A Random Beamforming Technique in MIMO Broadcast Channels

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Abstract

In systems with a large number of transmit antennas M and a large number of users N, it is not always reasonable to assume that perfect channel knowledge can be made available to the transmitter. Since lack of channel knowledge does not lead to multi-user gains, it is therefore of interest to investigate transmission schemes that employ only partial channel state information (CSI). In this project, in order to exploit having multiple antennas in the transmitter without having full CSI in the transmitter, we propose a scheme that constructs M random beams and transmits to the users with the highest signal to interference plus noise ratio (SINR). We show that how the ricean factor K and SINR affect on the sum rate capacity. We also show that, when M is large enough, the system becomes interference-dominated.

I. INTRODUCTION

There has been line of work studying the sum rate capacity, and in fact the capacity region, of MIMO broadcast channels. In many applications, however, it is not reasonable to assume that all the channel coefficient to every user can be made available to the transmitter. This is especially true if the number of transmit antennas M and/or the number of users N is large. Since perfect channel state information may be impractical, it is very important to devise and study transmission schemes that require only partial channel state information at the transmitter. In [1], D. Tse assumed only the base station has multiple antennas, and thought that it is optimum for sum rate capacity to concentrate whole resources to one best user according to information theory. However, in non-degraded channel (MIMO BC), concentrating whole power to one user is not generally optimal.

The scheme we propose is one that constructs M random orthonormal beams and transmits to users with the highest SINR. For $M > 1$, this is a vector Gaussian broadcast channel, and unlike the scalar Gaussian broadcast channel (M = 1), it is in general not degraded and the capacity region is unknown. The main result of this project is the following characterization of the sum rate capacity of this channel.

II. CHANNEL MODEL AND NOTATIONS

We consider a memoryless vector Gaussian broadcast channel to model the downlink of a wireless system with M antennas at the base station and N users with a single antenna at each receiver. Focusing on one particular time instant, denote the received symbol at receiver j by y_j and $\mathbf{y}_{\mathrm{dl}} \overset{\mathrm{def}}{=} (y_1,\ldots,y_{\mathrm{N}})^t$. They are related by

$$\mathbf{y}_{\mathrm{dl,n}} = \mathbf{H}_{\mathrm{n}}^{\dagger}\mathbf{x}_{\mathrm{dl}} + \mathbf{z}_{\mathrm{n}} \qquad \mathrm{n} = 1,\ldots,\mathrm{N}$$

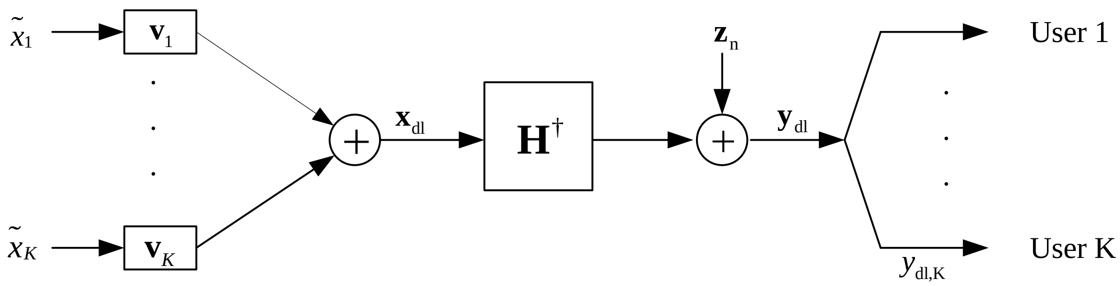

Fig. 1. Downlink with linear transmit beamforming

Denoting by K the Ricean factor of the channel, we rewrite the channel matrix $\mathbf{H}$ as

$$\mathbf{H} = \sqrt{\frac{\mathrm{K}}{\mathrm{K}+1}}\,\overline{\mathbf{H}} + \sqrt{\frac{1}{\mathrm{K}+1}}\,\mathbf{H}(w) \tag{1}$$

so as to separate the random component of the channel and the deterministic part:

- $\overline{\mathbf{H}}$ represent the line of sight component of the channel
- $\mathbf{H}(w)$ is the random component of the channel with Gaussian, independent and identically distributed entries. The complex element is circularly symmetric with zero mean and unit variance.

