Controlling the onset of turbulence by streamwise traveling waves

Mihailo Jovanović
www.umn.edu/∼mihailo

joint work with:
Rashad Moarref, Binh Lieu

University of Minnesota

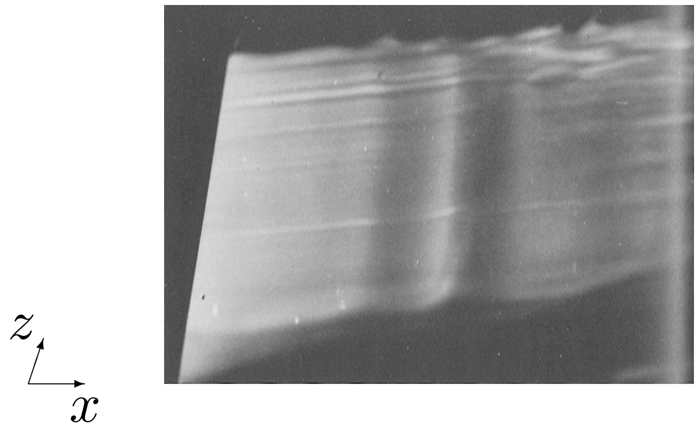
Rheology Research Seminar, University of Wisconsin; March 11, 2011

Transition to turbulence

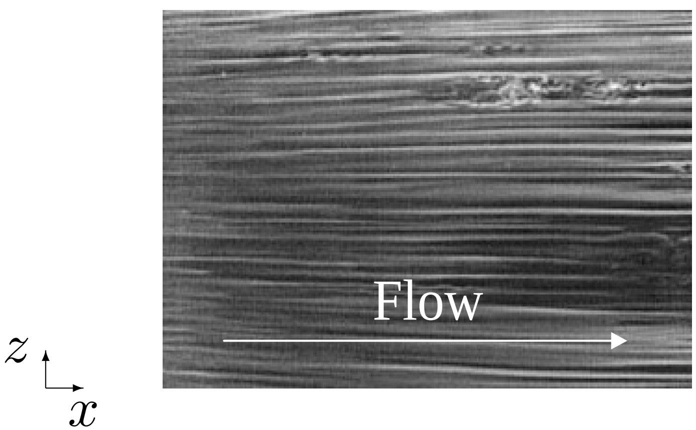
Linear hydrodynamic stability

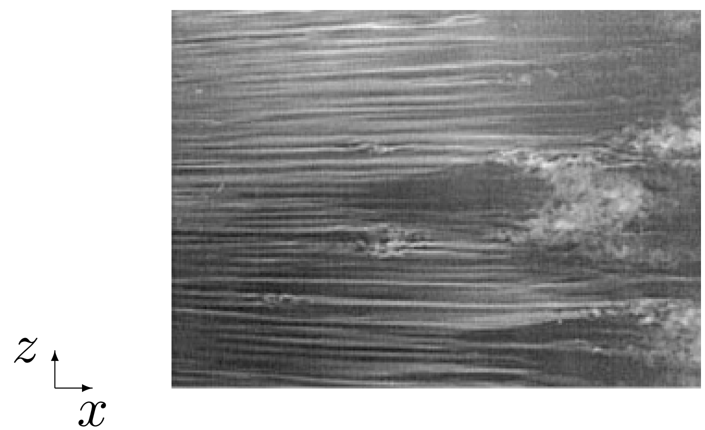
⋆ For $Re \geq Re_c \;\Rightarrow\;$ exp. growing normal modes

corresponding e-functions (TS-waves) } $:=$ exp. growing flow structures



Experiments: **streaky boundary layers and turbulent spots**





Matsubara & Alfredsson, *J. Fluid Mech. '01*

- FAILURE OF LINEAR HYDRODYNAMIC STABILITY
  caused by high flow sensitivity
  - ⋆ large transient responses
  - ⋆ large noise amplification
  - ⋆ small stability margins

TO COUNTER THIS SENSITIVITY: **must account for modeling imperfections**

TRANSITION $\approx$ STABILITY $+$ RECEPTIVITY $+$ ROBUSTNESS

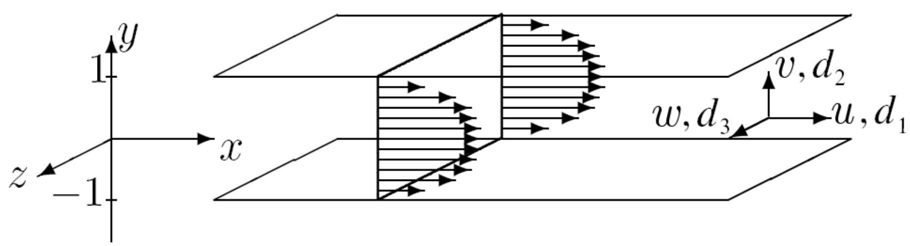
RECEPTIVITY $\downarrow$ **flow disturbances**

ROBUSTNESS $\downarrow$ **unmodeled dynamics**

Tools for quantifying sensitivity

- INPUT-OUTPUT ANALYSIS: **spatio-temporal frequency responses**

$\mathbf{d}$ { Free-stream turbulence; Surface roughness; Acoustic waves } → **Linearized Dynamics** → { Fluctuating velocity field } $\mathbf{v}$



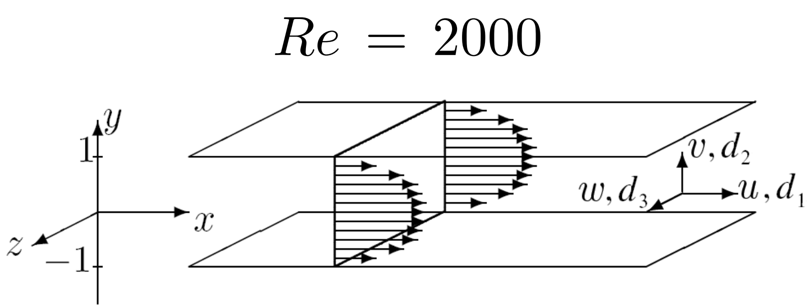
$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

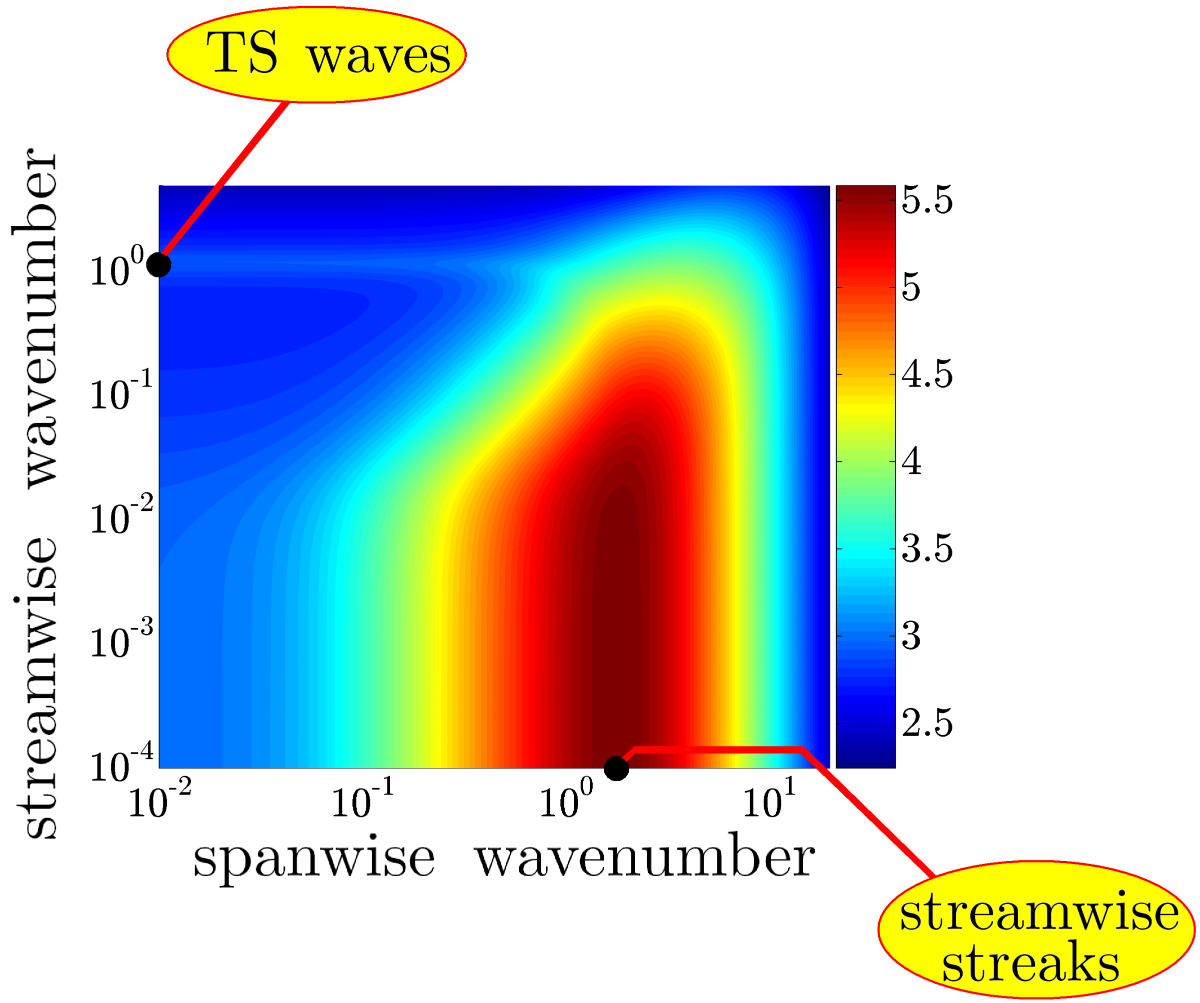
IMPLICATIONS FOR:

transition: insight into mechanisms

control: control-oriented modeling

Ensemble average energy density

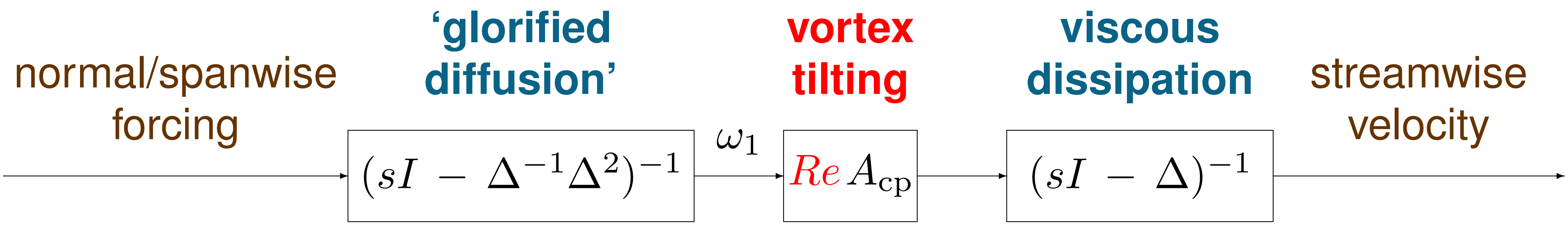




- **Dominance of streamwise elongated structures**
  streamwise streaks!

