EE 5501 Prof. Jindal
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Midterm 1

You have 75 minutes to complete this exam. You must show your work to receive credit.

1. **Baseband and Passband Signals** (30 pts)

   In this problem we will study the upconversion and downconversion processes in the frequency domain. For this problem it will be helpful to recall:

   $$\begin{aligned} \mathcal{FT}\{\cos(2\pi f_c t)\} &= \frac{1}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] \\ \mathcal{FT}\{\sin(2\pi f_c t)\} &= \frac{1}{2j}\left[\delta(f-f_c)-\delta(f+f_c)\right] \end{aligned}$$

   and that $1/j = -j$.

   (a) The real passband signal $s_p(t)$ is written in terms of the real baseband signals $s_c(t)$ and $s_s(t)$ as:

   $$s_p(t) = s_c(t)\sqrt{2}\cos(2\pi f_c t) - s_s(t)\sqrt{2}\sin(2\pi f_c t).$$

   Let $S_p(f)$, $S_c(f)$, and $S_s(f)$ denote the Fourier transforms of $s_p(t)$, $s_c(t)$, and $s_s(t)$, respectively. Show:

   $$S_p(f) = \frac{1}{\sqrt{2}}\left[S_c(f-f_c) + jS_s(f-f_c) + S_c(f+f_c) - jS_s(f+f_c)\right].$$

   (10 pts)

   (b) The first step in the downconversion process is multiplying the passband signal by a scaled version of $\cos(2\pi f_c t)$. Define

   $$a(t) = s_p(t)\sqrt{2}\cos(2\pi f_c t).$$

   Let $A(f)$ denote the Fourier transform of $a(t)$. Show:

   $$A(f) = S_c(f) + \frac{1}{2}\left[S_c(f-2f_c) + jS_s(f-2f_c) + S_c(f+2f_c) - jS_s(f+2f_c)\right].$$

   (10 pts)

   (c) Assume that $S_c(f)$ and $S_s(f)$ have frequency content only in $[-W/2, W/2]$, and that $a(t)$ is passed through an ideal low-pass filter that has unit gain for frequencies in $[-W/2, W/2]$, and zero gain for all other frequencies. What is the maximum value of W such that the output of the low-pass filter is $s_c(t)$? (10 pts)

2. **Performance Analysis of Continuous-time Waveforms** (40 pts)
Consider the following two waveforms:

$$s_0(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{else} \end{cases} \qquad s_1(t) = \begin{cases} -\frac{1}{\sqrt{\Delta}} & 0 \le t \le \Delta \\ 0 & \text{else} \end{cases}$$

Assume equal message priors, i.e., $\pi(0) = \pi(1)$, white Gaussian noise with variance $\sigma^2 = N_0/2$, and that Δ is a constant greater than or equal to 1.

(a) Find an orthonormal basis for the two waveforms and provide the signal space representation in terms of this basis. (10 pts)

(b) Compute E_s and E_b. (10 pts)

(c) Derive an expression for the probability of error in terms of Δ and σ. (10 pts)

(d) Is the probability of error increasing or decreasing in Δ? Explain. (10 pts)

3. **Performance Analysis** (30 pts)
Consider the following five point signal constellation: $(A, 0), (0, A), (-A, 0), (0, -A), (0, 0)$, and assume equal message probabilities.

(a) Sketch the optimal decision regions. (10 pts)

(b) Find the intelligent union bound on the probability of error. (10 pts)

(c) Prove that the probability of error for this five point constellation is greater than the probability of error for the four point constellation $(A, 0), (0, A), (-A, 0), (0, -A)$ (also with equal message probabilities). (10 pts)
Note: In this problem we compute probability of error in term of A - you should not consider energy per symbol or per bit.