Here $\mathbf{H}$ is a $\mathrm{M}\times\mathrm{N}$ matrix with $\mathbf{H}_{\mathrm{ij}}^{*}$ entry denoting the channel gain from the ith antenna to the jth user. $\mathbf{x}_{\mathrm{dl}} \in \mathbf{C}^{\mathrm{M}\times 1}$ is the vector input to the antenna array with an average total power constraint of P. The additive noise $\mathbf{z}_{\mathrm{n}}$ is zero mean, unit variance, complex circular symmetric Gaussian.

Note that the model is general enough to take into account line of sight (LOS) and non line of sight (NLOS) cases. Indeed, as $\mathrm{K} \to \infty$, (1) models a deterministic fading channel, whereas for $\mathrm{K} = 0$ it describes a Rayleigh fading channel.

III. MAIN RESULT

Transmitter beamforming is a sub-optimal technique that supports simultaneous transmission to multiple users on a broadcast channel. Each active user is assigned a beamforming direction by the transmitter and multi-user interference is treated as noise. Let $S(t)$ be the $\mathrm{M}\times 1$ vector of the transmit symbols at time slot *t*, and let $\mathbf{Y}_i(t)$ be the $\mathrm{N}\times 1$ vector of the received signal at the i th receiver. We choose M normalized random vectors $\mathbf{v}_m(\mathrm{M}\times 1)$ for $m=1,\ldots,\mathrm{M}$, where $\mathbf{v}_m$'s are generated according to an isotropic distribution. Then at time slot t, the m^{th} vector is multiplied by the m^{th} transmit symbol s_m, so that the transmitted signal is,

$$S(t)=\sum_{m=1}^{\mathrm{M}}\mathbf{v}_m s_m, \qquad t=1,\ldots,\mathrm{T}. \tag{2}$$

From now on for simplicity, we drop the time index, and therefore, the received signal at the i'th receiver is,

$$\mathbf{Y}_i=\sum_{m=1}^{\mathrm{M}}\mathbf{H}_i\mathbf{v}_m s_m+\mathbf{z}_i \qquad i=1,\ldots,\mathrm{N} \tag{3}$$

We further assume that the i'th receiver knows $\mathbf{H}_i\mathbf{v}_m$ for $m=1,\ldots,\mathrm{M}$. Therefore, the i'th receiver can compute the following M SINRs by assuming that s_m is the desired signal and the other s_i's are interference as follows,

$$\mathrm{SINR}_{i,m}=\frac{\mathrm{P}_m\left|\mathbf{H}_i\mathbf{v}_m\right|^2}{1+\sum_{k\neq m}^{\mathrm{M}}\mathrm{P}_k\left|\mathbf{H}_i\mathbf{v}_k\right|^2}, \qquad m=1,\ldots,\mathrm{M} \tag{4}$$

where $p_k \overset{def}{=} E[\tilde{x}_k^2]$ is the power allocated to stream k.

Note that on average the $\mathrm{SINR}_{i,m}\approx\frac{1}{(\mathrm{M}-1)}$. Thus if we randomly assign beams to users,

$$\begin{aligned}\mathrm{R}=\mathrm{E}\left\{\sum_{i=1}^{\mathrm{M}}\log(1+\mathrm{SINR}_{i,m})\right\}&=\mathrm{ME}\left\{\log(1+\mathrm{SINR}_{i,m})\right\}\overset{(a)}{\leq}\\ \mathrm{M}\log(1+\mathrm{E}\left\{\mathrm{SINR}_{i,m}\right\})&\approx\mathrm{M}\log(1+\frac{1}{\mathrm{M}-1})<1.\end{aligned} \tag{5}$$

(a) follows from the Jensen's inequality.

