Lecture 6

Tuesday, February 08, 2011
8:11 AM

Closed Specs

- Bandwidth ω_{BW}
- $PM = 100\zeta$
- GM
- $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
- $t_s = \frac{4}{\zeta\omega_0}$
- $t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_0\sqrt{1-\zeta^2}}$
- Steady state behavior

open-loop

$\omega_{BW} = (1.2 - 1.6)\,\omega_{gc}$

Can be read from the Bode of L

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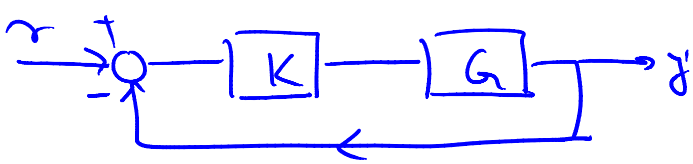
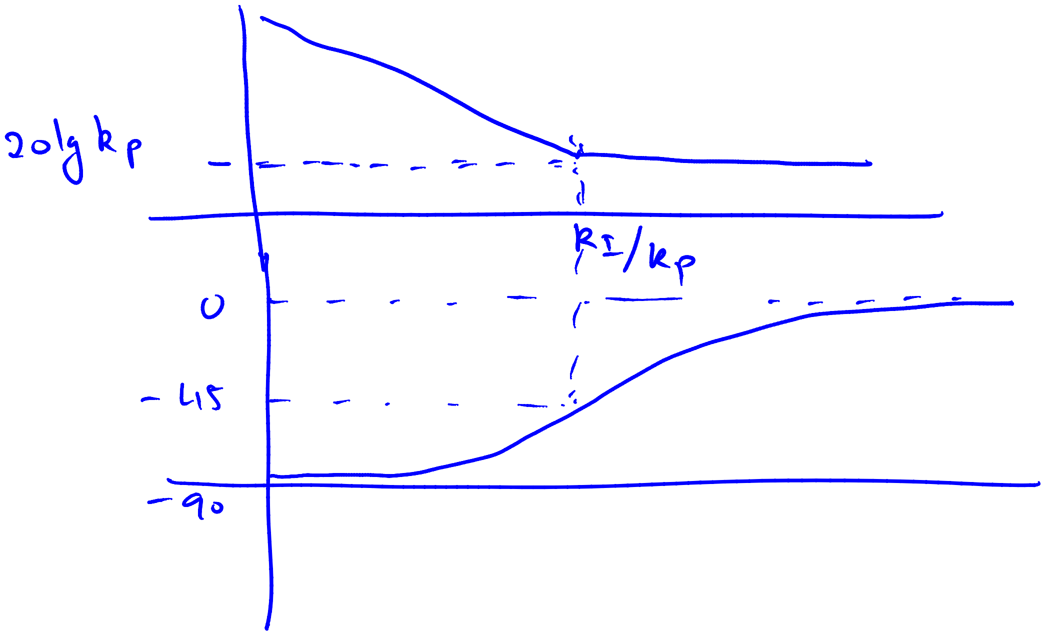
$\zeta = \frac{PM \text{ in deg}}{100}$

is dictated by the slope of the magnitude part of the Bode plot near $\omega = 0$

Proportional-Integral Controllers:



$$K = k_P + \frac{k_I}{s} = \frac{k_I}{s}\left[1 + \frac{s}{k_I/k_P}\right]$$



① PI controllers are primarily used to better the steady state behavior; in particular they increase the type of the system

Example of PI Controller design

$$G(s) = \frac{500}{s^2 + 6s + 5}$$

Design a PI controller to meet the following specifications

(1) $M_p \le 16\%$

(2) e_{ss} (steady state error) due to ramp input ≤ 0.1

Solution:

$$M_p \le 16\%$$

$$\Rightarrow \quad e^{-\pi\zeta/\sqrt{1-\zeta^2}} \le 0.16$$

intuition being carried from a 2nd order prototype

$$\Rightarrow \quad \zeta \ge 0.5039$$

PM rule of thumb

$$\therefore \; PM_d \simeq 100\zeta = 50.39 \text{ degrees}$$

$$PM_d = 100\zeta + (\text{safety margin})$$

$= 1009 + 7 = \underline{57 \text{ degrees}}$

e_{ss} due to ramps:

$$e(s) = \left(\frac{1}{1+L}\right) r(s)$$

$$= \left(\frac{1}{1+L}\right)\frac{1}{s^2} \quad ; \; r(s) = 1/s^2$$

$$e_{ss} = \lim_{s\to 0} s\, e(s) = \lim_{s\to 0} \frac{1}{s + sL(s)} = \frac{1}{\lim_{s\to 0} s\,L(s)}$$

