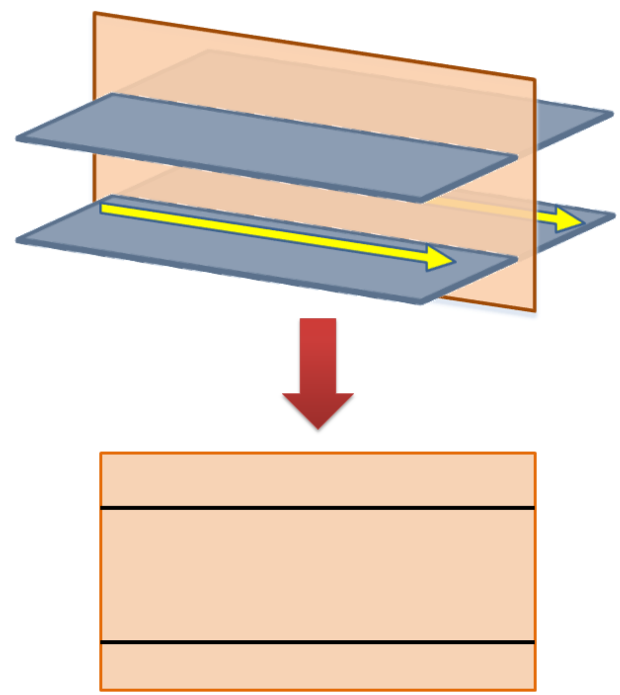


cross-sectional dynamics

STABILITY:

$$\psi_t \;=\; A\,\psi$$

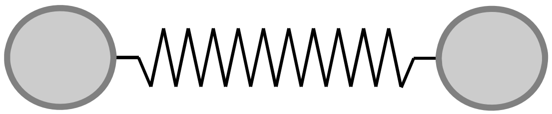
e-values of A



2D models

Oldroyd-B fluids

HOOKEAN SPRING:



$$
\begin{aligned}
(Re/We)\,\mathbf{u}_t &= -Re\,(\mathbf{u}\cdot\nabla)\,\mathbf{u} - \nabla p + \beta\,\Delta\mathbf{u} + (1-\beta)\,\nabla\cdot\boldsymbol{\tau} + \mathbf{d} \\
0 &= \nabla\cdot\mathbf{u} \\
\boldsymbol{\tau}_t &= -\boldsymbol{\tau} + \nabla\mathbf{u} + (\nabla\mathbf{u})^T + We\left(\boldsymbol{\tau}\cdot\nabla\mathbf{u} + (\nabla\mathbf{u})^T\cdot\boldsymbol{\tau} - (\mathbf{u}\cdot\nabla)\boldsymbol{\tau}\right)
\end{aligned}
$$

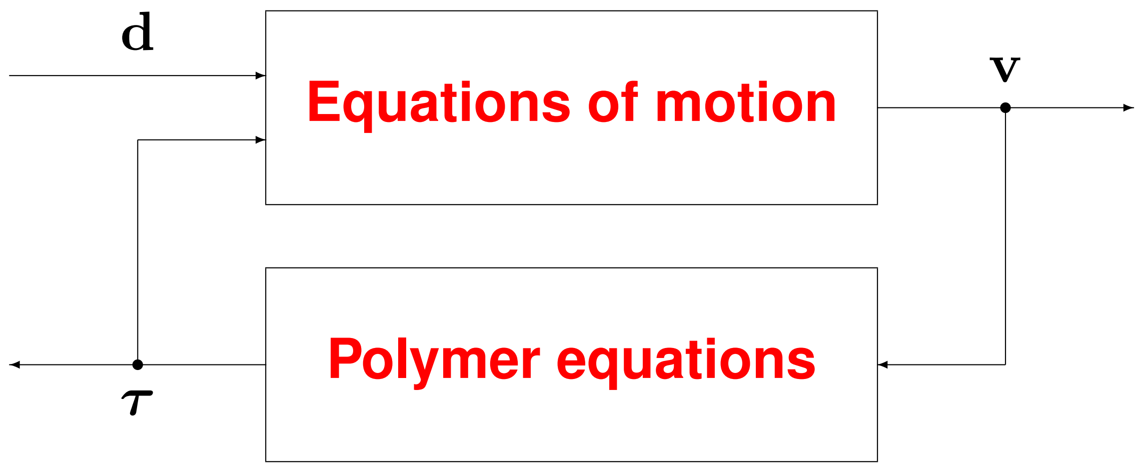
VISCOSITY RATIO: $\beta := \dfrac{\text{solvent viscosity}}{\text{total viscosity}}$