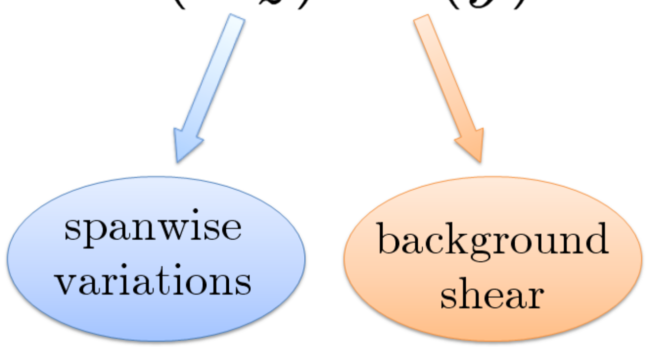
Amplification mechanism in streamwise-constant flows

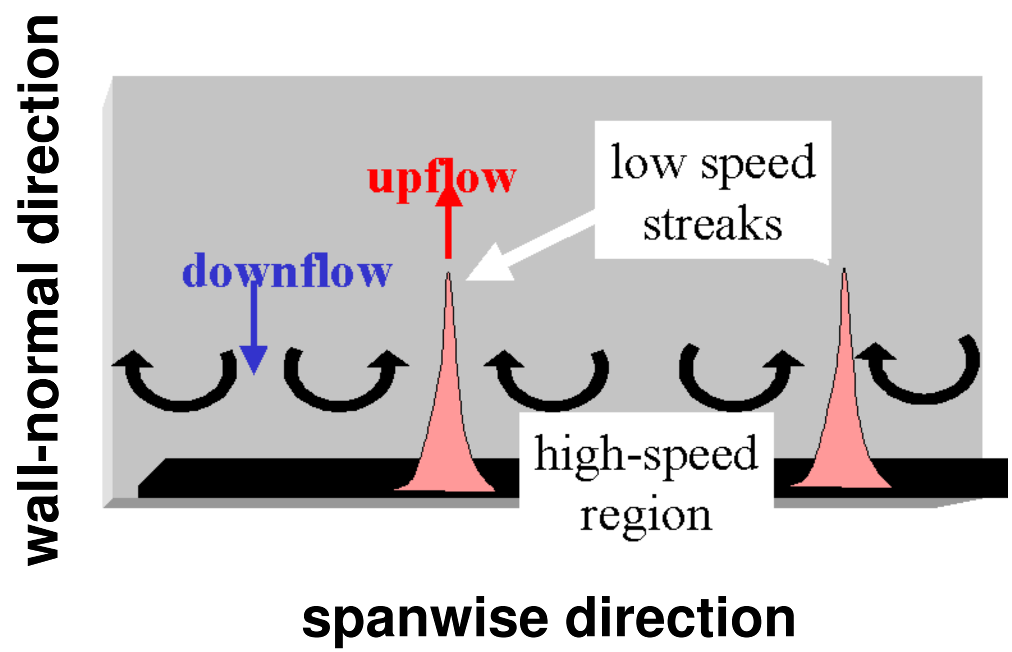
- HIGHEST AMPLIFICATION: $(d_2, d_3) \rightarrow u$



☞ AMPLIFICATION MECHANISM: **vortex tilting** or **lift-up**

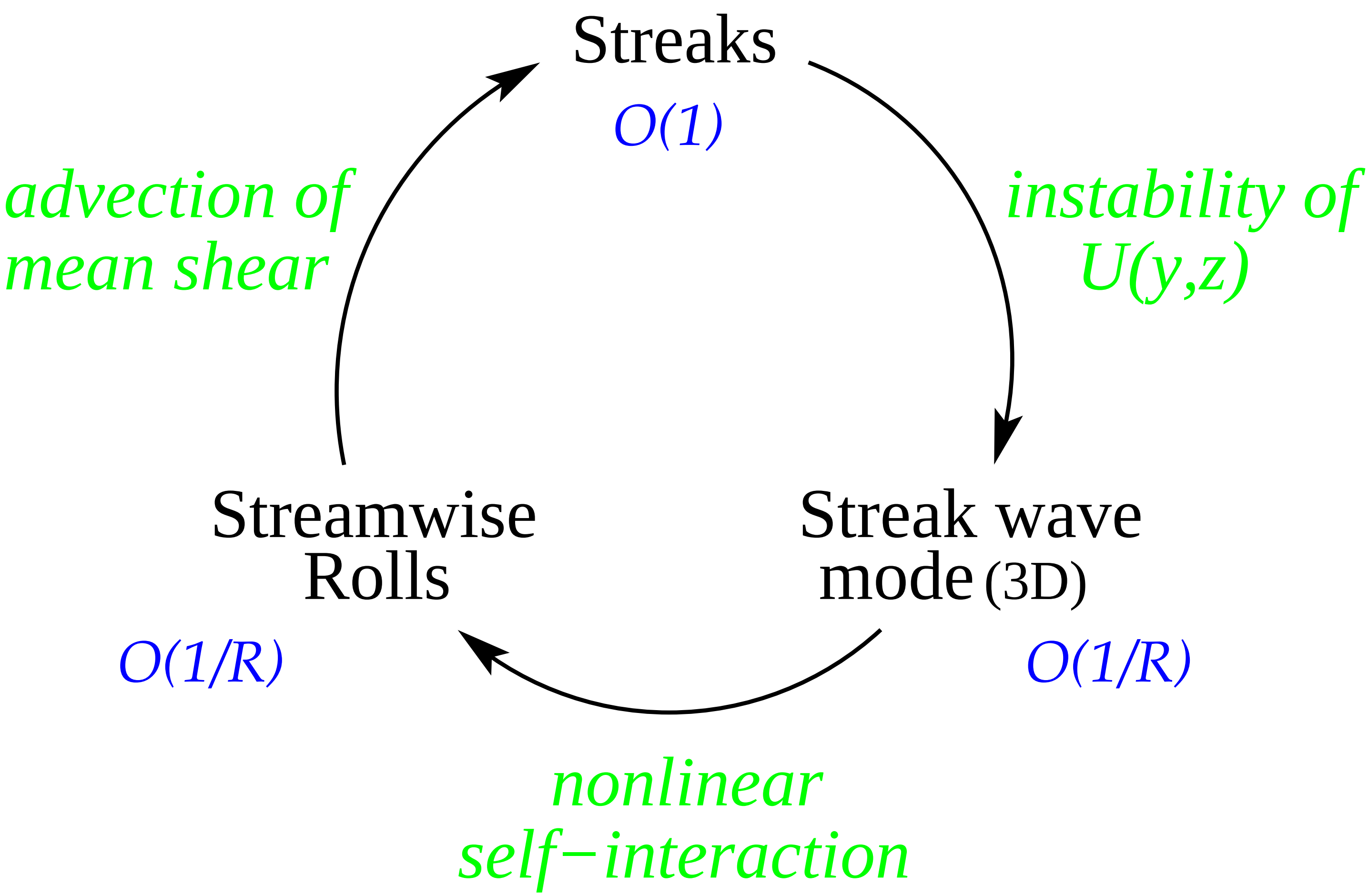
$$A_{\rm cp} = -(\mathrm{i}k_z)\; U'(y)$$





The later stages of transition

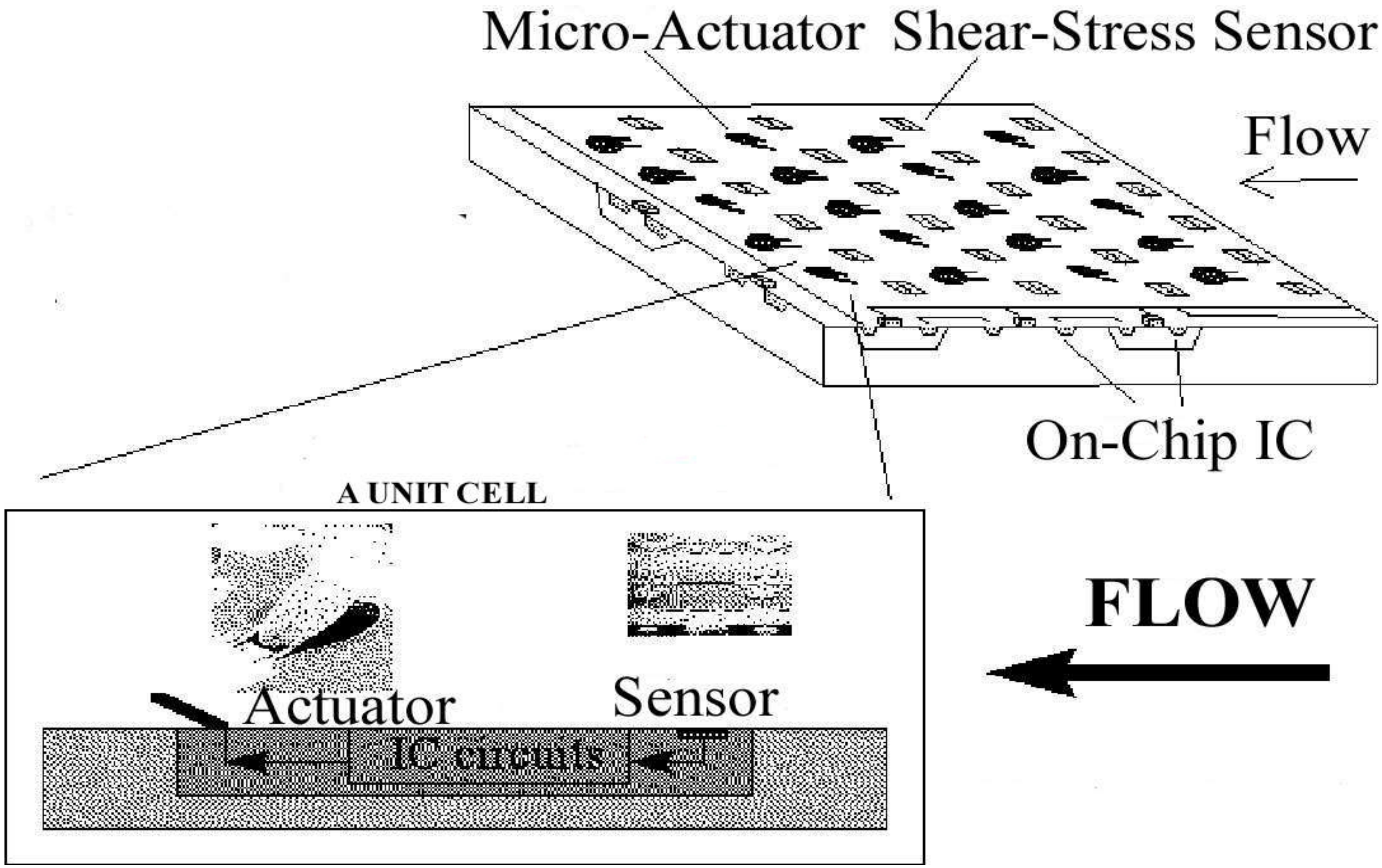
- QUADRATIC INTERACTIONS: **self-sustaining process**
  Waleffe, *Phys. Fluids '97*



FLOW CONTROL

- **Objective**
  - ⋆ **controlling the onset of turbulence**
- **Transition initiated by**
  - ⋆ **high flow sensitivity**
- **Control strategy**
  - ⋆ **reduce flow sensitivity**

Feedback flow control



technology: shear-stress sensors & magnetically actuated micro-flaps

application: turbulence suppression or skin-friction drag reduction

challenge: distributed controller design for complex flow dynamics (Bewley, Henningson, Kim, Krstić, Speyer, McKeon, . . .)

