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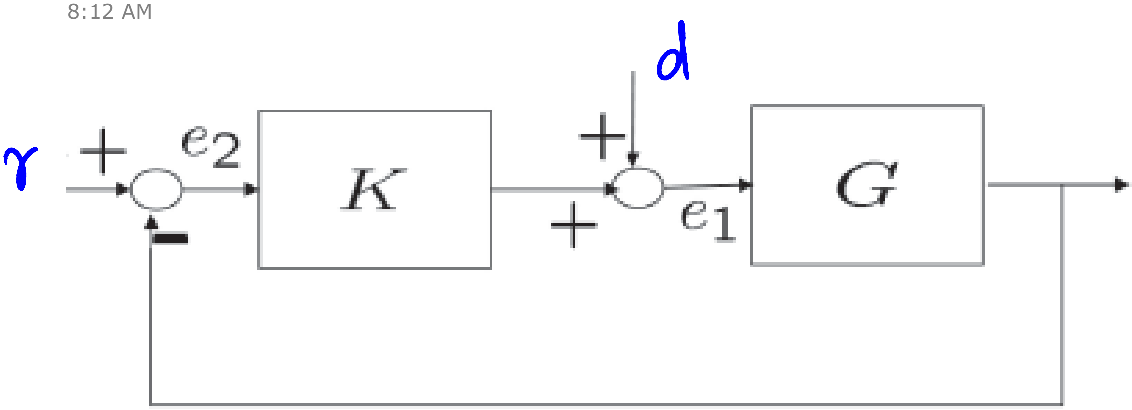

(a)

$$\left.\begin{aligned} e_2 &= r - G e_1 \\ e_1 &= K e_2 + d \end{aligned}\right\}$$

Well posedness:

$$e_2 + G e_1 = r$$
$$e_1 - K e_2 = d$$

$$\begin{bmatrix} G & I \\ I & -K \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} r \\ d \end{bmatrix}$$

$\det(I + GK)$ should not be 0.

$$\det(I+GK) \not\equiv 0$$

$\exists\, s_0 \in \mathbb{C}$ such that $(I+GK)(s_0) \neq 0$

Stability: (Definition)

The interconnection is stable

if and only if

$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ are uniformly bounded

for with respect to input $\begin{pmatrix} r \\ d \end{pmatrix}$

Theorem: Interconnection is stable if and only if

$$\begin{pmatrix} -G & I \\ I & -K \end{pmatrix}^{-1} \text{ is stable}$$

as a transfer matrix.

Proof:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} G & I \\ I & -K \end{pmatrix}^{-1} \begin{bmatrix} r \\ d \end{bmatrix}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$= \frac{1}{1+GK} \begin{bmatrix} -K & -I \\ -I & G \end{bmatrix}$$

$\frac{K}{1+GK}, \frac{I}{1+GK}, \frac{G}{1+GK}$ to be stable.

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The interconnection is stable if and only if $d_u d_K + n_u n_K$ has no zeros in the rhp where

$$G = \frac{n_u}{d_u} \; ; \quad K = \frac{n_K}{d_K}$$ are

coprime representations of G and K. $n_u, n_K; d_u, d_K$ are polynomials in s.

→ $\underline{n}$ and d polynomials are coprime if n and d have no common factors.

$$\frac{(S-1)^2}{S-1} \neq (S-1)$$

→ $d_u d_K + n_u n_K$ is called the characteristic polynomial.

→ Proof: closed-loop map is

$$\Big(\quad\quad 1 - K \quad\quad -1 \Big)$$

$$\frac{}{1+GK} \begin{pmatrix} -I & G \end{pmatrix}$$

$$= \frac{1}{1+\frac{n_G}{d_G}\frac{n_K}{d_K}} \begin{pmatrix} -\frac{n_K}{d_K} & -I \\ -I & \frac{n_G}{d_G} \end{pmatrix}$$

$$= \frac{d_G d_K}{d_G d_K + n_G n_K} \begin{bmatrix} -\frac{n_K}{d_K} & -I \\ -I & \frac{n_G}{d_G} \end{bmatrix}$$

$$= \frac{1}{d_G d_K + n_G n_K} \begin{bmatrix} -n_K d_G & -d_G d_K \\ -d_G d_K & d_K n_G \end{bmatrix}$$

If $d_G d_K + n_G n_K$ has no zeros in rhp then system is stable

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Suppose $(d_u d_K + n_u n_K)(s_0) = 0$
with $s_0 \in$ RHP.

$\Rightarrow \dfrac{d_u d_K}{d_u d_K + n_u n_K}$; $\dfrac{d_u n_K}{d_u d_K + n_u n_K}$

$\dfrac{d_K n_u}{d_u d_K + n_u n_K}$ are stable

transfer function.

This is possible only if

$(d_u d_K)(s_0) = 0 \Rightarrow d_u = 0$ or $d_K = 0$
or both.

$(d_u n_K)(s_0) = 0$

$(d_K n_u)(s_0) = 0$

Case 1: $d_u = 0$

$(d_u n_K)(s_0) = 0$

$(d_K n_u)(s_0) = 0$

$d_K(s_0) = 0$ or $n_u(s_0) = 0$

$\downarrow$

$d_K(s_0) = 0$

$(n_u n_K)(s_0) = 0$

$n_c(s_0) = 0$ or $\underline{\underline{n_{ic}(s_0) = 0}}$

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Summary:

The interconnection is stable if and only if the characteristic polynomial $d_G d_K + n_G n_K$ has no rhp zeros. ($G = \frac{n_G}{d_G}$; $K = \frac{n_K}{d_K}$ are coprime representations).

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Interconnection is stable if and only if

(1) $(I+L) = I+GK$ has no zeros in the RHP

(2) There is no unstable pole-zero cancellation in forming $GK = \frac{n_G n_K}{d_G d_K}$.

$\Rightarrow$ ~~Th~~ Stability $\Rightarrow$ (1) and (2)

Pf: stability $\Rightarrow$ $d_G d_K + n_G n_K$ has no zero in rhp

$\Downarrow$ $I + GK = \frac{d_G d_K + n_G n_K}{d_G d_K}$

has no zeros in rhp

$\Downarrow$ (1)

Stability $\Rightarrow$ $d_G d_K + n_G n_K$ has no zero in the rhp

Suppose $(d_G d_K)(s_0) = n_G n_K(s_0)$ with $s_0 \in$ RHP

$$\Downarrow \quad (d_{\llcorner} d_{IK} + \cap_{\llcorner} \cap_{IK})(\delta\omega) = 0$$

$$\Downarrow \quad (2).$$

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Suppose $(I+L)$ has no rhp zeros and no unstable pole-zero cancellation while forming $\frac{n_L n_K}{d_L d_K}$.

$\Leftarrow$ $I+L = \dfrac{d_L d_K + n_L n_K}{d_L d_K}$ has no rhp zeros.

$\Rightarrow$ $(d_L d_K + n_L n_K)(s_0) = 0$ for some $s_0 \in RHP$

$\Downarrow$

$$\left.\begin{aligned} &(d_L d_K)(s_0) = 0 \\ \Downarrow\ &(n_L n_K)(s_0) = 0 \end{aligned}\right\} \Rightarrow$$

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GK

Summary:

The feedback interconnection is stable $\Leftrightarrow$ (1) (I+L has no zeros in rhp)

(2) (There are no unstable pole-zero cancellations while forming $\frac{N_G N_K}{D_G D_K}$.)