Thus, even though we are sending M different signals, we do not get M fold increase in the capacity. For instance, we construct N = 2 orthonormal beams. Fig.1 and Fig.2 show that sum rate capacity never exceed 2 for the different M and SINR, respectively.

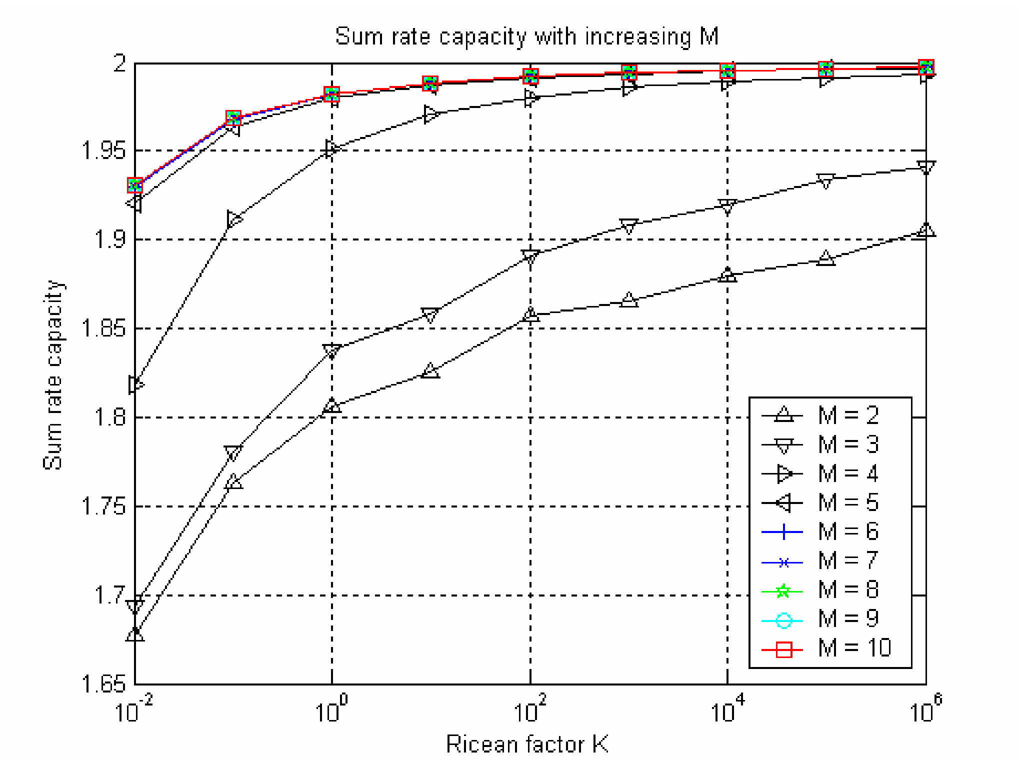

Fig. 1 Sum rate capacity versus the Ricean factor K for different Ms

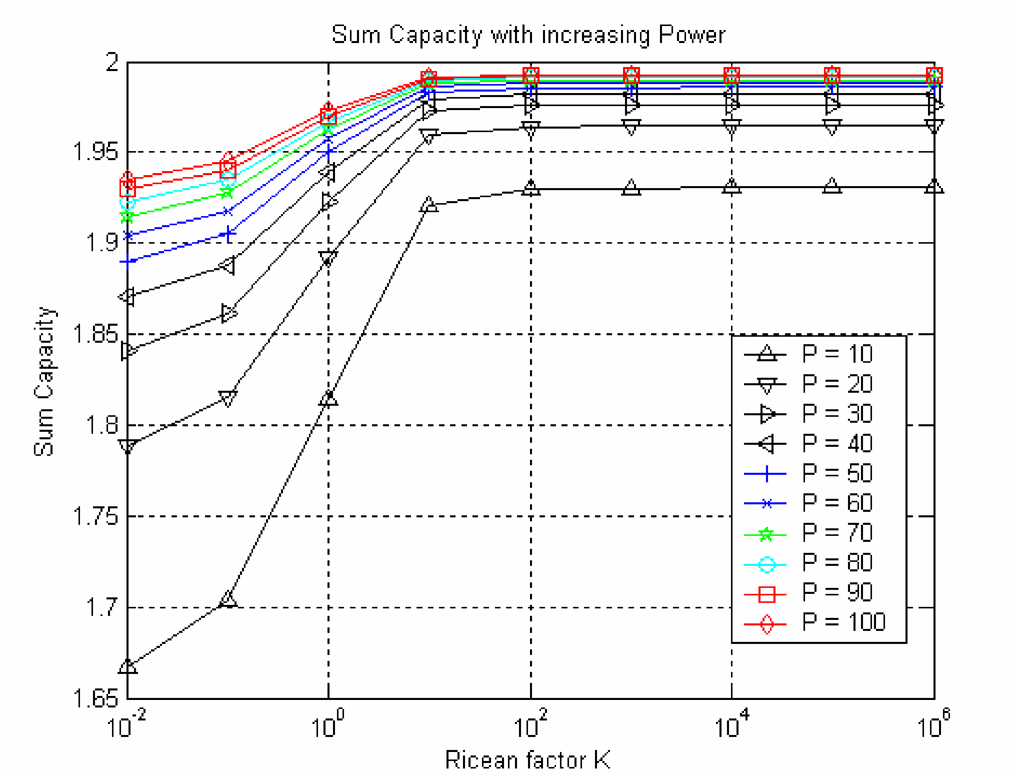

Fig.2 Sum rate capacity versus the Ricean factor K for different SINRs

Suppose now each receiver feeds back its maximum SINR, i. e. $\max_{1\le m\le \mathrm{M}} \mathrm{SINR}_{i,m}$, along with the index m in which the SINR is maximized. Therefore, in the transmitter, instead of randomly assigning the beam to the users, the transmitter assigns s_m to the users with the highest corresponding SINR, i.e. $\max_{1\le i\le \mathrm{N}} \mathrm{SINR}_{i,m}$. So if we do the above the sum rate capacity

$$\mathrm{R} \approx \mathrm{E}\left\{\sum_{m=1}^{\mathrm{M}} \log(1+\max_{1\le i\le \mathrm{N}} \mathrm{SINR}_{i,m})\right\} = \mathrm{M}\,\mathrm{E}\left\{\log(1+\max_{1\le i\le \mathrm{N}} \mathrm{SINR}_{i,m})\right\} \tag{6}$$

