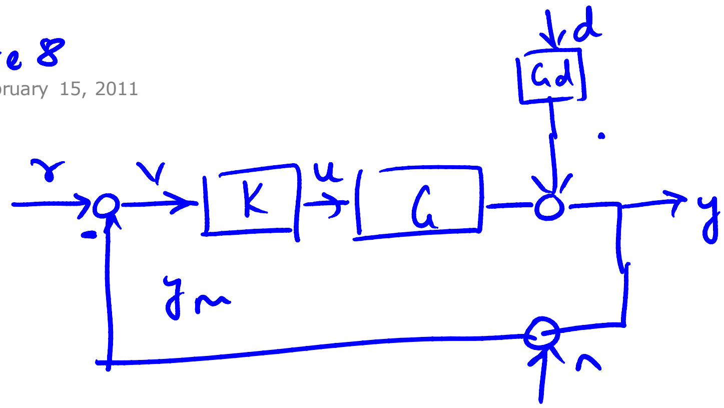
Lecture 8

Tuesday, February 15, 2011
8:13 AM



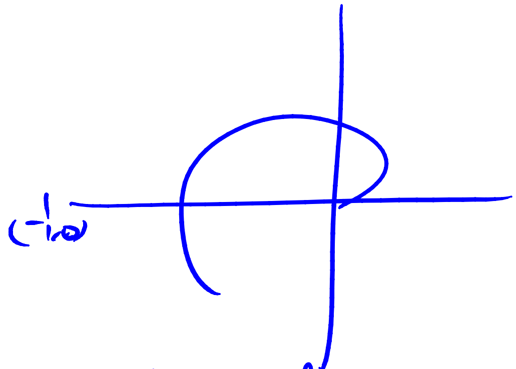
- $e = y - r = -Sr - Tn + SG_d d$
- $S = (I+GK)^{-1}$; $T = GK(I+GK)^{-1}$

Specifications on S:

- Tracking bandwidth, ω_B is the value of the frequency where $|S|$ crosses +3dB from below i.e. $|S(j\omega)| \leq 3\text{dB} \ \forall \omega \in [0, \omega_B]$.

- $|S(j\omega)|$ cannot exceed prespecified values at certain prespecified frequences $\omega_1, \omega_2, \ldots \omega_n$.

- $|S(j\omega)| \leq M$ for all $\omega \in [0, \infty)$
  ↳ Robustness criteria



(*) $\min_{\omega \in [0,\infty)} |1+L(j\omega)| = \alpha$ is the closest approach of the Nyquist plot to the point $(-1,0)$ and therefore characterizes robustness to uncertainties.

(*) Note that we would like to have α to be large in other words

$$\max_{\omega \in [0,\infty)} |S(j\omega)| = \max_{\omega \in [0,\infty)} \frac{1}{|1+L(j\omega)|}$$ to be small.

→ Lets assumes that $W_p(s)$ is a filter that is such that $\left|\frac{1}{W_p(j\omega)}\right|$ satisfies all the specifications

sensitivity weight is derived from Specs on S.

→ The controller K will satisfy all the performance Specs if

$$|S(j\omega)| \le \left|\frac{1}{W_p(j\omega)}\right| \quad \forall\, \omega \in [0,\infty)$$

$$\Leftrightarrow \quad |S(j\omega)|\,|W_p(j\omega)| \le 1 \quad \forall\, \omega \in [0,\infty)$$

$$\Leftrightarrow \quad |S(j\omega)|\,|W_p(j\omega)| \le 1 \quad \forall\, \omega \in [0,\infty)$$

$$\Leftrightarrow \quad \sup_{\omega\in[0,\infty)} |S(j\omega)\, W_p(j\omega)| \le 1$$

Note that S has to be a stable transfer function; furthermore the weight W_p is typically chosen to be stable and therefore SW_p is a stable transfer (i.e. it is analytic in the RHP).

