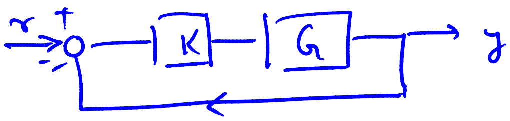
Lecture 7

Thursday, February 10, 2011
8:08 AM



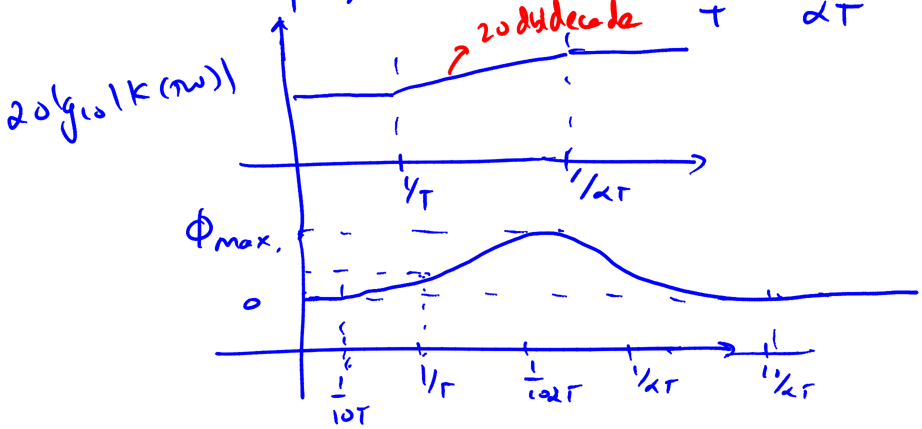
Lead Controller:

$$K(s) = k \, \frac{Ts+1}{\alpha Ts+1} \quad ; \quad 0<\alpha<1, \quad T>0, \quad k>0$$

Bode plot of the lead controller

$$K(s) = k \, \frac{\left(\frac{s}{1/T}+1\right)}{\left(\frac{s}{1/\alpha T}+1\right)}$$

The break frequencies are at $\frac{1}{T} < \frac{1}{\alpha T}$



- features: positive phase throughout
- Let ω_m be the frequency where $\underline{|K(j\omega)|}$ is the maximum then it can be shown that

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

- The maximum phase ϕ_m satisfies

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}.$$

- $$|K(j\omega_m)| = \frac{k}{}$$

$\sqrt{\alpha}$

Steps for lead Controller design

1. Choose k to satisfy steady state error requirements
2. Let $G_1(s) = k\,G(s)$; draw the bode of $G_1(s)$ and determine the phase margin
3. From the desired PM requirement, determine the extra phase needed to satisfy PM desired
4. $\phi_m = \text{PM required} + \underbrace{\text{safety margin}}_{(6-10^\circ)}$;
5. determine α by $\sin\phi_M = \dfrac{1-\alpha}{1+\alpha}$ ✓
6. We will set $\omega_m = \omega_{gc}$; ω_m is given by $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$; Note that at $\omega_m = \omega_{gc}$

$$\left|\frac{Ts+1}{\alpha Ts+1}\right|_{s=j\omega_m} = \frac{1}{\sqrt{\alpha}}$$

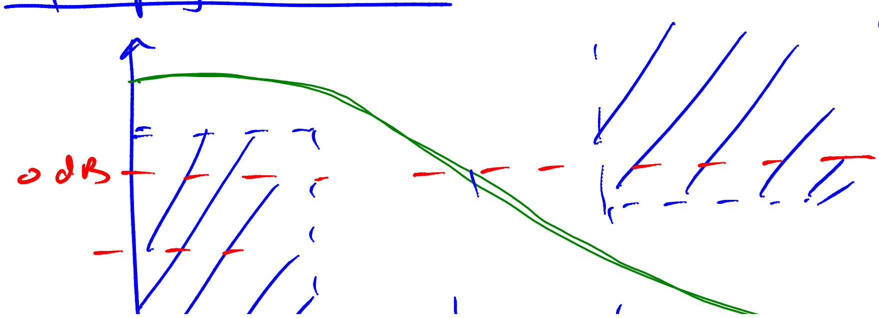
k is determin

$$\therefore\ \boxed{1 = |L(j\omega_m)| = \Big|k\,\underbrace{\frac{Ts+1}{\alpha Ts+1}}_{=\frac{1}{\sqrt{\alpha}}}\,G(s)\Big|_{s=j\omega_m}}$$