Fig. 3 and Fig. 4 show the sum rate capacity when the random beamforming has the maximum SINR for the different M and SINR, respectively. Remarkably, the capacity depends only on a few meaningful parameters, namely, the number of antennas (M), the signal to interference plus noise ratio(SINR), and the Ricean factor K. In order to evaluate the lower and upper bounds, we have to obtain the distribution of $\mathrm{SINR}_{i,m}$. Since $\mathbf{v}=(\mathbf{v}_1 \ldots \mathbf{v}_{\mathbf{M}})$ is a unitary matrix, and so $\mathbf{H}_i\mathbf{v}$ is a vector with i.i.d. $CN(0,1)$ entries. This implies that $|\mathbf{H}_i\mathbf{v}_m|^2$ are i.i.d. over m (and also over i) with $\chi^2(2)$ distribution. Therefore $\mathrm{SINR}_{i,m}$ for $i=1,\ldots,\mathrm{N}$, are i.i.d. but not independent over $m=1,\ldots,\mathrm{M}$. Thus,

$$\mathrm{SINR}_{i,m} = \frac{|\mathbf{H}_i\mathbf{v}_m|^2}{1+\sum_{k\ne m}|\mathbf{H}_i\mathbf{v}_k|^2} = \frac{z}{1+y} \tag{7}$$