Sensor-free flow control

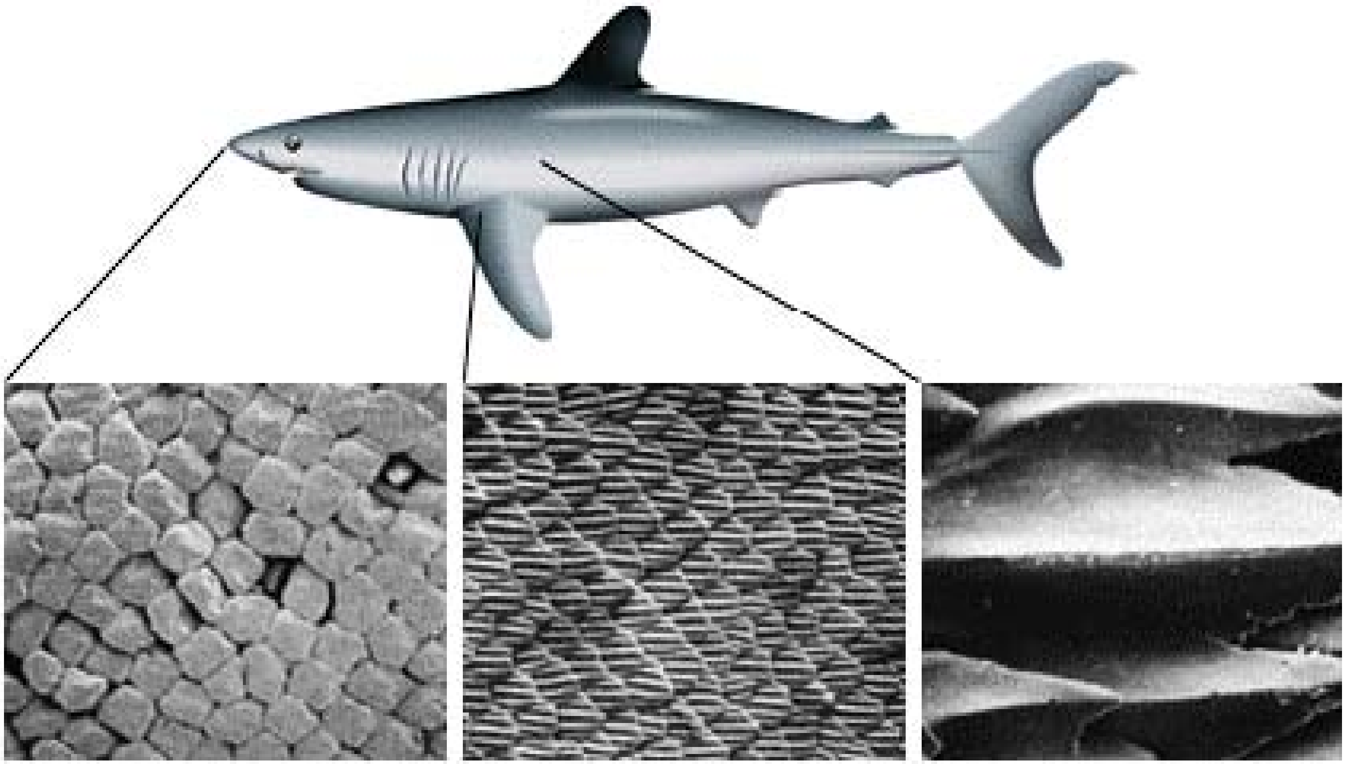
- **Geometry modifications**
  - ⋆ riblets
  - ⋆ super-hydrophobic surfaces
  - ⋆ surface roughness

- **Body forces**
  - ⋆ temporally/spatially oscillatory forces
  - ⋆ traveling waves

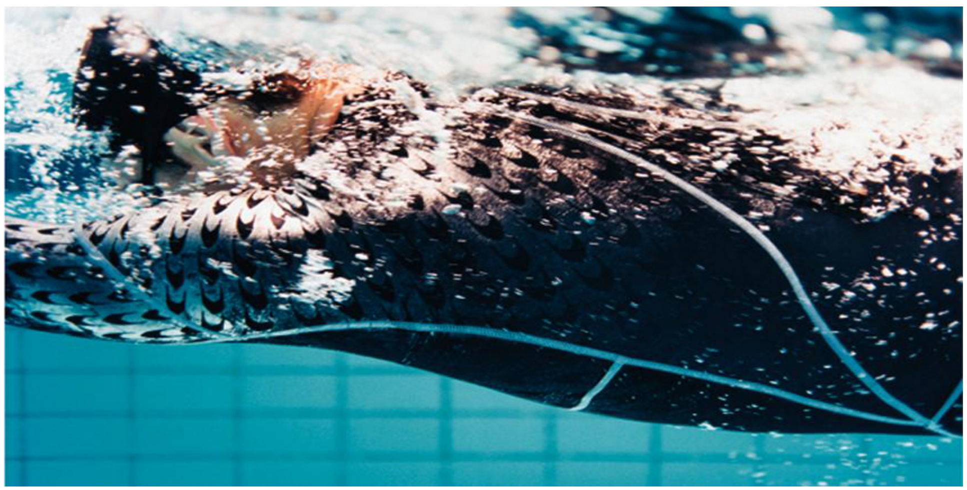
- **Wall oscillations**
  - ⋆ transverse wall oscillations

common theme: PDEs with spatially or temporally periodic coefficients

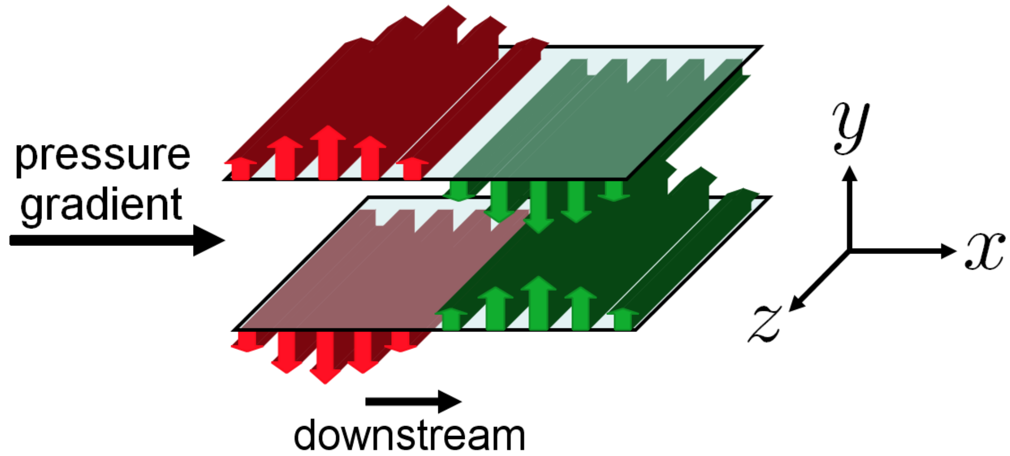
Flow control in nature . . .



. . . and in swimming competitions



Blowing and suction along the walls



NORMAL VELOCITY: $V(y = \pm 1) \;=\; \mp\,\alpha\,\cos\,(\omega_x(x \,-\, ct))$

- TRAVELING WAVE PARAMETERS:

spatial frequency: ω_x

speed: c $\quad \begin{cases} c > 0 & \text{downstream} \\ c < 0 & \text{upstream} \end{cases}$

amplitude: α

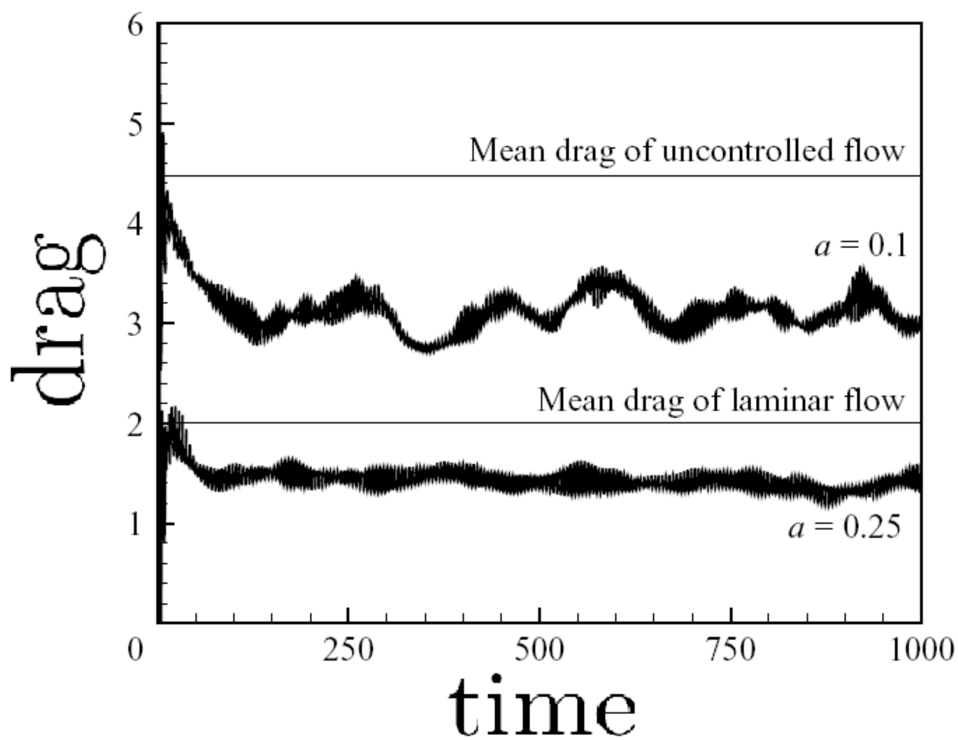
- INVESTIGATE THE EFFECTS OF c, ω_x, α ON:

⋆ **cost of control**

⋆ **onset of turbulence**

Min, Kang, Speyer, Kim, *J. Fluid Mech. '06*:

⇒ SUSTAINED SUB-LAMINAR DRAG



CHALLENGE: **selection of wave parameters**

- THIS TALK:
  - ⋆ **cost of control**
  - ⋆ **onset of turbulence**

Effects of blowing and suction?

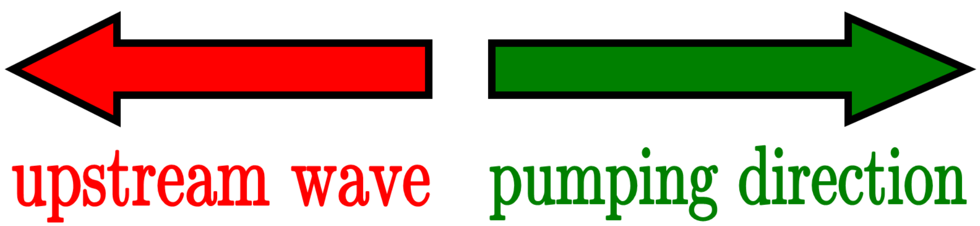
- DESIRED EFFECTS OF CONTROL:
  - ⋆ **bulk flux** $\nearrow$
  - ⋆ **net efficiency** $\nearrow$
  - ⋆ **fluctuations' energy** $\searrow$