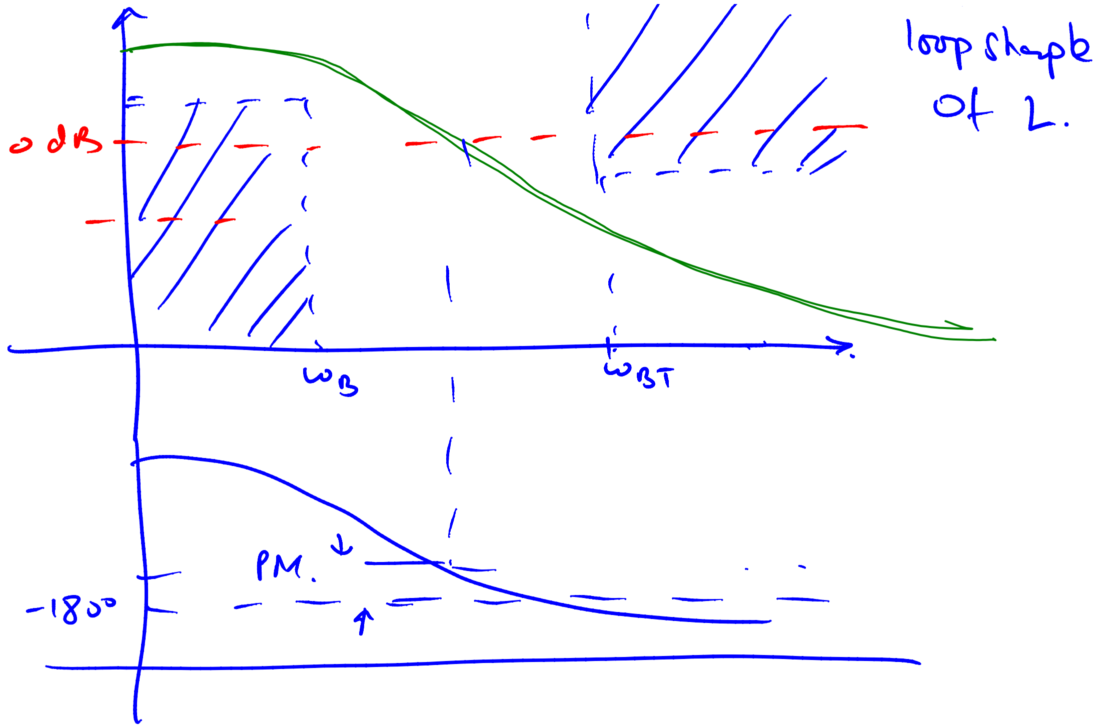
$\Rightarrow$ This provides ω_m

7. $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$. determine T.

Loopshaping objectives



typical desired loop shape of L.



Typical objectives are

① Increase $20\log_{10}|L(j\omega)|$ for $\omega \in [0, \omega_B)$ as much as possible. Also increase the slope

② Decrease $20\log_{10}|L(j\omega)|$ for $\omega \in [\omega_{BT}, \infty)$ as much as possible

③ Make sure there is enough phase Margin

④ Decrease the width of (ω_B, ω_{BT})

$\Downarrow$

you will require a sharp drop of $20\log_{10}|L(j\omega)|$ $\underbrace{\text{between } \omega_B \text{ and } \omega_{BT}}_{\text{This region characterizes gain cross over region}}$

- x -

Gain-Phase Relationship for LTI Causal Systems that are stable.