WEISSENBERG NUMBER: $We := \dfrac{\text{fluid relaxation time}}{\text{characteristic flow time}}$

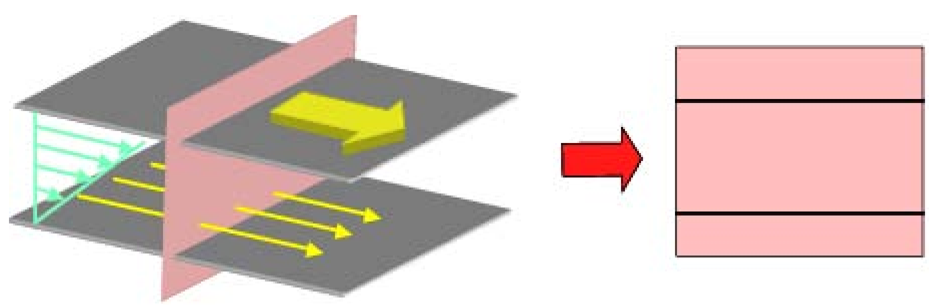
- TRANSIENT GROWTH ANALYSIS

  Sureshkumar *et al.*, *JNNFM '99*; Atalik & Keunings, *JNNFM '02*; Kupferman, *JNNFM '05*; Doering *et al.*, *JNNFM '06*; Renardy, *JNNFM '09*

- INPUT-OUTPUT ANALYSIS

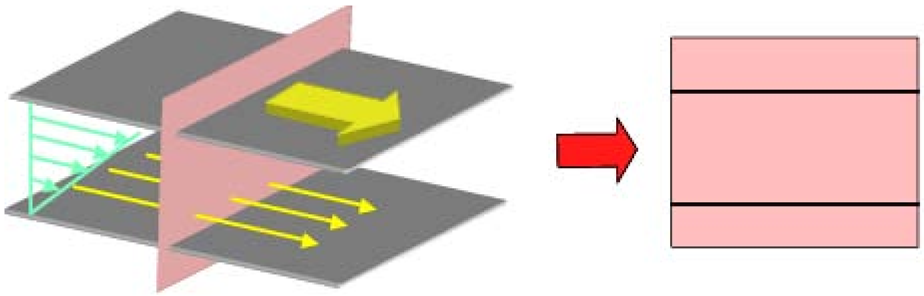


importance of streamwise elongated structures



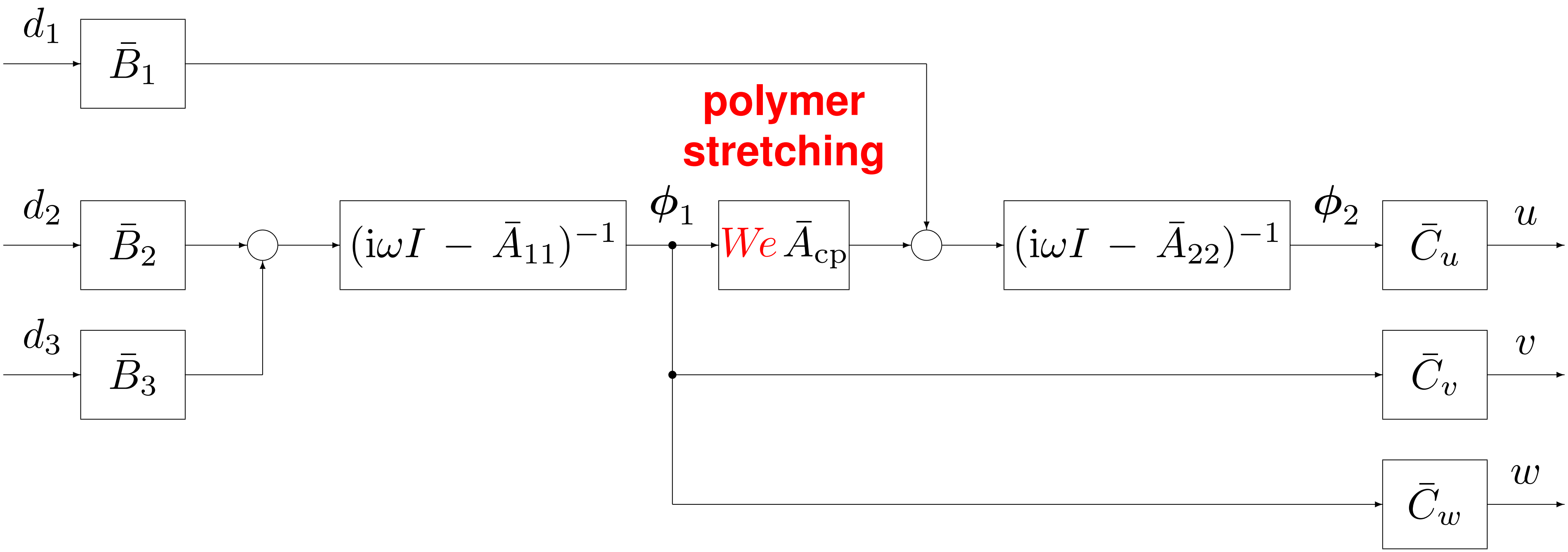
Hoda, Jovanović, Kumar, *J. Fluid Mech. '08*
Hoda, Jovanović, Kumar, *J. Fluid Mech. '09*

