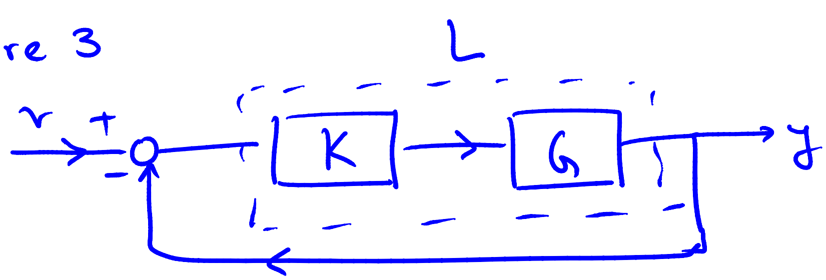
Lecture 3



L : Loop gain.

① The interconnection is stable if and only if

(i) $1+L(s)$ has no zeros in the rhp

(ii) There are no unstable pole-zero cancellations while forming the product $L = GK$; i.e there are no rhp common factors between $n_G n_K$ and $d_G d_K$.

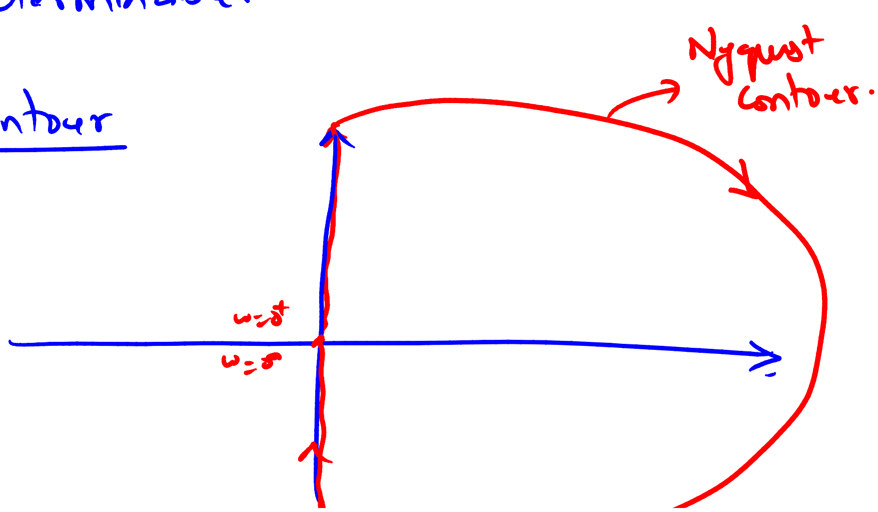
② Note $1+L(s) = 1 + \frac{n_G n_K}{d_G d_K} = \frac{d_G d_K + n_G n_K}{d_G d_K}$

has the same unstable poles as $GK = L$. assuming (ii) is satisfied.

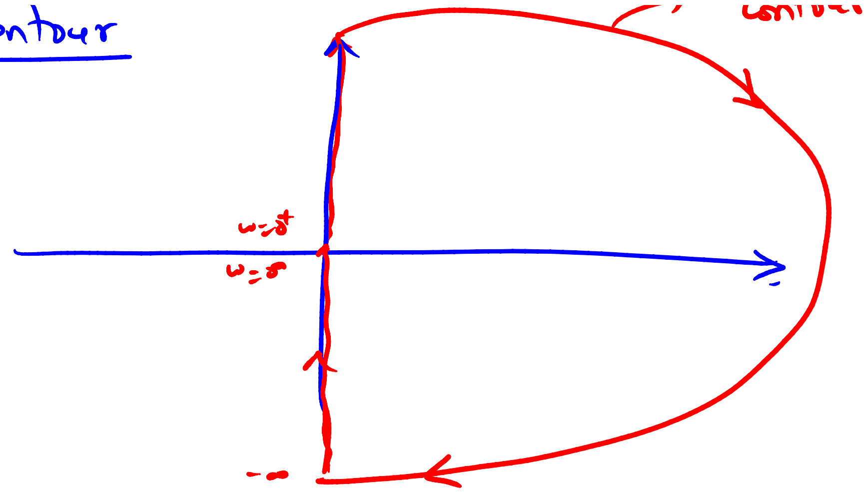
P : # of poles of $1+L$ in the rhp.