K_v

$$K_v = \lim_{s\to 0} s\,L(s)$$

$$= \lim_{s\to 0} s\,(GK) = \lim_{s\to 0} s\left(\frac{500}{s^2+6s+5}\right)\left(k_p + \frac{k_I}{s}\right)$$

$$= \lim_{s\to 0} \left(\frac{500}{s^2+6s+5}\right)(s k_p + k_I)$$

$$= G(0)\, k_I = K_v$$

$\therefore \quad e_{ss} \leq 0.1$

$$\Rightarrow \frac{1}{K_v} \leq 0.1 \quad \Rightarrow K_v \geq 10$$

$$\Rightarrow G(0)\, k_I \geq 10$$

$$\Rightarrow k_I \geq \frac{10}{G(0)}$$

$$= \frac{10}{100} = 0.1$$

$$\boxed{k_I \geq 0.1}$$

In summary the closed-loop specs translate to

1. $PM_d = 57^\circ$
2. $k_I \geq 0.1$

} on the open loop part.

Step 1: Lets meet the PM_d requirement.

$$K = k_p + \frac{k_I}{s} = \frac{k_p}{s}\left(1 + \frac{s}{k_I/k_p}\right)$$

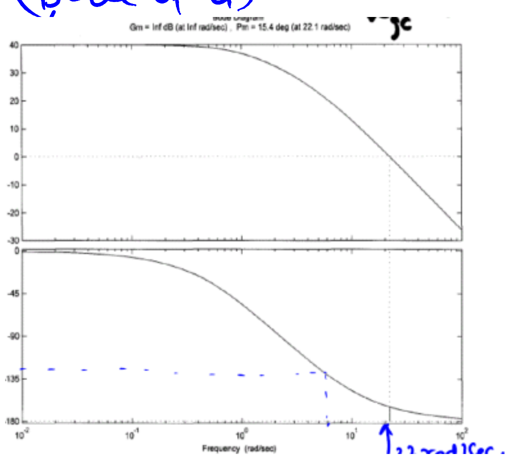
Find the frequency ω_{gcd} where the phase is such that $\angle G(j\omega_{gcd}) + 180$

$= PM_d = 57$

Find ω_{gcd} where $\angle G(j\omega_{gcd})$

$= 57 - 180 = -123°$

(Bode of G)



at about 4.39 rad/sec

$\angle G(4.39j) \simeq -119°$.

$$\boxed{\omega_{gcd} \simeq 4.39 \text{ rad/sec.}}$$

① Choose k_p to shift the gain crossover to 4.39 rad/sec

$$|k_p G(j\omega_{gcd})| = 1$$

$$k_p = \frac{1}{|G(j\omega_{gcd})|} = 0.06.$$

② $$\frac{k_I}{k_p} = \frac{\omega_{gcd}}{\alpha} = \left(\frac{4.39}{\alpha}\right) \quad ; \quad \alpha = 6$$

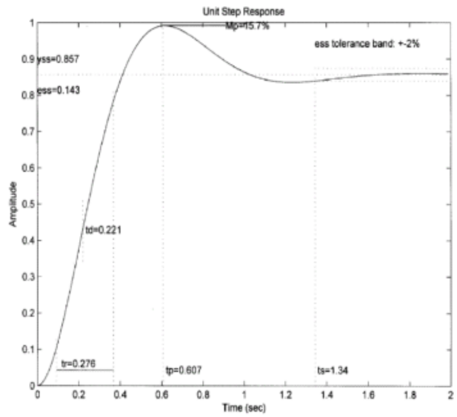
$$k_I = \left(\frac{4.39}{6}\right) k_p = \left(\frac{4.39}{6}\right)(0.06)$$

$$k_I \simeq 0.044.$$

Note that

$k_I \geq 0.1$

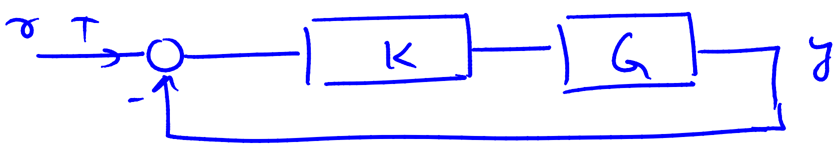
is being compromised.