Inertialess channel flow: streamwise-constant model



$$
\begin{aligned}
0 &= -\nabla p + \beta\,\Delta\mathbf{u} + (1-\beta)\,\nabla\cdot\boldsymbol{\tau} + \mathbf{d}\\
0 &= \nabla\cdot\mathbf{u}\\
\boldsymbol{\tau}_t &= -\boldsymbol{\tau} + \nabla\mathbf{u} + (\nabla\mathbf{u})^T + We\left(\boldsymbol{\tau}\cdot\nabla\mathbf{u} + (\nabla\mathbf{u})^T\cdot\boldsymbol{\tau} - (\mathbf{u}\cdot\nabla)\boldsymbol{\tau}\right)
\end{aligned}
$$

Inertialess Oldroyd-B vs. Inertial Newtonian



OLDROYD-B W/O INERTIA: **Weissenberg number & Polymer Stretching**

$\approx$

NEWTONIAN WITH INERTIA: **Reynolds number & Vortex Tilting**

Jovanović & Kumar, *Phys. Fluids '10*

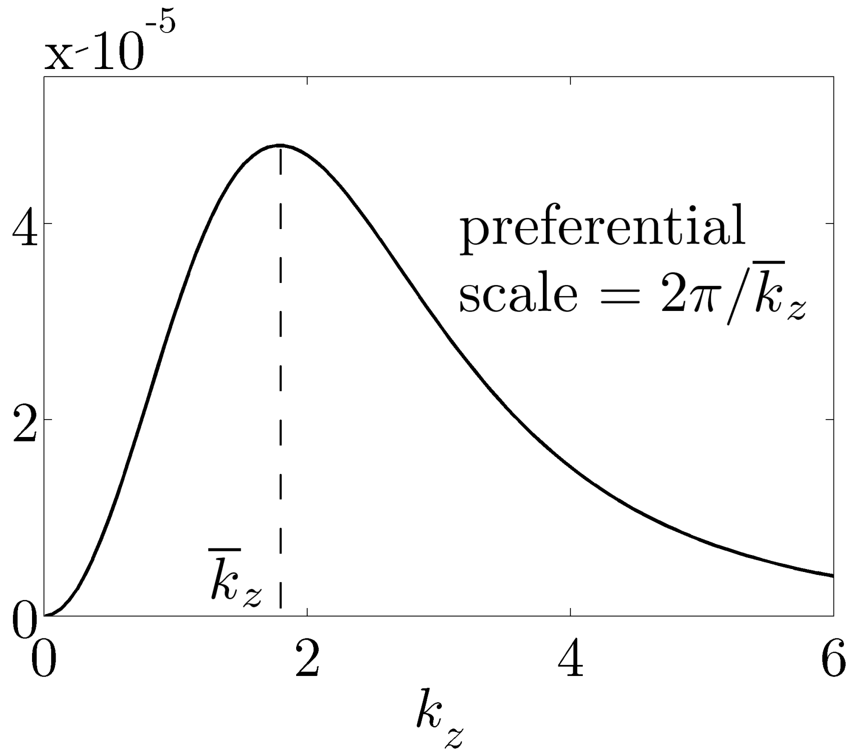
Jovanović & Kumar '10, (submitted)