TRAVELING WAVE

- ⋆ **induces a bulk flux (pumping)**

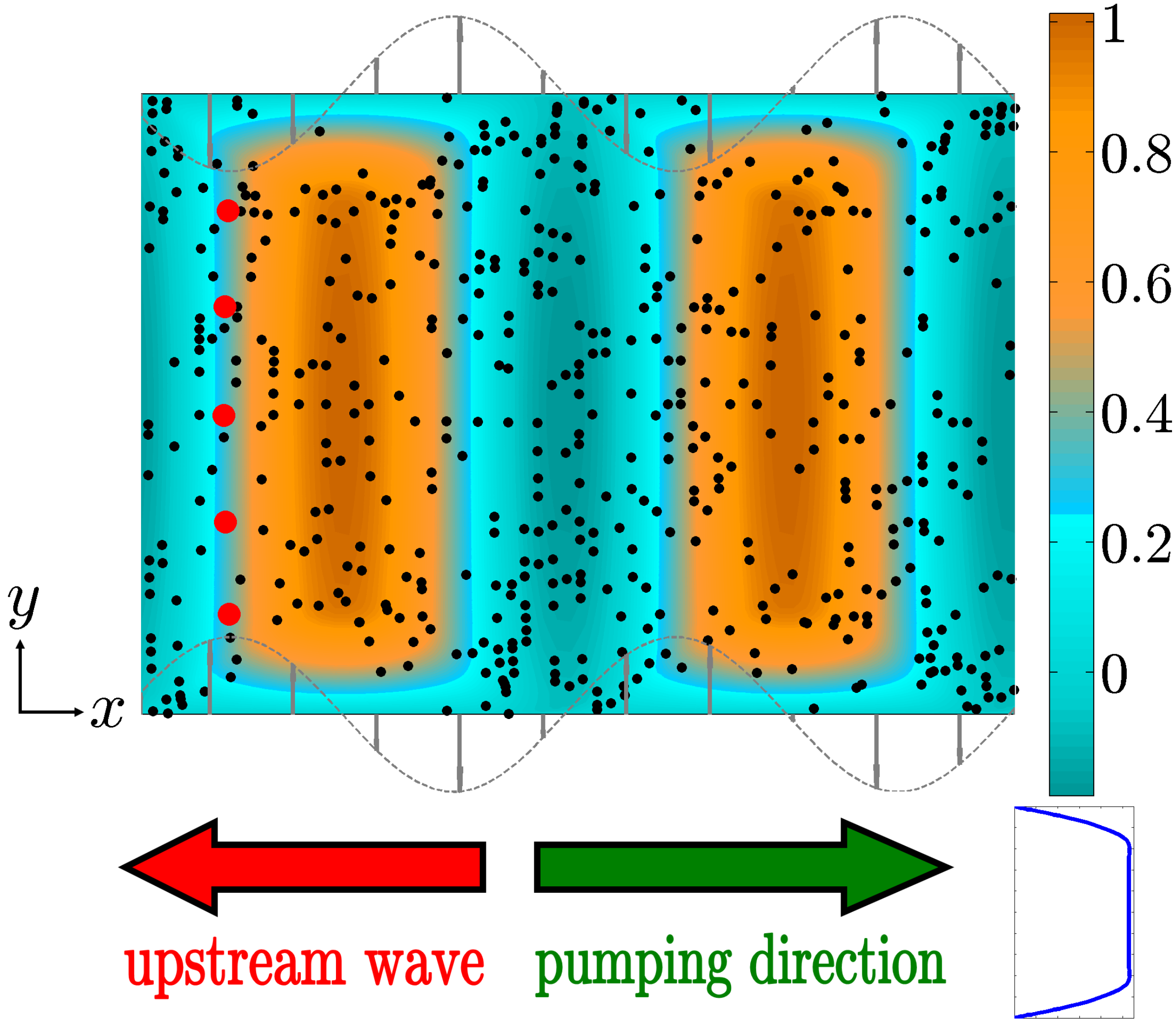
PUMPING DIRECTION

- ⋆ **opposite to a traveling wave direction**



Downstream vs. Upstream

STREAMWISE VELOCITY; NO PRESSURE GRADIENT:

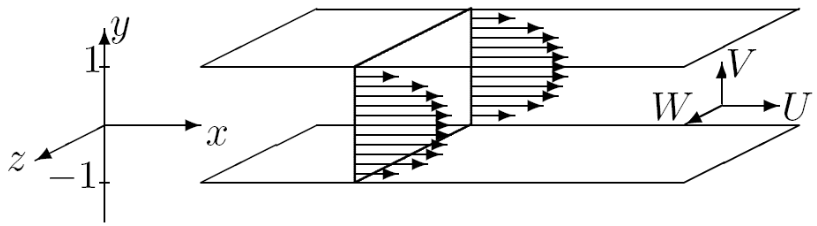


with pressure gradient:

DOWNSTREAM: **bulk** ↘

UPSTREAM: **bulk** ↗

Nominal velocity



$$V(y = \pm 1) = \mp\alpha\cos\left(\omega_x(x - ct)\right) = \mp\alpha\cos\left(\omega_x\bar{x}\right) \Rightarrow$$

$\bar{\mathbf{u}} = (U(\bar{x},y),\ V(\bar{x},y),\ 0)$

steady in a traveling wave frame

periodic in $\bar{x}$

- SMALL AMPLITUDE BLOWING/SUCTION

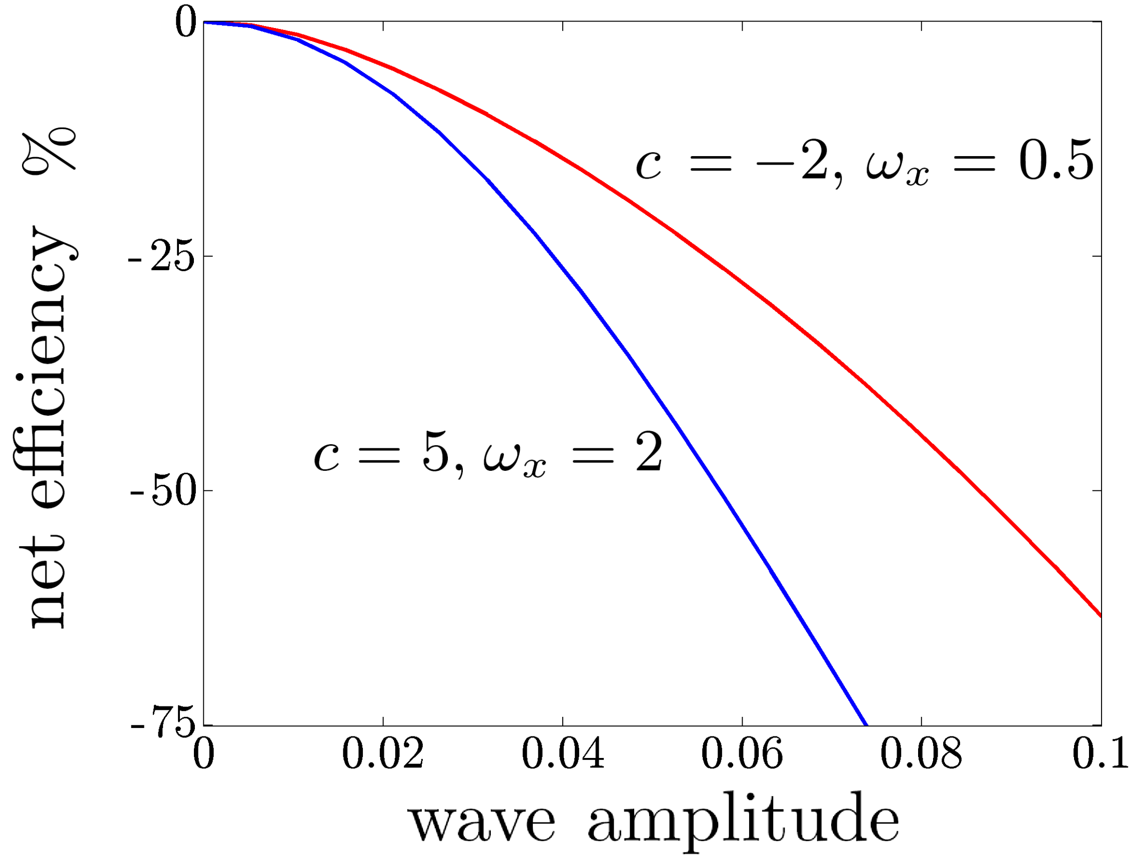
  weakly-nonlinear analysis

$$U(\bar{x},y) = \overbrace{U_0(y)}^{\text{parabola}} + \alpha^2\,\overbrace{U_{2,0}(y)}^{\text{mean drift}} + \alpha\,\overbrace{\left(U_{1,-1}(y)\,\mathrm{e}^{-\mathrm{i}\omega_x\bar{x}} + U_{1,1}(y)\,\mathrm{e}^{\mathrm{i}\omega_x\bar{x}}\right)}^{\text{oscillatory: no mean drift}}$$
$$+ \alpha^2\left(U_{2,-2}(y)\,\mathrm{e}^{-2\mathrm{i}\omega_x\bar{x}} + U_{2,2}(y)\,\mathrm{e}^{2\mathrm{i}\omega_x\bar{x}}\right)$$
$$+ \mathcal{O}(\alpha^3)$$

Nominal net efficiency

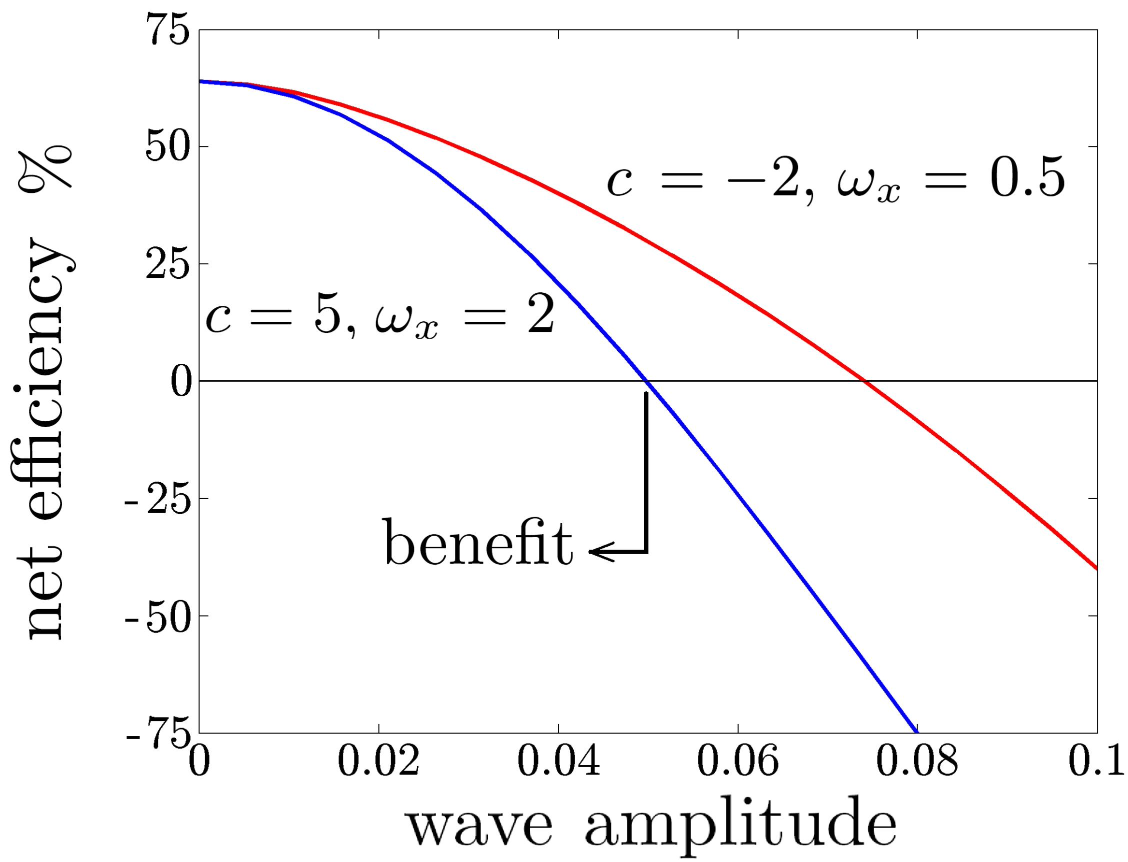
ASSUME: no control: laminar
with control: laminar

$$\frac{\text{Net Efficiency}}{\text{Power Required to Drive Laminar Flow}} \approx -\alpha^2 \left|\pi_2(Re; \omega_x, c)\right| < 0$$