$\equiv$ # of poles of L in the rhp which is easily determinable.

③ Nyquist Contour



Nyquist Contour



- Suppose $(1+L(s))$ has Z zeros in the rhp and has P poles in the rhp then the # of clockwise encirclements of the origin in the plot of $Im(1+L)$ vs $Re(1+L)$ as given N satisfies

$$N = Z - P$$

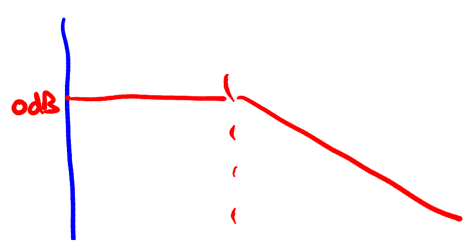
$$\Rightarrow \quad Z = N + P$$

→ The clockwise encirclements of the origin of $1+L$ = the # of clockwise encirclements of L of the point $(-1, 0)$ in the complex place

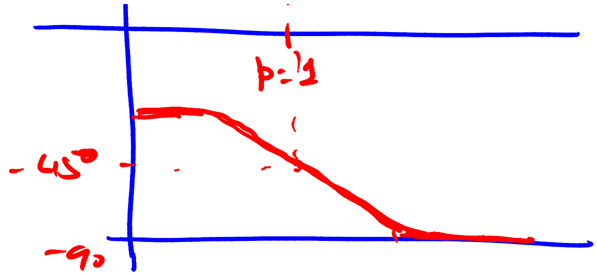
Examples of Nyquist plot:

Example 1

$$G(s) = \frac{1}{s+1}$$



$$G(j\omega) = \frac{1}{j\omega+1} = \frac{1-j\omega}{1+\omega^2}$$



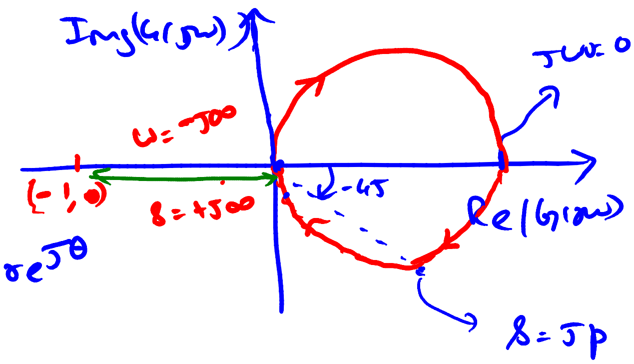
$\omega = 0$; $G(j0) = 1$

$\omega = p = 1$; $|G(j\omega)| = \frac{1}{\sqrt{2}}$

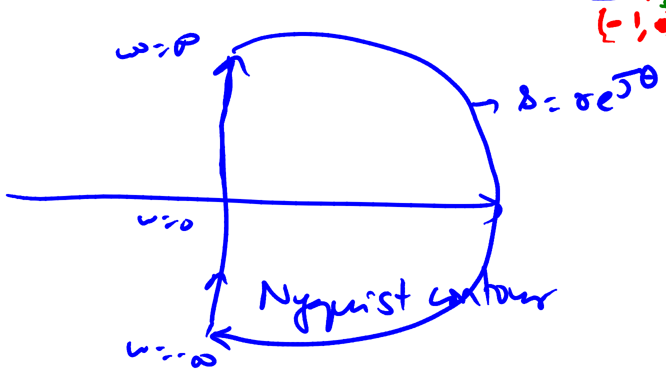
$\angle G(j\omega) = -45^\circ$

$\omega \gg p$; $\angle G(j\omega) = -90$

$|G(j\omega)| \approx 0$.