Spatial frequency responses

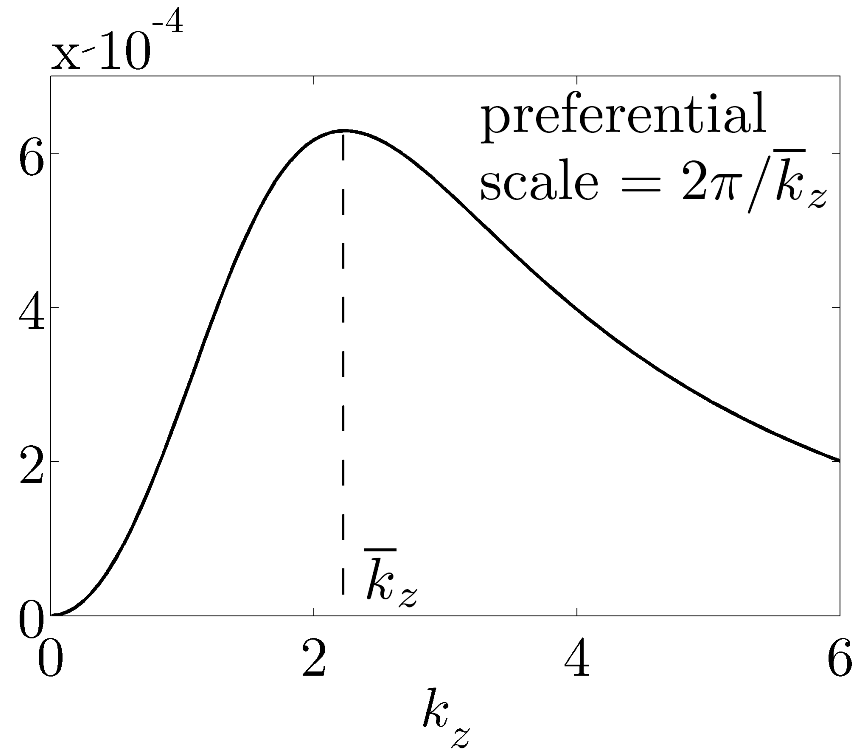
$$(d_2, d_3) \xrightarrow{\textbf{amplification}} u$$

INERTIAL NEWTONIAN:	$E(k_z; Re) = Re^2 f(k_z)$
INERTIALESS OLDROYD-B:	$E(k_z; We, \beta) = We^2 g(k_z) (1-\beta)^2 / \beta$