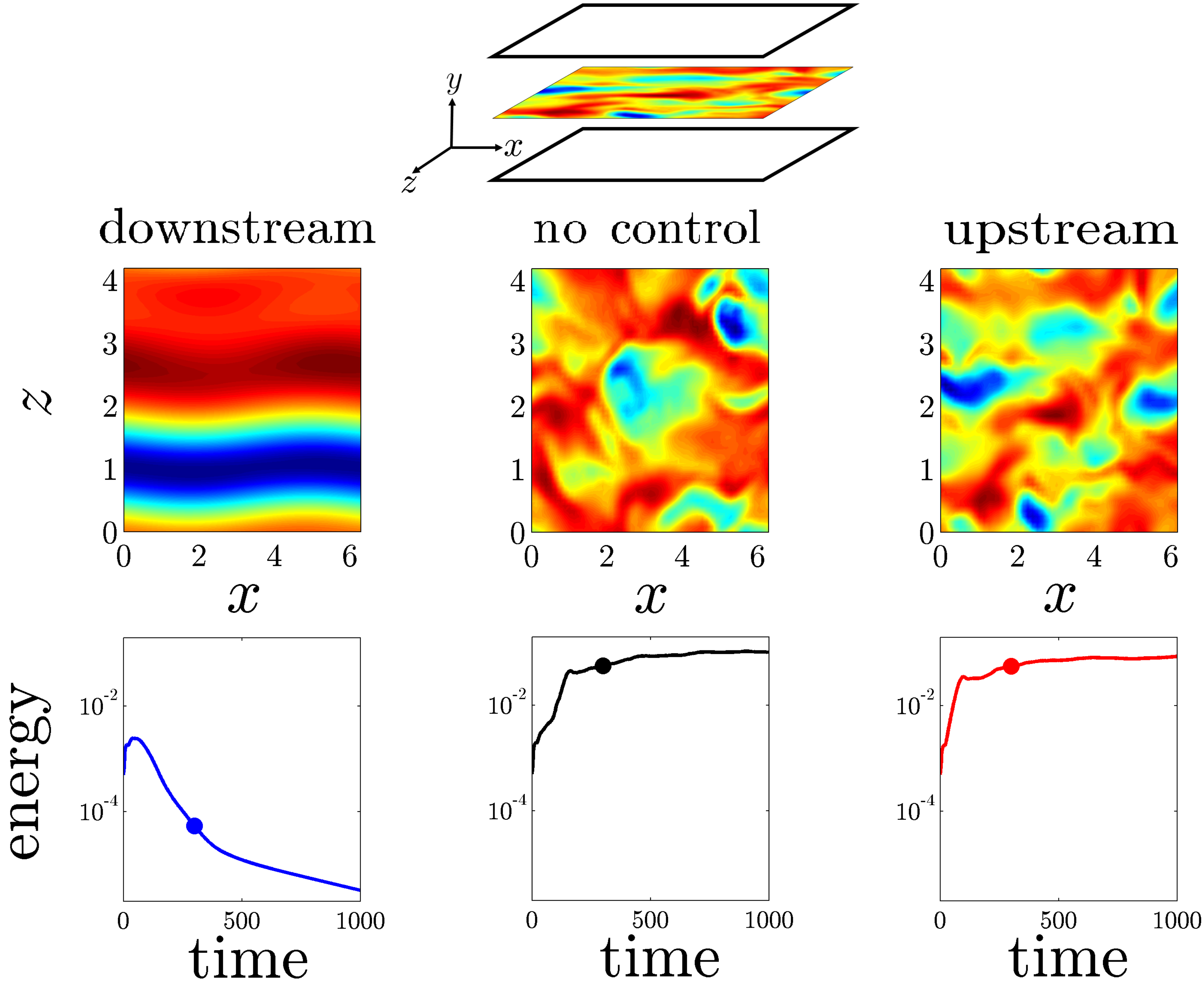


ASSUME: $\begin{cases} \text{no control:} & \text{turbulent} \\ \text{with control:} & \text{laminar} \end{cases}$

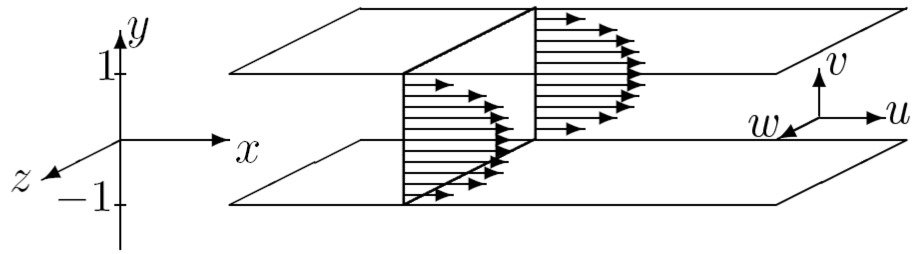
$$\frac{\text{Net Efficiency}}{\text{Power Required to Drive Turbulent Flow}} \approx \frac{U_{\mathrm{B,lam}}}{U_{\mathrm{B,turb}}} \left(\underbrace{1 - \frac{U_{\mathrm{B,turb}}}{U_{\mathrm{B,lam}}}}_{>0} - \alpha^2 \, |\pi_2(Re; \omega_x, c)| \right)$$



Velocity fluctuations: DNS preview



Ensemble average energy density



EVOLUTION MODEL: **PDEs with periodic coefficients in $\bar{x}$**

$$\left.\begin{aligned} \boldsymbol{\psi}_t &= A\,\boldsymbol{\psi} + B\,\mathbf{d} \\ \mathbf{v} &= C\,\boldsymbol{\psi} \end{aligned}\right\}$$

$\mathbf{d} = \mathbf{d}(\bar{x}, y, z, t) \rightsquigarrow$ stochastic body forcing

$\mathbf{v} = (u,\ v,\ w) \rightsquigarrow$ velocity fluctuations

$\boldsymbol{\psi} = (v,\ \eta) \rightsquigarrow$ normal velocity/vorticity

- SIMULATION-FREE APPROACH TO DETERMINING ENERGY DENSITY

effect of small wave amplitude:

$$E(\theta, k_z; Re, \omega_x, c, \alpha) \;=\; E_0(\theta, k_z; Re, \omega_x) \;+\; \underbrace{\alpha^2}_{\text{small}} E_2(\theta, k_z; Re, \omega_x, c) \;+\; \mathcal{O}(\alpha^4)$$

$E(\theta, k_z; Re, \omega_x, c, \alpha)$ $\downarrow$ **with control**

$E_0(\theta, k_z; Re, \omega_x)$ $\downarrow$ **w/o control**

$(\theta,\ k_z) \rightsquigarrow$ spatial wavenumbers

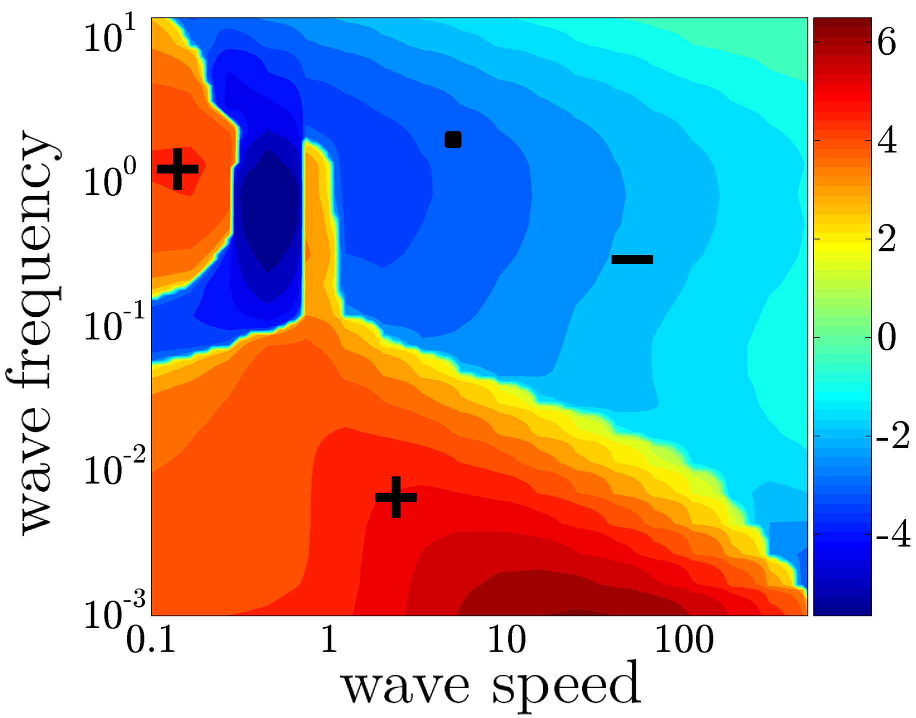
Energy amplification: controlled flow with $Re = 2000$

explicit formula:

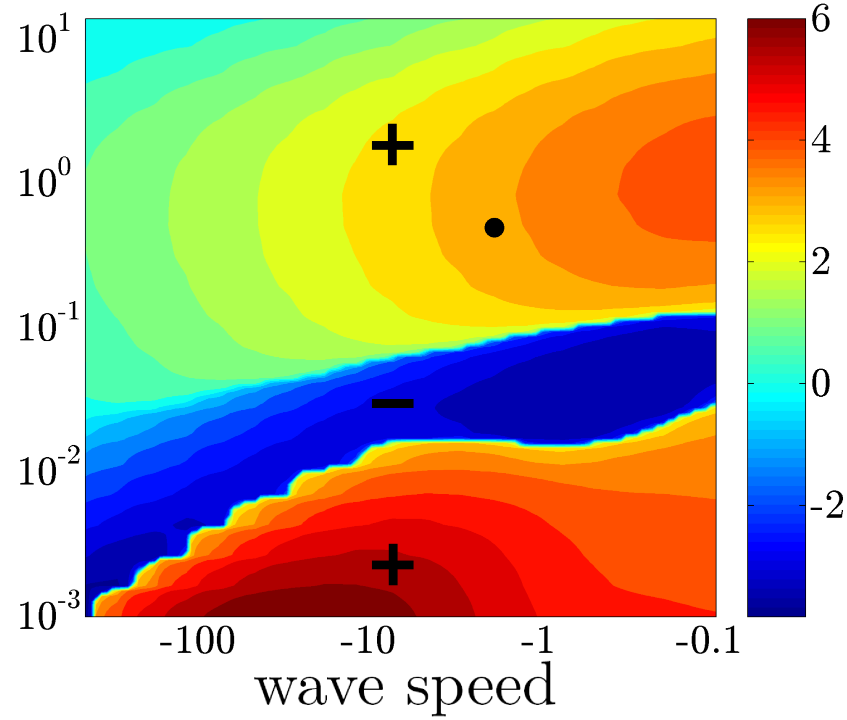
$$\frac{\text{energy density with control}}{\text{energy density w/o control}} \approx 1 + \alpha^2 g_2(\theta, k_z; \omega_x, c)$$

- $(\theta = 0, \; k_z = 1.78)$: **most energy w/o control**

g_2, downstream:



