HW#1

Show all work for full credit!

0. Reading Exercise: Read the CourseNotes for the last two weeks

1. Obtain a realization (the A, B, C, D matrices) of the following transfer functions

   (a) $G(s) = \frac{s^2+2s+3}{s^3+3s^2+4s+5}$

   (b) $G(s) = \frac{s^3+3s^2+2s+4}{2s^3+2s^2+3s+2}$

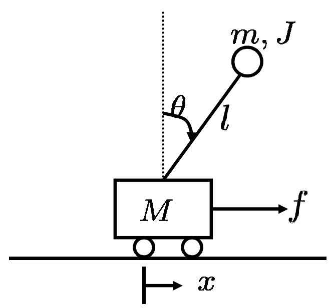
   Provide the analog computer simulation schematic also (in terms of adders, integrators and amplifiers).

2. A system with input $u(t)$ and output $y(t)$ is given by

$$\dot{y}(t) = u(t), \quad y(0) = 0.$$

   Prove that this system is linear, time-invariant, and causal (Assume that for all inputs are causal signals, that is, $u(t) = 0$ for $t < 0$).

3. A model for the inverted pendulum on a cart shown in figure below ...



   ... is described by the following set of equations

$$\begin{aligned} (M+m)\ddot{x} + ml\ddot{\theta}\cos\theta &= -b\dot{x} + ml\dot{\theta}^2\sin\theta + f \\ (J+ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta &= -mgl\sin\theta \end{aligned}$$

   Assume $M = m = 1Kg$, $J = 1\text{Kg}m^2$, $l = 1\text{m}$ and $b = 1$ Kg/s.

   (a) Obtain a four-dimensional nonlinear state-space representation with output $y = x$, input $u = f$, and states $[x_1\ x_2\ x_3\ x_4] = [x\ \theta\ \dot{x}\ \dot{\theta}]$.

   (b) An *equilibrium point* of a system of the form $\dot{x} = f(x, u)$ is obtained by solving for x that satisfies $f(x, 0) = 0$. Determine equilibrium points of this system (You can revise the notes on *equilibrium points*).

   (c) Linearize this system of equations around its around the trajectory about the equilibrium point $x_{eq} = [0\ \pi\ 0\ 0]$ (when $f(t) = 0$). Write it in state-space form.

   (d) Find the transfer function for the linear system obtained above.

4. **Discrete-time systems.** In class we have studied about continuous-time systems. We can make following analogous statements about the discrete-time systems. Read them and then solve the problem given below these statements.

(S1.) A nonlinear discrete-time system is described by a model of the form

$$\begin{aligned} x[k+1] &= f(x[k], u[k]]), x(0) = x_0 \\ y(k) &= h(x[k], u[k]), \end{aligned}$$

where $k \in \mathbb{N}$ denotes the discrete time. A similar linearization scheme (as in the continuous time case) would lead to a linearized model

$$\begin{aligned} \tilde{x}[k+1] &= A[k]\tilde{x}[k] + B[k]\tilde{u}[k], \\ \tilde{y}[k] &= C[k]\tilde{x}[k] + D[k]\tilde{u}[k] \end{aligned}$$

where, again, the $\{A[k], B[k], C[k], D[k]\}$ are obtained by evaluating the Jacobian matrices about he nominal state $\{x_{nom}[k], u_{nom}[k]\}$.

(S2.) An *equilibrium point* of a discrete-time system of the form $x[k+1] = f(x[k], u[k])$ is obtained by solving for x that satisfies $f(x, 0) = x$.

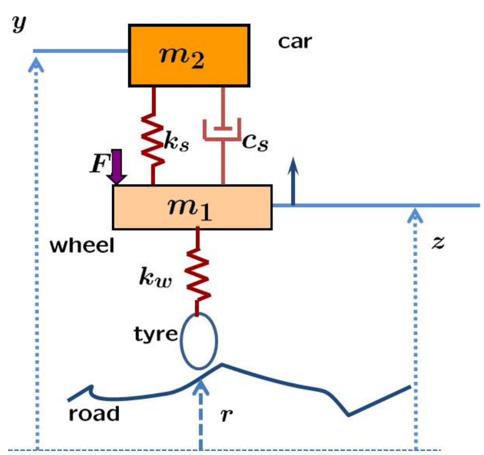
Consider the logistics equation, which is a basic model used in studying population dynamics,

$$x[k+1] = \alpha x[k] - \alpha x^2[k], x[0] = x_0, \alpha > 1$$

(a) Compute the two equilibrium points of this system.

(b) Determine the linearized system approximation about each equilibrium point.

(c) Let $\alpha = 1.5$. Determine $\tilde{x}[10]$ for each linearized system in terms of $\tilde{x}[0]$. State about which equilibrium is the corresponding linearized system a better approximation of the nonlinear logistics equation.



5. Consider the quarter car model shown, r, y and z denote the road height, car position and the wheel position respectively. F is a force acting on the wheel. Obtain a state space realization of the quarter car model with input being $[r,\ F]^T$ and output being the car position y. Obtain the transfer functions from the input to the output.