Nyquist plot



$N = 0$

$P = 0$

$Z = N + P = 0$

$\therefore$ the unity -ve feed back interconnection is stable

Example 2:

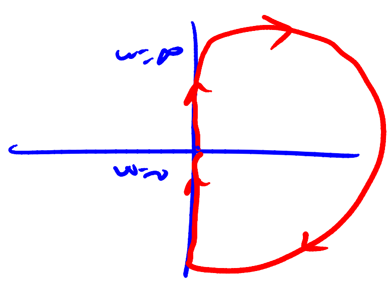
$$G(s) = \frac{2}{s-1}\ ; \quad G(j\omega) = \frac{2}{j\omega - 1} = \frac{2(1+j\omega)}{(j\omega)^2 - 1} = -2\,\frac{(1+j\omega)}{1+\omega^2}$$

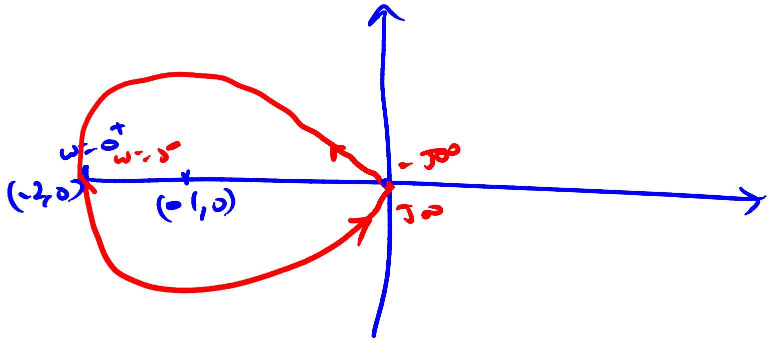
$$Re(G(j\omega)) = -\frac{2}{1+\omega^2}\ ; \quad Img(G(j\omega)) = -\frac{2\omega}{1+\omega^2}$$

$\omega = 0$; $G(j0) = -2$;

$\omega = |p| = 1$; $\angle G(jp) = -135^\circ$

$\omega \gg p$; $\angle G(j\infty) = -90$.





$$N = -1 \; ; \qquad P = 1$$

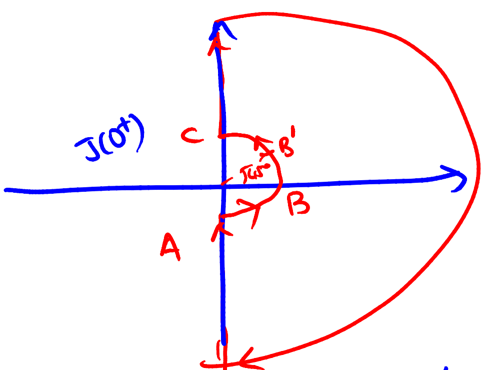
$$Z = N + P = -1 + 1 = 0$$

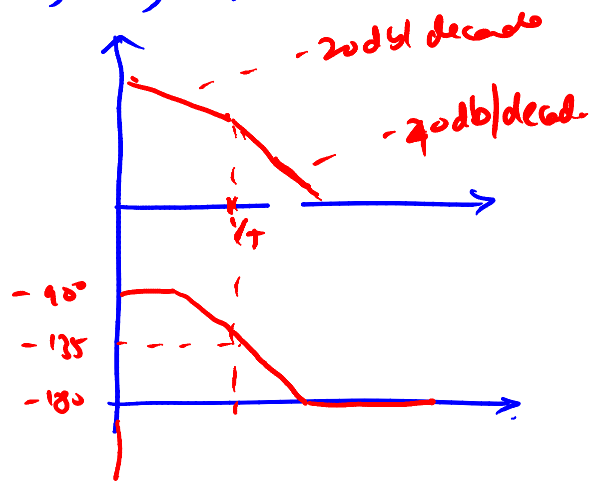
$\therefore$ The interconnection is stable.

