Survey on capacity results and limits on MIMO systems

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Why MIMO?

- MIMO techniques increase spectral efficiency
- Better control of transmit power
- Reduction of co-channel interference

When do we have such an increase in spectral efficiency?

When we have rich scattering

MIMO idea and research explosion

- [Foscini,1996] “Layered space time architecture for wireless communication in fading environments, when using multi-antenna elements”
- [Telatar,1999] “Capacity of multi antenna Gaussian channels”

General channel model for the single user case

$$\underline{y} = H \underline{x} + \underline{z}$$

$$\underline{x} \in C^{M}, \underline{y} \in C^{N}$$

$$\underline{z} \in C^{N}, H \in C^{NxM}$$

Channel statistics and channel matrices

- Rayleigh fading model: the channel matrix H contains i.i.d zero mean complex,gaussian channel gains
- Ricean fading: H contains independent nonzero mean complex gaussian channel gains.

In the Ricean case the channel matrix is written as follows:

$$H = \sqrt{\frac{K}{K+1}} H_{LOS} + \sqrt{\frac{1}{K+1}} H_{NLOS} = D + \tilde{H}$$

Implicit assumption:The transmission scheme is subject to an average power constraint

$$E[xx^{+}] \leq P$$

$+$ *denotes hermirian transposition*

Different definitions of capacity for operation in a wireless environment

- “Ergodic capacity”: C=E[C(θ)] where θ is parameter characterizing the pdf of the channel behavior
- “Outage Capacity”: Achievable rate for which the probability of outage is minimized.

The following results based on a tutorial paper

[Andrea Goldsmith,Syed Ali Jafar, Nihar Jindal, Siram Vishwanath,2003]: “Capacity Limits of MIMO channels”

In this presentation:use of ergodic capacity

Computation of C(θ) for a general p_θ(.) is a difficult problem.Research interest in the following 3 models:

(a) Zero mean spatial white (ZMSW): $E[H]=0, H=H^{W}$**;** H^{W} **is a white channel matrix.**

(b) Channel mean information (CMI): $E[H]=\bar{H}, H=\bar{H}+\sqrt{a}H^{W}$**; this model can naturally fit in the Ricean channel case, where** a **is the estimation error.**

(c) Channel covariance information (CCI): $E[H]=0, H=R_r^{1/2}H^{W}R_t^{1/2}$**, where** R_r, R_t **are the receive and transmit fade covariance matrices, which are assumed to be known.**

Brief report of the main Capacity results

When the channel is *constant* **and** *known perfectly at the transmitter* **and the receiver, the capacity is:**

$$C = \max_{Q:tr(Q)=P} \log | I_N + HQH^{+} |$$

where Q **is the input covariance matrix.**

Optimal Q**: waterfilling over the eigenmodes of the channel. Power allocation to the ith eigenmode is given by:**

$$P_i = (\mu - \frac{1}{\sigma_i^2})^{+}, i = 1, \ldots, \min\{M, N\}$$

In this case,the capacity becomes

$$C = \sum_{i=1}^{\min\{M,N\}} (\log(\mu\sigma_i^2))^+$$

Doesn’t depend on the EIGENSTRUCTURE of the channel matrix

Nonconstant channel matrix:

Assumption:Random variations according to a stationary known distribution

Capacity with perfect channel state at the Rx(CSIR) and at the Tx(CSIT)

$$C = E_H[\max_{Q:tr(Q)=P} \log| I_N + HQH^{+} |]$$

Maximization prior to expectation operator denotes that the Tx has the ability to use the channel state information(CSIT)

Capacity with perfect channel state at Rx(CSIR) and channel distribution knowledge at the Tx(CDIT)

$$C = \max_{Q:tr(Q)=P} E_H[\log|I_N + HQH^{+}|]$$

Maximization AFTER the expectation operator denotes that the Tx doesn't have any specific knowledge of the channel state.

“Transmitter optimization yields the intuitive optimal transmission strategy: Split the power equally to all directions”

$$C = E_H[\log|I_N + \frac{P}{M} HH^+|]$$

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Capacity with perfect CSIR and CDIT: CMI and CCI models

In both these cases nothing changes in comparison to the previous case, except from the different model set up for the channel matrices. The capacity expression is unaffected so:

$$C = \max_{Q:tr(Q)=P} E_H[\log| I_N + HQH^+ |]$$

Optimal transmit covariance can in general be full(high)rank.In the case that the rank is unity, capacity can be achieved via BEAMFORMING

The above results apply both to Rayleigh fading and Ricean fading.For the Ricean fading, we mention just a monotonicity result to highlight the effect of the channel mean

[Hösli,Lapidoth]

- “Capacity in a Ricean fading channel is monotinic in the singular values of the channel mean.”
- “The optimal covariance must have the same eigenvectors with the channel mean”

THANK YOU!