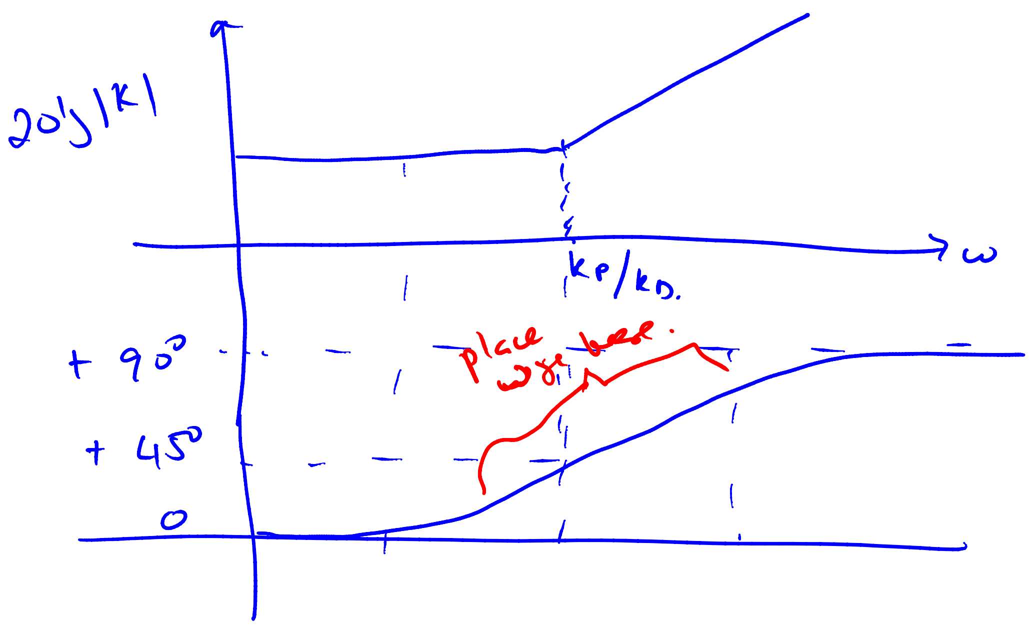
Note that $K_r = 4.4$ instead of 10.

(a). Note that the $M_p = 0.157$ that barely meets $M_p \le 0.16$ requirement. Thus, making K_I larger will lead to sacrificing M_p specification.

Proportional - differential Controller: (PD)



$$K = (k_p + k_D s) = k_p \left[1 + \frac{s}{(k_p/k_D)} \right]$$



Example: $G(s) = \dfrac{200}{s^2+4s+4}$

It is desired that

(a) $M_p \leq 16\%$

(b) ω_{gcd}; the desired gain crossover frequency is 14 rad/sec.

Solution: $M_p \leq 16\% \Rightarrow PM_d \approx 57$ degrees

$\omega_{gcd} = 14$ rad/sec.

(a) we want $\angle L(j\omega_{gcd}) + 180 \simeq 57$ degrees

$$\angle K(j\omega_{gcd})\, G(j\omega_{gcd}) + 180 = 57 \text{ degrees}$$

$$\Rightarrow \angle K(j\omega_{gcd}) + \angle G(j\omega_{gcd}) + 180 = 57 \text{ deg}$$

$$\Rightarrow \angle K(j\omega_{gcd}) \simeq 47 \text{ degrees}$$

$$\angle k_p + k_D (j\omega_{gcd}) \simeq 47 \text{ degrees.}$$

$$\tan^{-1}\left[\frac{k_D\,\omega_{gcd}}{k_p}\right] \simeq 47 \text{ degr}$$

$$\Rightarrow \boxed{\frac{k_D}{k_p} = \frac{\tan(47)}{\omega_{gcd}} = 0.077}$$

Also we need

$$|K(j\omega_{gcd})\, G(j\omega_{gcd})| = 1$$

$$\Rightarrow k_p = \frac{1}{\left|1 + \frac{k_D}{k_p}(j\omega_{gcd})\, G(j\omega_{gcd})\right|}$$

$\frac{k_D}{k_p} \rightarrow 0.077$

$$= 0.682$$

$$k_D = (k_D)(k_p) = 0.522.$$

$$k_D = \left(\frac{k_D}{k_p}\right)(k_p) = 0.522.$$

$$k(s) = 0.682 + 0.522\,s$$

Look at the week 10 notes on the EE4235 weblink.