<u>Example</u>:

$$G(s) = \frac{K}{s(Ts+1)} \; ; \quad K > 0 \; ; \; T > 0$$

→ there is a pole at $s = 0$





→ (B B' C) is described by $s = re^{j\theta}$
θ increases from 0 to $\theta = 90°$

→ $r > 0$ is a really small number

$$G(re^{j\theta}) = \frac{K}{(re^{j\theta})[Tre^{j\theta} + 1]}$$

$r << 1/T$;

$$G(re^{j\theta}) \approx K \quad = \quad K r^{-1} e^{-j\theta}$$

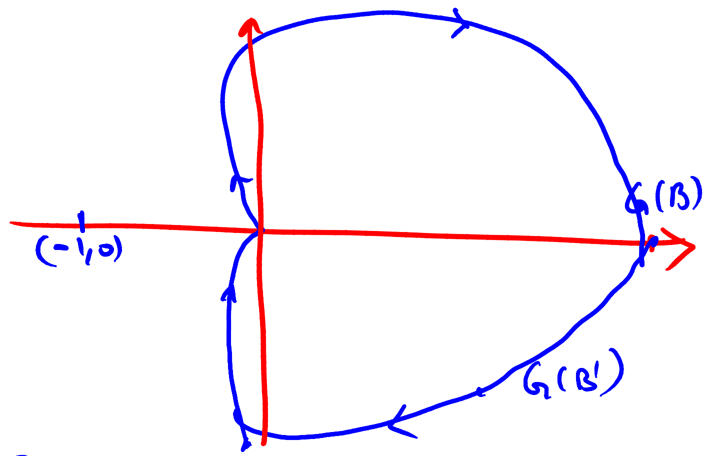
$$\frac{\quad}{re^{j\theta}}$$

$$G(B) = K r^{-1} e^{j\angle 0} = \frac{K}{r}$$

$$G(B') = \frac{K}{re^{j\theta}} = K r^{-1} e^{j\angle +45} = \frac{K}{r}(\cos 45 - j\sin 45)$$

$$= \frac{K}{\sqrt{2}r} - j\frac{K}{\sqrt{2}r}$$

$$G(C) = \frac{K}{re^{j\theta}} = \frac{K}{r} e^{j\angle 90^\circ} = \frac{K}{r}(-j).$$

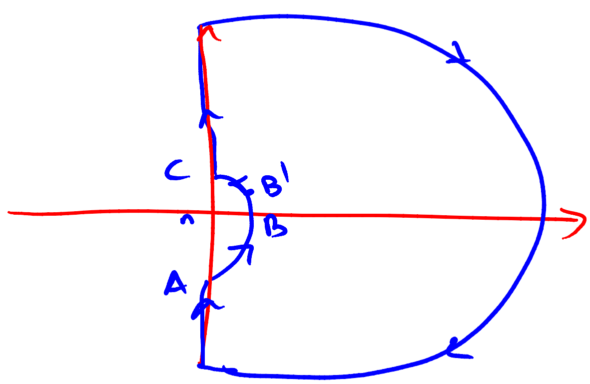


$P = 0$, $N = 0$ $\Rightarrow$ $Z = 0$

$\therefore$ interconnection is stable.

<u>Example</u>: $G(s) = \dfrac{1}{s(s-1)}$; pole at $s = +1$

pole at $s = 0$.



