Problem 1

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11:58 PM

All questions carry equal points.

Problem:

The block diagram of a control system is shown in Figure 1. In Figure 1, $r$ is the reference input, $n$ is measurement noise, $y$ is the measured output, the controller $C(s) = \frac{s+\alpha}{s}$ and the plant is $G(s) = \frac{K(s+2)}{s^2-1}$.

1. Find the relationship between $K$ versus $\alpha$ for the system to be stable.

2. Find the transfer function $S$ from reference $r$ to error $e = r - y$ when $\alpha = 1$ and $K = 10$; assume that $n = 0$. Determine if $S$ is stable. Find the steady state of the error $e$ when $r = \sin 3t$. The transfer function $S$ captures the error in tracking a reference input $r$ and is desired to be small in the frequency region where good tracking is desired.
Consider the unity negative feedback interconnection

\[ \rightarrow Q \rightarrow K \rightarrow G \rightarrow y \]

(c) Give an example of \( G \) and \( K \) transfer functions, if possible, such that \( \frac{1}{1 + KG} \) and \( \frac{G}{1 + GK} \) are stable but \( \frac{K}{1 + KG} \) is

(b) Give an example of \( G \) and \( K \) transfer functions, if possible, such that \( \frac{K}{1 + KG} \) and \( \frac{G}{1 + GK} \) are stable but \( \frac{1}{1 + KG} \) is not

(c) Give an example of \( G \) and \( K \) transfer functions, if possible, such that \( \frac{1}{1 + KG} \) and \( \frac{K}{1 + GK} \) are stable but \( \frac{G}{1 + GK} \) is
Obtain the Bode plot of

\[ H(\omega) = \frac{10}{1 + \frac{\omega}{100}} \frac{1 + \frac{\omega}{10}}{1 + \frac{\omega}{10}} \]

Using MATLAB, with the range of the magnitude plot between -10dB to 80dB. On the same Bode plot, sketch the Bode plot

\[ 100 \frac{1 + \frac{\omega}{100}}{1 + \frac{\omega}{10}} \]

by hand.
Obtain the asymptotes of the Bode plots of the following transfer functions:

(a) \( \frac{100s + 100}{s^2 + 110s + 1000} \)

(b) \( \frac{10}{s^2 + 3s} \)

(c) \( -\frac{100}{(s+1)^2 (s+10)} \)

(d) \( \frac{30}{s^2 + 3s + 50} \)

(e) \( \frac{4(s^2 + 8 + 25)}{s^3 + 100s^2} \)

Plot using MATLAB the Bode plots of the above transfer functions and compare with asymptotic Bode plots.