→ H_∞ norm is defined for stable transfer functions f and is defined as

$$\|f\|_{H_\infty} := \sup_{\omega\in[0,\infty)} |f(j\omega)|.$$

→ Another way of stating the specification that

$$\sup_{\omega\in[0,\infty)} |S(j\omega)\, W_p(j\omega)| \le 1$$

is

$$\boxed{\|S W_p\|_{H_\infty} \le 1}$$

<u>Typical Weights on the Sensitivity transfer function:</u>

Specs required are

① $\|S\|_{H_\infty} \le M.$

↓ tracking bandwidth

② $|S(j\omega)| \le m_p$ for $\omega \in [0, \omega_p)$

Let $W_p(s) = \dfrac{\frac{s}{M_p} + \omega_p}{s + \omega_p m_p}$

$$\frac{1}{W_p(s)} = \frac{s + \omega_p m_p}{\frac{s}{M_p} + \omega_p}$$

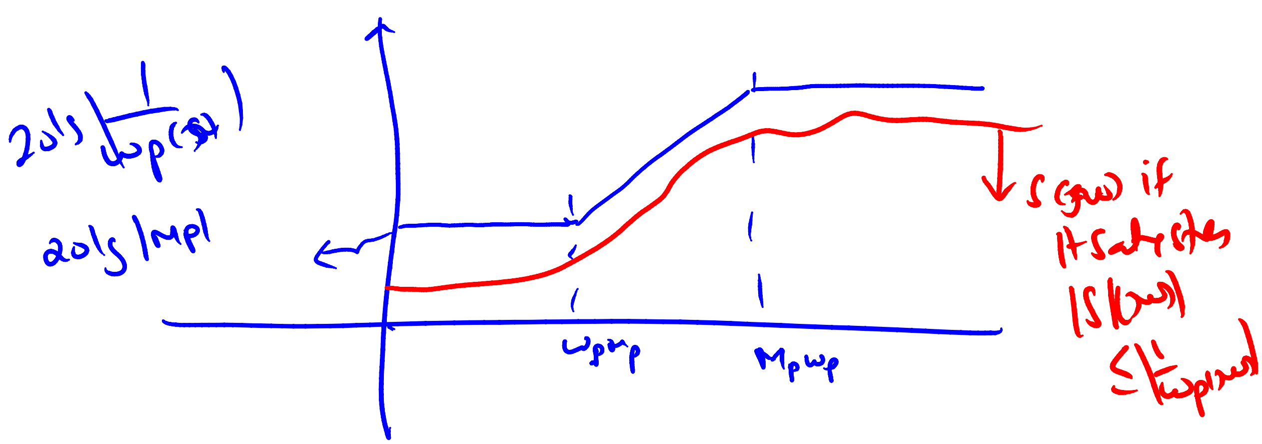
$$= \frac{\omega_p m_p}{\omega_p} \frac{\left[\frac{s}{\omega_p m_p} + 1\right]}{\left[\frac{s}{M_p \omega_p} + 1\right]}$$

$$= m_p \frac{\left[\frac{s}{\omega_p m_p} + 1\right]}{\left[\frac{s}{M_p \omega_p} + 1\right]}$$

The corner frequency of the zero is at $\omega_p m_p$

The corner frequency of the pole is at $\omega_p M_p$

Note that $M_p > m_p$



One can show that both specs are met by imposing the condition

$$|S(j\omega)| \le \left|\frac{1}{W_p(j\omega)}\right|$$

$$\Leftrightarrow \quad \| S W_p \|_{H_\infty} \leq 1.$$

If the controller K can be designed so that the closed-loop interconnection is stable and

$$\| (I + GK)^{-1} W_p \|_{H_\infty} \leq 1 \quad \text{then}$$

the robustness plus tracking objectives would be achieved.

- <u>Specifications on Complimentary Sensitivity:</u>
- $|T(j\omega)| < \frac{1}{A_l}$ for all $\omega < \omega_T - \Delta\omega$
- $|T(j\omega)| < A_h$ for all $\omega > \omega_T + \Delta\omega$

where $\frac{1}{A_l} \simeq 1$ and A_h is small.

A weight that satisfies these two specs is

$$W_T(s) = \frac{s + \left(\frac{1}{A_l}\right)\omega_T}{A_h s + \omega_T}$$

$\therefore$ The Specifications on Complimentary Sensitivity are met by imposing the condition that

$$\| T W_T \|_{H_\infty} \leq 1$$

$$\Rightarrow \| GK (I + GK)^{-1} W_T \|_{H_\infty} \leq 1$$

<u>Specifications on the transfer function KS.</u>

The transfer function between $r \mapsto u$

is KS.

$$\|K(s)\,S(s)\,W_u(s)\|_{H_\infty} < 1.$$

$\frac{1}{W_u}$ to be a constant weight.

Summary of the Controller Design Problem:

Given weights W_p, W_T, W_u design a controller K that stabilizes the feedback interconnection and satisfies the performance requirements of

* $\|W_p S\|_{H_\infty} \le 1$
* $\|W_T T\|_{H_\infty} \le 1$
* $\|W_u KS\|_{H_\infty} \le 1$

The H_∞ software solves the following problem

$$\min_{K \text{ stabilizing}} \left\| \begin{matrix} W_p S \\ W_T T \\ W_u KS \end{matrix} \right\|_{H_\infty}$$

If $f : \mathbb{C} \to \mathbb{C}^n$ is a vector valued complex function analytic in the RHP then we define

$$\|f\|_{H_\infty} := \sup_{\omega \in (0,\infty)} \bar{\sigma}(f(j\omega))$$