Theorem:

Suppose G is a transfer function that is stable (no unstable poles) and has no unstable zeros (no rhp zeros); Assume that G is rational. Then the following identity holds

$$\boxed{\angle G(j\omega_0) = \int_{-\infty}^{\infty} \underbrace{\left(\frac{d\ln|G(j\omega)|}{d\ln\omega}\right)}_{N(\omega)} \ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| \frac{1}{\omega}\, d\omega}$$

Slope of the magnitude part of Bode at frequency ω

This integral can be approximated as follows

$$\angle G(j\omega_0) \cong \int_{-\infty}^{\infty} N(\omega_0) \ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| \frac{1}{\omega}\, d\omega$$

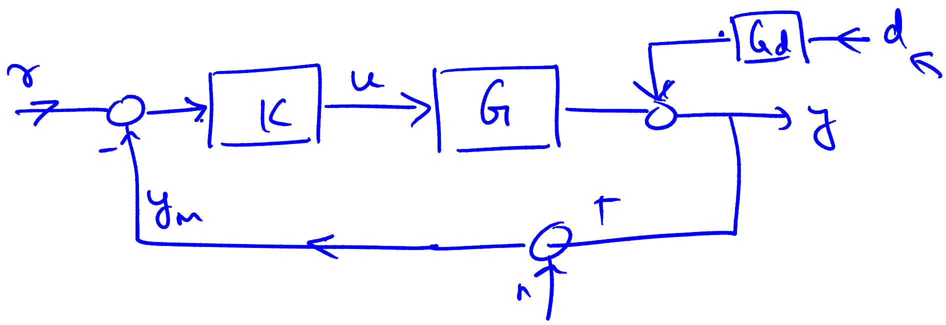
$$= N(\omega_0) \underbrace{\int_{-\infty}^{\infty} \frac{1}{\omega} \ln\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right| d\omega}_{\pi/2}$$

$$= N(\omega_0)\, \pi/2 .$$

Note: The phase of a system with rhp zero is necc. larger than if the rhp zeros are not present.

This is why transfer functions with no rhp zeros are termed "minimum phase system"

Unity Negative Feedback Configuration:



- r is the ref
- y_m is the measured y
- u is the control effort
- d is output disturbance (low frequency content)
- n is the measurement noise. (frequency content over a large range of frequencies).

→

Tracking error: $e = y - r$; note that $e \neq y_m - r$

let $v = y_m - r$ is the input to the controller

One can show that

$$y = \overbrace{(I+GK)^{-1} GK}^{T = \text{Complimentary sensitivity}} r - (I+GK)^{-1} GK n + \underbrace{(I+GK)^{-1}}_{\text{Sensitivity transfer function}} G_d d$$

$$e = y - r = -Sr - Tn + S G_d d$$

- Note that $S(j\omega) + T(j\omega) = 1 \;\; \forall \omega$ is a fundamental limitation.

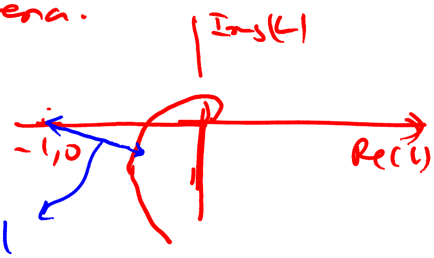
Typical requirement on S:

- Minimum tracking bandwidth ω_B^* defined as the frequency where $|S(j\omega)|$ crosses $+3\text{dB}$

from below. ($|S(j\omega)|$ crosses 0.707 from below 0.707)

(3) $S(j\omega)$ not to exceed certain prespecified value at given frequencies $\omega_1, \omega_2, \ldots \omega_m$.

(4) $|S(j\omega)|$ to have a maximum value below M ↑ captures robustness criteria.



min $|1+L(j\omega)|$

(5) $\max\limits_{\omega} |S(j\omega)| = \max\limits_{\omega \in \mathbb{R}} \dfrac{1}{|1+L(j\omega)|}$

and this number has to be $\leq M$.

→ $$\frac{dT/T}{dG/G} = S$$

↑ This is the reason for calling S the sensitivity transfer function.