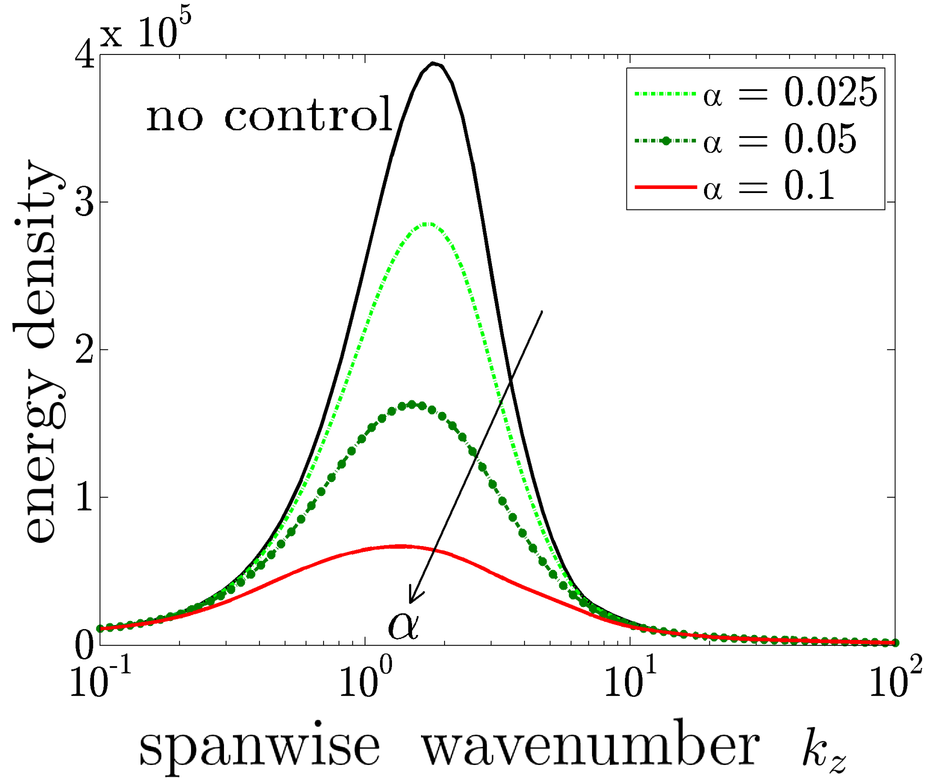
g_2, upstream:



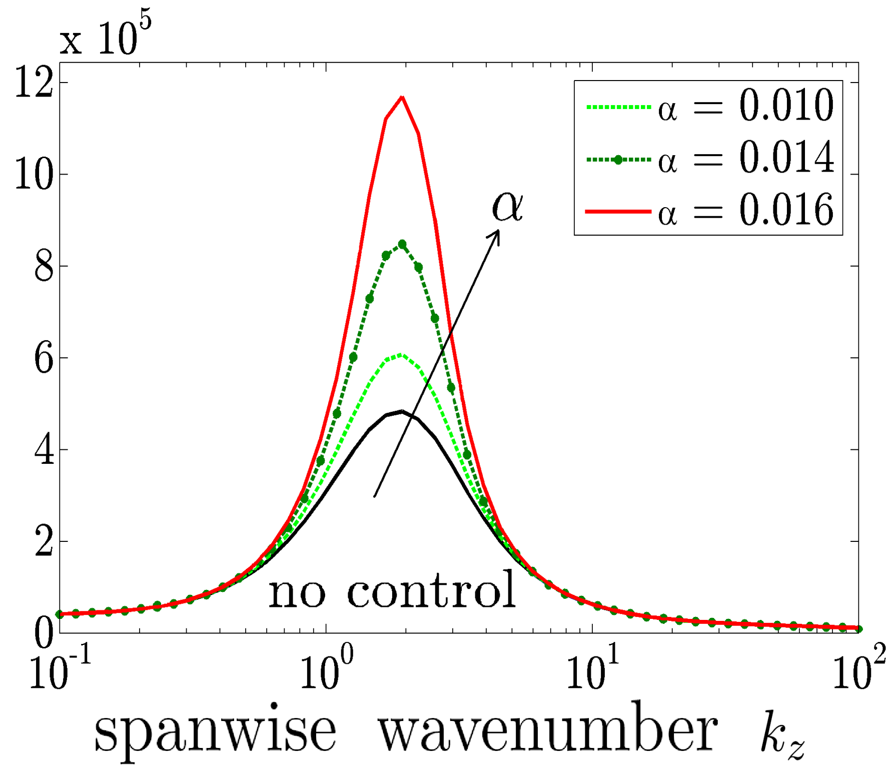
Effect of amplitude α on energy density

$(\theta = 0, k_z = 1.78)$

downstream $(c = 5, \omega_x = 2)$:



upstream $(c = -2, \omega_x = 0.5)$:



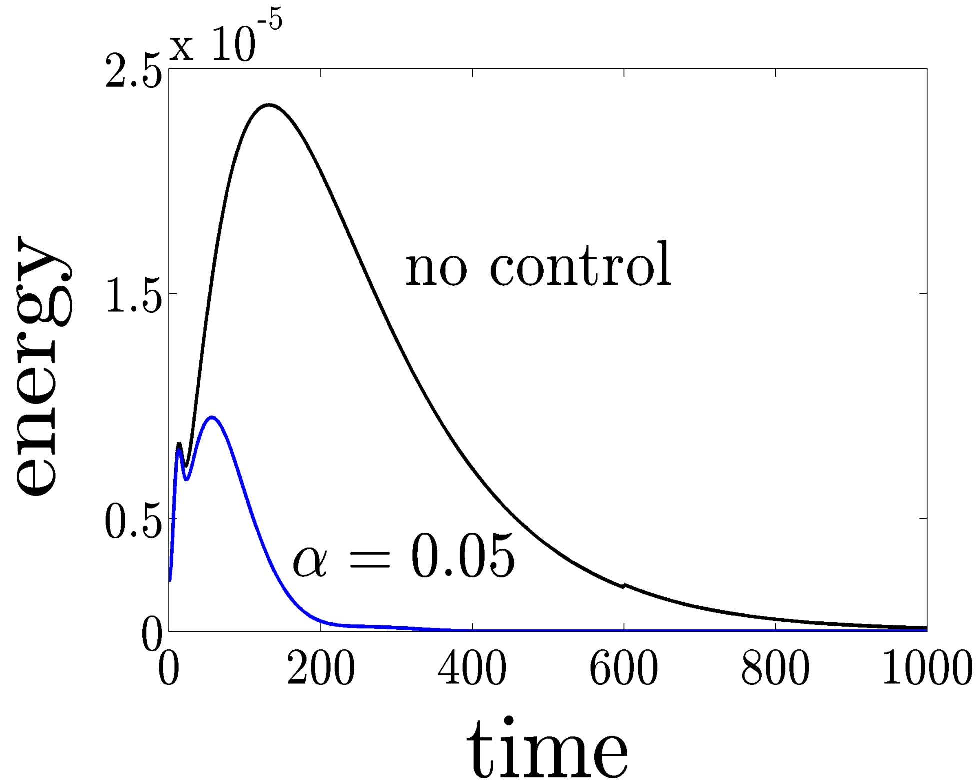
PEAK TO PEAK: $\approx \begin{cases} 60\,\% \text{ reduction}, & \alpha = 0.05 \\ 80\,\% \text{ reduction}, & \alpha = 0.1 \end{cases}$

DNS results: avoidance/promotion of turbulence

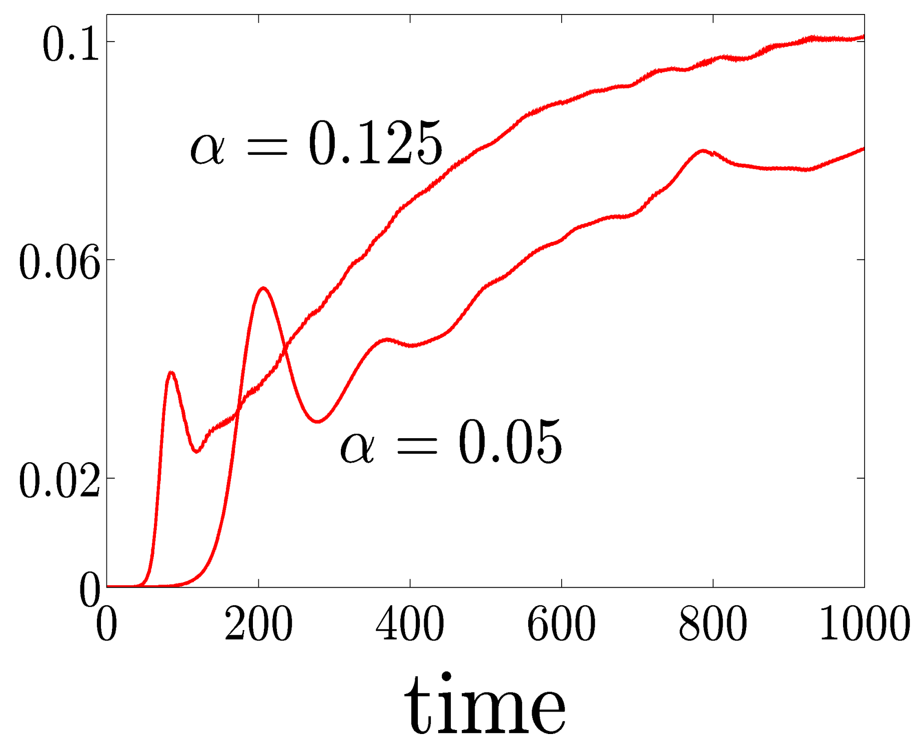
small initial energy: $E(0) = 2.25 \times 10^{-6}$

(flow with no control stays laminar)