vortex tilting: $f(k_z)$



polymer stretching: $g(k_z)$

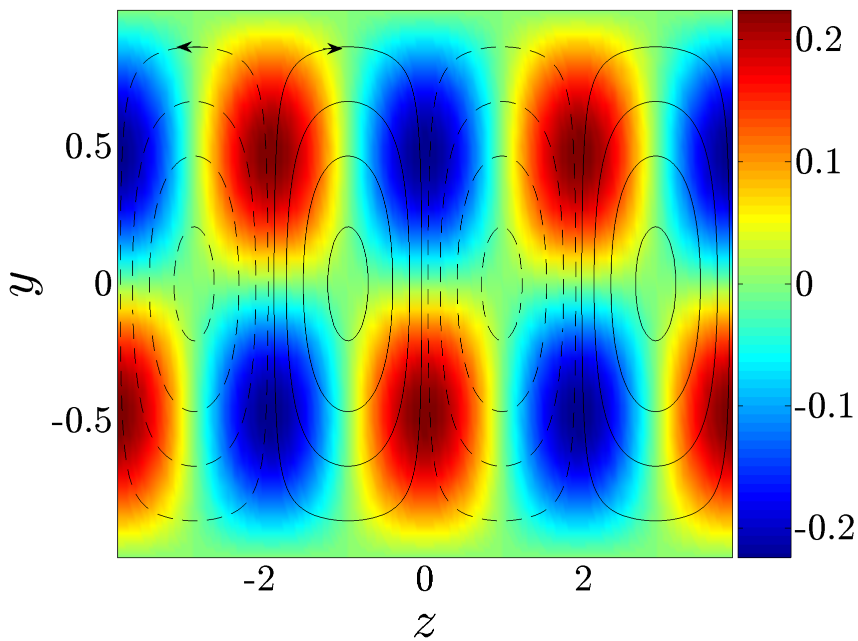


Dominant flow patterns

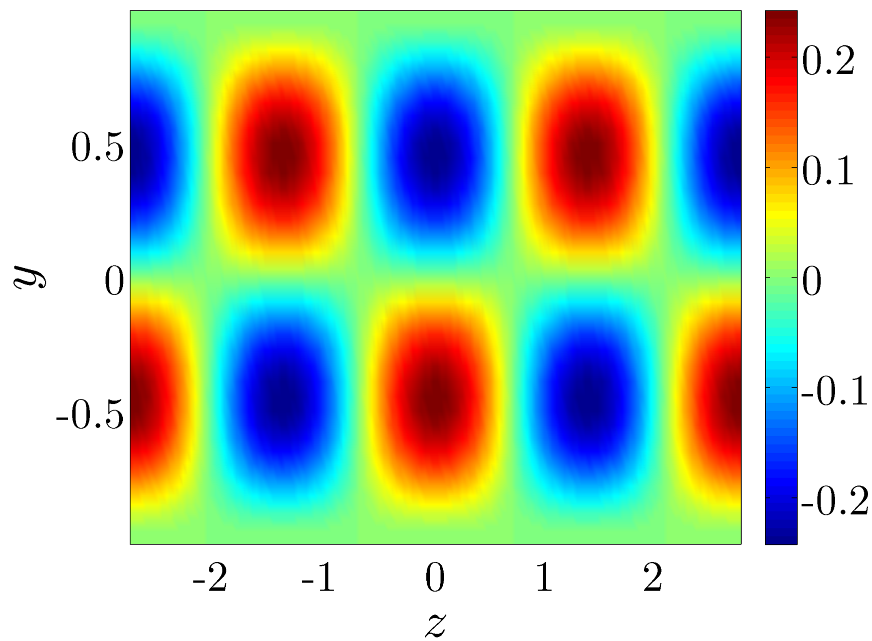
- FREQUENCY RESPONSE PEAKS

☞ **streamwise vortices and streaks**

Inertial Newtonian:



Inertialess Oldroyd-B:

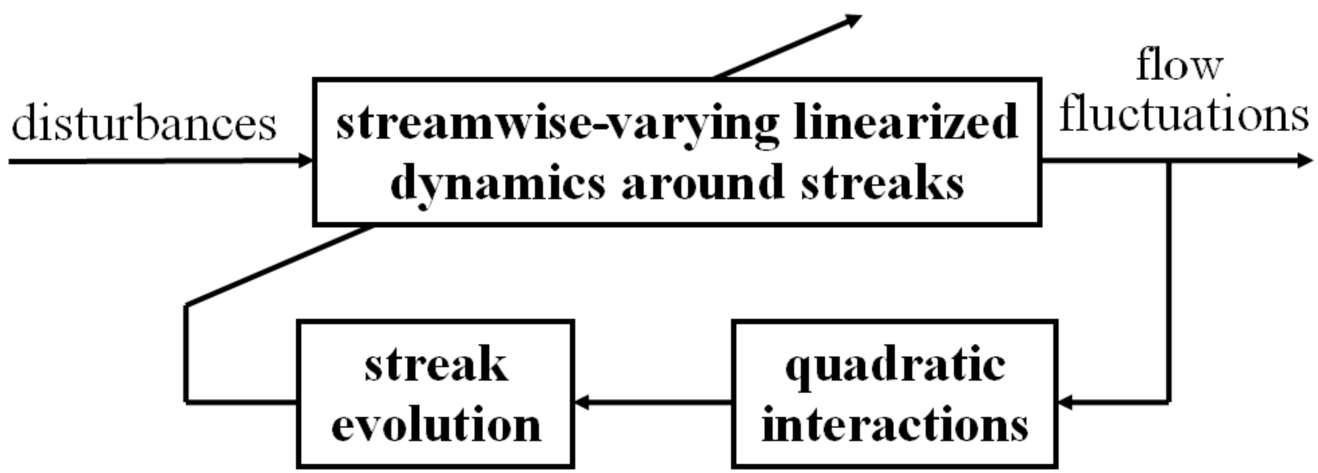


- CHANNEL CROSS-SECTION VIEW: color plots: streamwise velocity; contour lines: stream-function

Outlook

- WISH LIST
  - ⋆ **direct numerical simulations of stochastically forced flows**
    track 'linear' and 'nonlinear' stages of disturbance development
  - ⋆ **secondary sensitivity analysis**
    study influence of streamwise-varying disturbances on streaks

- **Challenge: relative roles of flow sensitivity and nonlinearity**



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Minnesota Supercomputing Institute