Lag Controller:

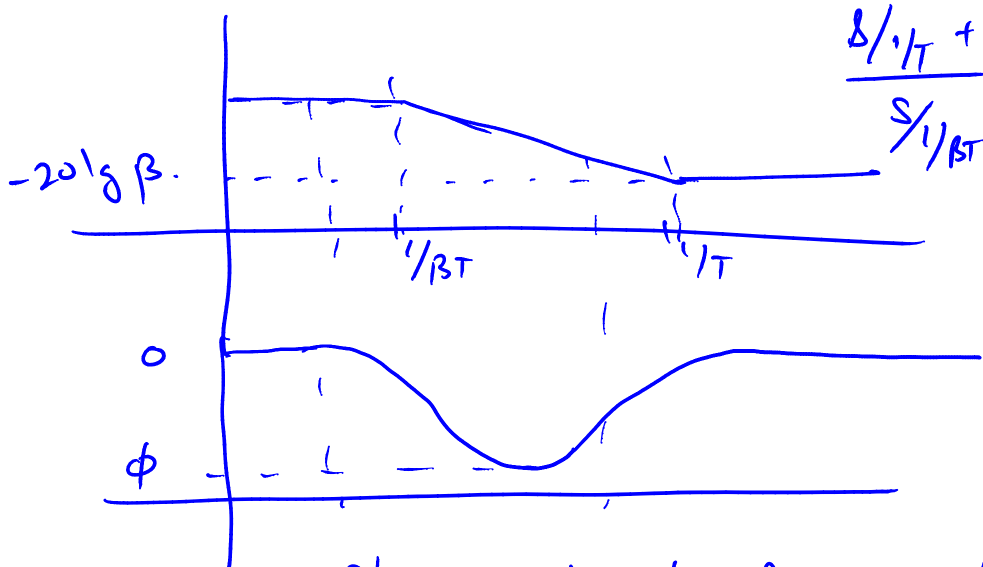
General form of a lag controller

$$K(s) = k\,\frac{Ts+1}{\beta Ts+1} \quad ; \quad \beta > 1.$$

$$= k\left\{\frac{\left[\frac{s}{1/T}+1\right]}{\left(\frac{s}{1/\beta T}+1\right)}\right\}$$

$$\beta > 1 \Rightarrow \frac{1}{\beta T} < \frac{1}{T}$$

$$\frac{s/\tfrac{1}{T}+1}{s/\tfrac{1}{\beta T}+1}$$



Phase of the lag design is always -ve.

$\therefore$ choose $1/T$ to be much below ω_{gcd}.

Example: Let $G(s) = \dfrac{1}{s(s+1)(0.5s+1)}$

design a lag controller to ~~st~~ satisfy

ⓐ $k_v \geq 5$ $\rightarrow$ $\lim_{s\to} sL(s)$

ⓑ $PM \geq 40^\circ$

ⓒ $GM \geq 10$.

Solution: step 1) $\lim_{s\to 0} s\left(G(s)\left[\dfrac{Ts+1}{\beta Ts+1}\right]k\right) = 5$

$\Rightarrow \quad k = 5$

Step 2: Consider the new plant to be

$G_1 = k\,G(s) = 5\,G(s)$.

Find ω_{gcd} such that

Safety margin

$$\angle G_1(j\omega_{gcd}) + 180 = (40 + 10^\circ) = 50^\circ$$

$\omega_{gcd} \approx 0.5$ rad/s.

Step 3:

Choose $1/T \approx \dfrac{\omega_{gcd}}{10} = 0.05$

$\Rightarrow T \approx 20$.

Step 4: Choose β to satisfy

$T = 20$.

$$\left| G(j\omega_{gcd})\, k\, \frac{Ts+1}{\beta Ts+1} \right|_{s=j\omega_{gcd}} = 1$$

$\beta T s + 1 \;\Big|_{s = j\omega_{sd}}.$

$k = 5$

$\beta = 10.$