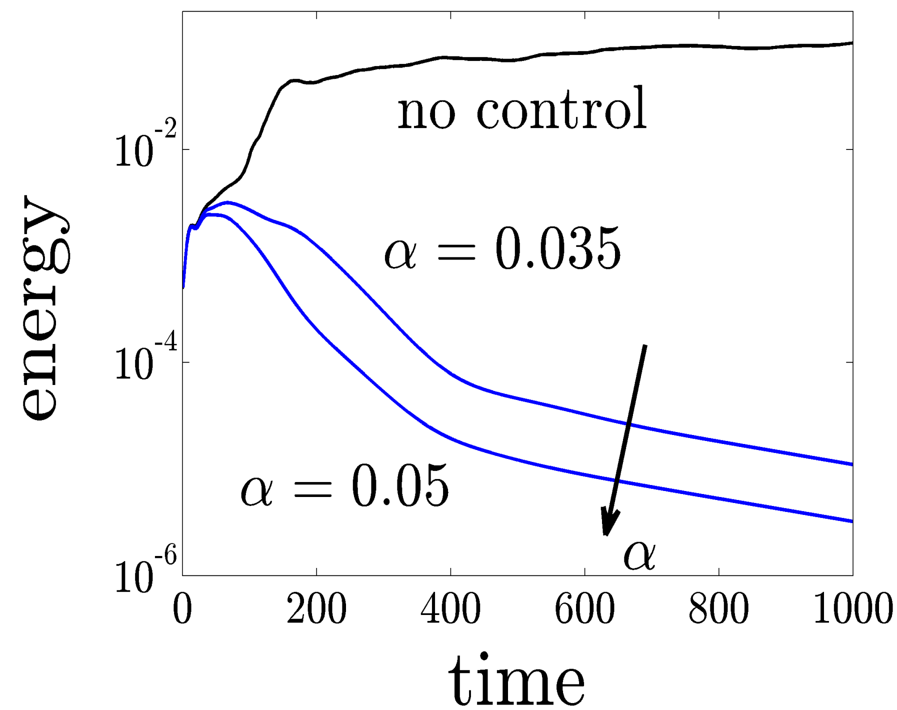
DOWNSTREAM: NO TURBULENCE



UPSTREAM: PROMOTES TURBULENCE



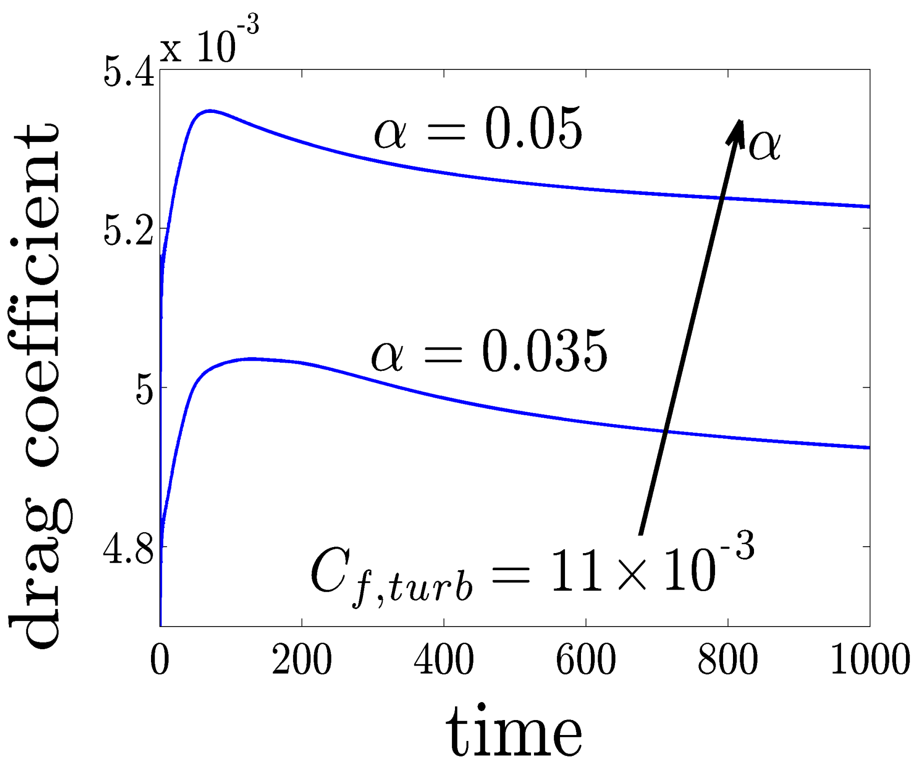
NO TURBULENCE:

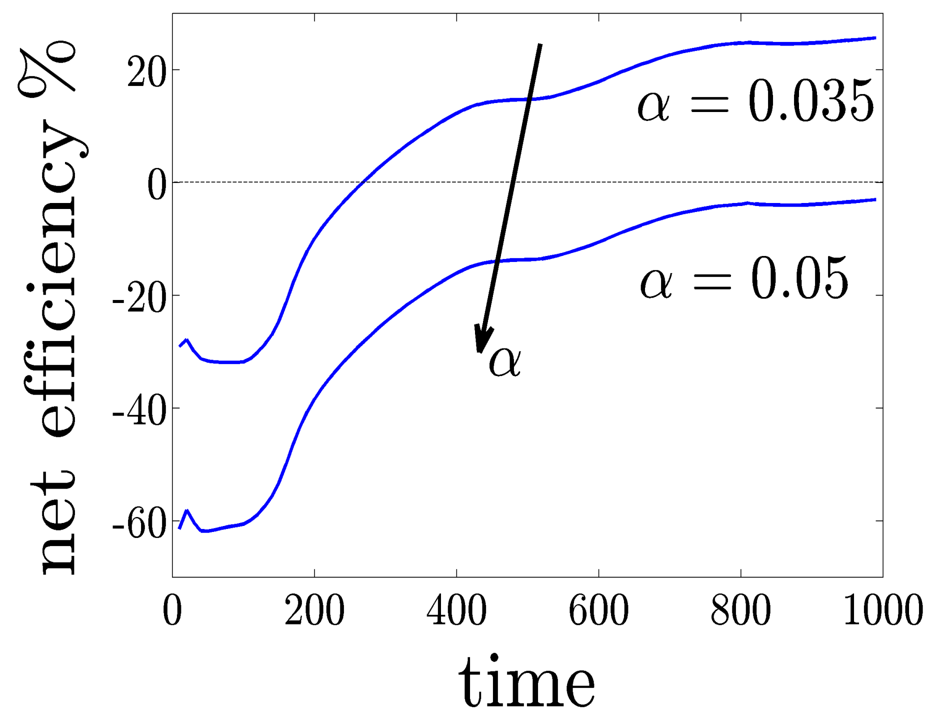


DOWNSTREAM:

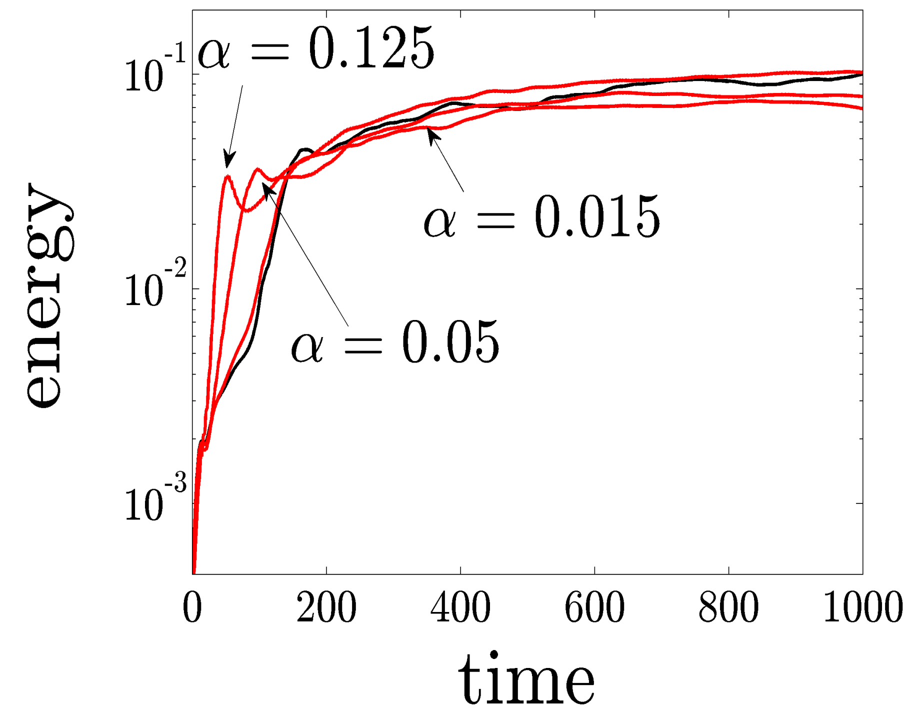
moderate initial energy:

$E(0) \; = \; 5.06 \times 10^{-4}$





TURBULENCE:



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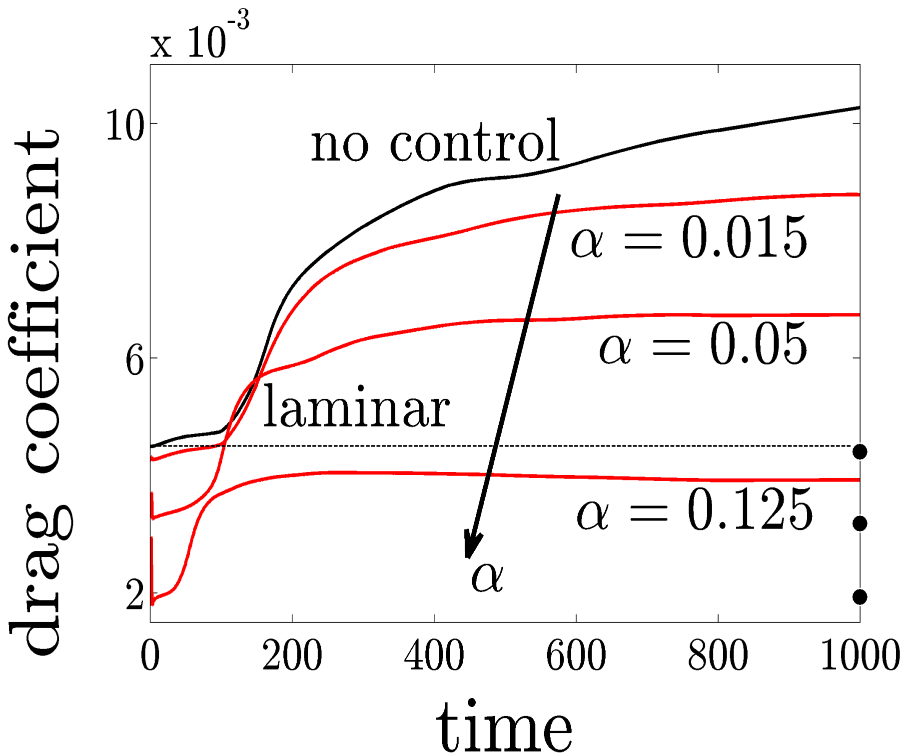
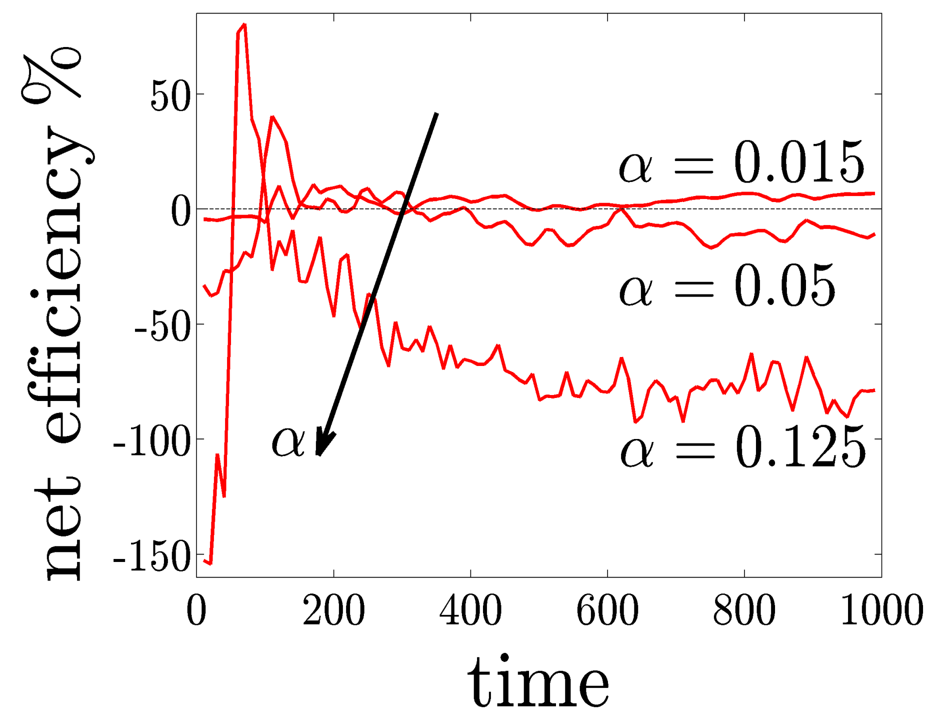