on $BB'C$

$$G(re^{j\theta})$$

$$= \frac{1}{re^{j\theta}(re^{j\theta} - 1)}.$$

$$|re^{j\theta}| << 1$$

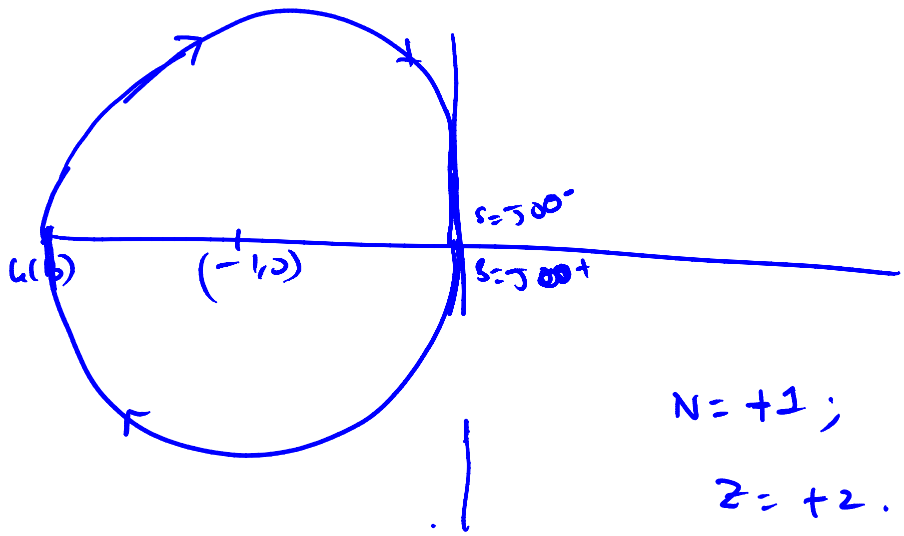
$$G(re^{j\theta}) = \frac{-1}{re^{j\theta}}$$

at B ; $G(B) = -\dfrac{1}{r}$

at B' ; $G(B') = -\frac{1}{\gamma} e^{-j\underline{45}} = -\frac{1}{\gamma}\left[\cos(-45) - j\sin(-45)\right]$

$= -\frac{1}{\sqrt{2}\gamma} + \frac{j}{\sqrt{2}\gamma}$

at C ; $G(C) = -\frac{1}{\gamma} e^{-j\underline{180}}$



$Z = N + P$

$N = +1$; $P = 1$

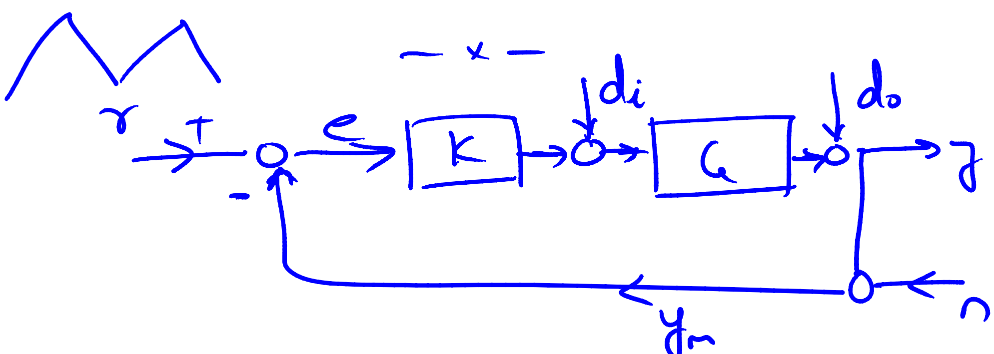
$Z = +2.$

feedback interconnection is unstable.

— x —

→ We have analyzed the open loop transfer function L to gain insights into the stability of the closed-loop system.

— x —



$r \equiv$ reference trajectory

$y_m \equiv$ the measurement of y

$n \equiv$ measurement noise.

$u \equiv$ control effort.

Typical Engineering requirement

Typical Engineering requirements

(1) good tracking of reference r

(2) good noise rejection.

(3) good disturbance rejection.

Tracking Requirements

→ good tracking of reference required within a bandwidth ω_B.

→ the error e should be small for all $r(j\omega)$ with $\omega \in [0, \omega_B)$.

~~Eh~~ to $\frac{e(j\omega)}{r(j\omega)}$ to be small in the range $\omega \in [0, \omega_B]$

tracking bandwidth

(*) $r \mapsto e$ transfer function is called the sensitivity transfer function

$$S = \frac{1}{1+GK} = \frac{1}{1+L}; \quad e = Sr$$

$|S(j\omega)|$ has to be small $\forall\ \omega \in [0, \omega_B]$

(*) $n \mapsto y$; is called complimentary transfer function; T

$$T = \frac{GK}{1+GK} = \frac{L}{1+L}$$

$|T|$ to be small in the range $\omega \in [\omega_{BT}, \infty)$

$\int S + T = \frac{1}{1+L} + \frac{L}{1+L} = 1.$