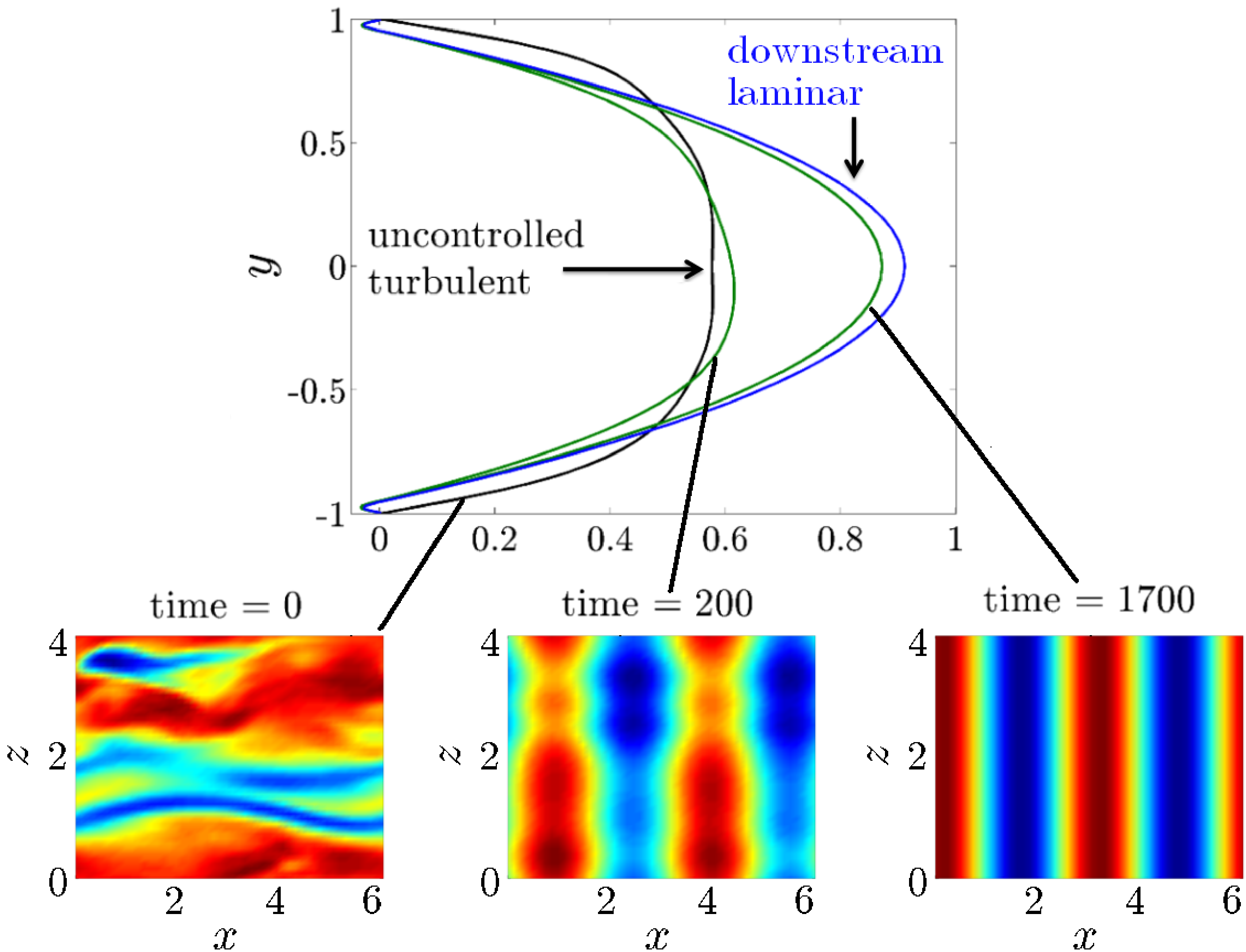


DNS results: summary

Initial Energy	c	ω_x	α	$10^3\ C_f$	$\%\Pi_{net}$
small	–	–	–	4.5	0
	5	2	0.050	5.2	-38.5
	-2	0.5	0.050	6.0	-39.7
	-2	0.5	0.125	3.7	-96.2
moderate	–	–	–	10.3	0
	5	2	0.035	4.9	25.6
	5	2	0.050	5.2	-3.1
	-2	0.5	0.015	8.8	6.8
	-2	0.5	0.050	6.7	-10.8
	-2	0.5	0.125	3.9	-78.8
large	–	–	–	11.2	0
	5	2	0.050	11.9	-51.3
	5	2	0.125	12.1	-208.2
	-2	0.5	0.050	7.4	-20.5
	-2	0.5	0.125	4.0	-85.2

Relaminarization by downstream wave

MEAN VELOCITY:



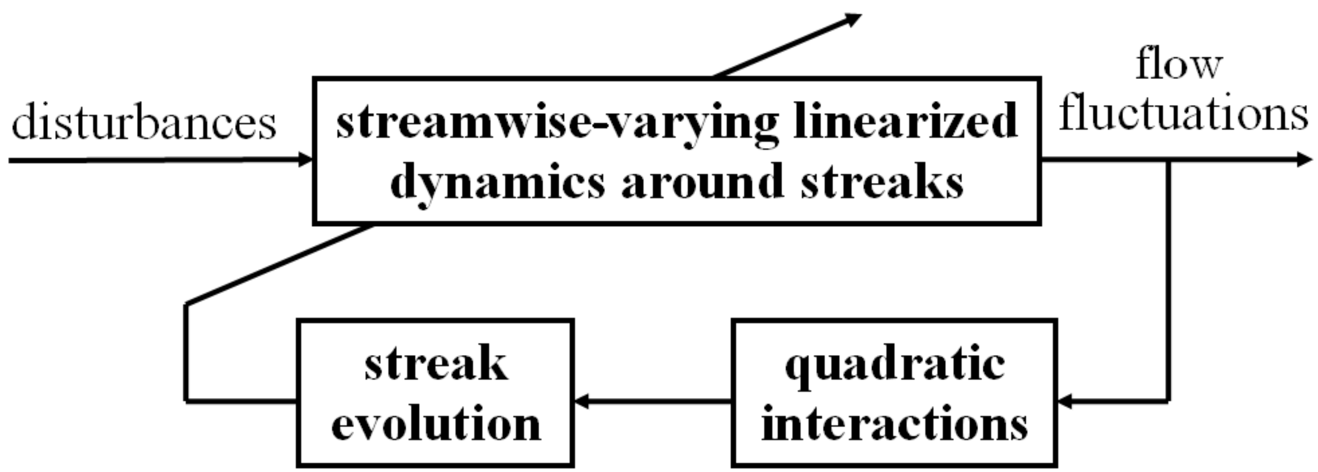
Summary: the early stages of transition

- INPUT-OUTPUT ANALYSIS
  - ⋆ quantifies flow sensitivity
  - ⋆ reveals distinct mechanisms for subcritical transition

    streamwise streaks, oblique waves, TS-waves
  - ⋆ exemplifies the importance of streamwise elongated flow structures

    Jovanović & Bamieh, *J. Fluid Mech. '05*

- LATER STAGES OF TRANSITION
  - ⋆ **challenge: relative roles of flow sensitivity and nonlinearity**



Summary: flow control

- FACTS REVEALED BY PERTURBATION ANALYSIS

Blowing/Suction Type	Nominal flow analysis	Energy amplification analysis
Downstream	reduce bulk flux	reduce amplification ✓
Upstream	increase bulk flux ✓	promote amplification

- POWERFUL SIMULATION-FREE APPROACH TO PREDICTING FULL-SCALE RESULTS
  - ⋆ DNS verification

Moarref & Jovanović, *J. Fluid Mech.* ’10

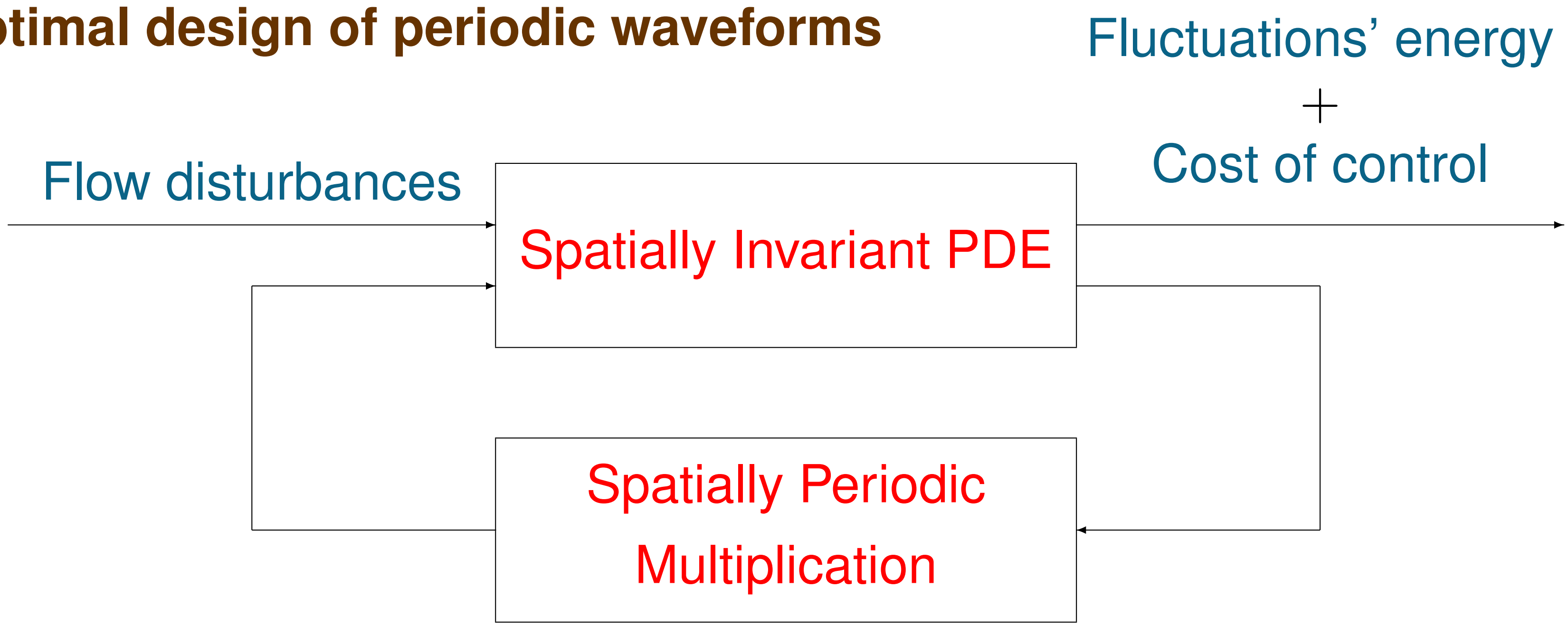
Lieu, Moarref, Jovanović, *J. Fluid Mech.* ’10

Outlook: model-based sensor-free flow control

GEOMETRY MODIFICATIONS	WALL OSCILLATIONS	BODY FORCES
riblets super-hydrophobic surfaces	transverse oscillations	oscillatory forces traveling waves

- USE DEVELOPED THEORY TO DESIGN GEOMETRIES AND WAVEFORMS FOR
  - ⋆ control of transition/skin-friction drag reduction

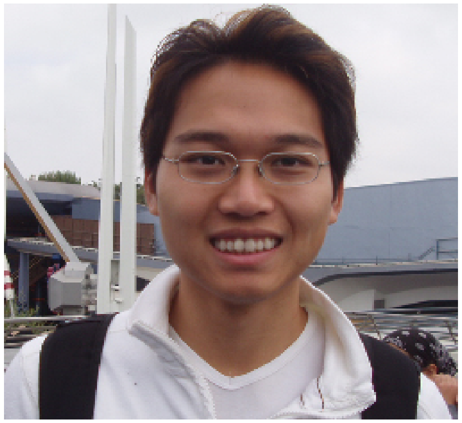
- CHALLENGES
  - ⋆ **control-oriented modeling of turbulent flows**
  - ⋆ **optimal design of periodic waveforms**



Acknowledgments

TEAM:

Rashad



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SUPPORT:

NSF CAREER Award CMMI-06-44793

DNS CODE:

www.channelflow.org

COMPUTING RESOURCES:

Minnesota Supercomputing Institute