Conditioning on y, the probability distribution function(PDF) of $\mathrm{SINR}_{i,m}$, $f(x)$, can be written as,

$$\begin{aligned} f(x) &= \int_0^\infty f(xy)f(y)dy \\ &= \int_0^\infty (1+y)e^{-(1+y)x} \times \frac{y^{\mathrm{M}-2}e^{-y}}{(\mathrm{M}-2)!}dy \\ &= \frac{e^{-x}}{(1+x)^{\mathrm{M}}}(x+M) \end{aligned} \tag{8}$$

We can also calculate the cumulative distribution function (CDF) of $\mathrm{SINR}_{i,m}$, $F(x)$, as,

$$F(x) = \int_0^x \frac{e^{-x}}{(1+x)^{\mathrm{M}}}(x+\mathrm{M})dx = 1-\frac{e^{-x}}{(1+x)^{\mathrm{M}-1}}, \qquad x\ge 0 \tag{9}$$

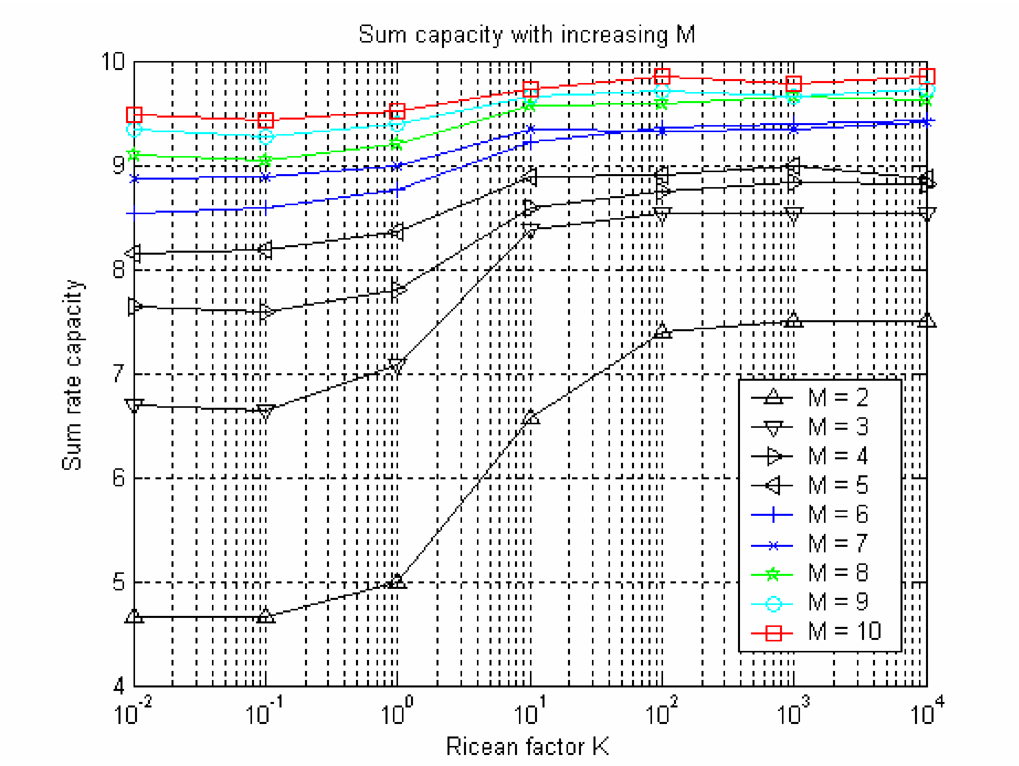

Fig 3. Sum rate capacity versus the Ricean factor K for different M when random beamforming has maximum SINR

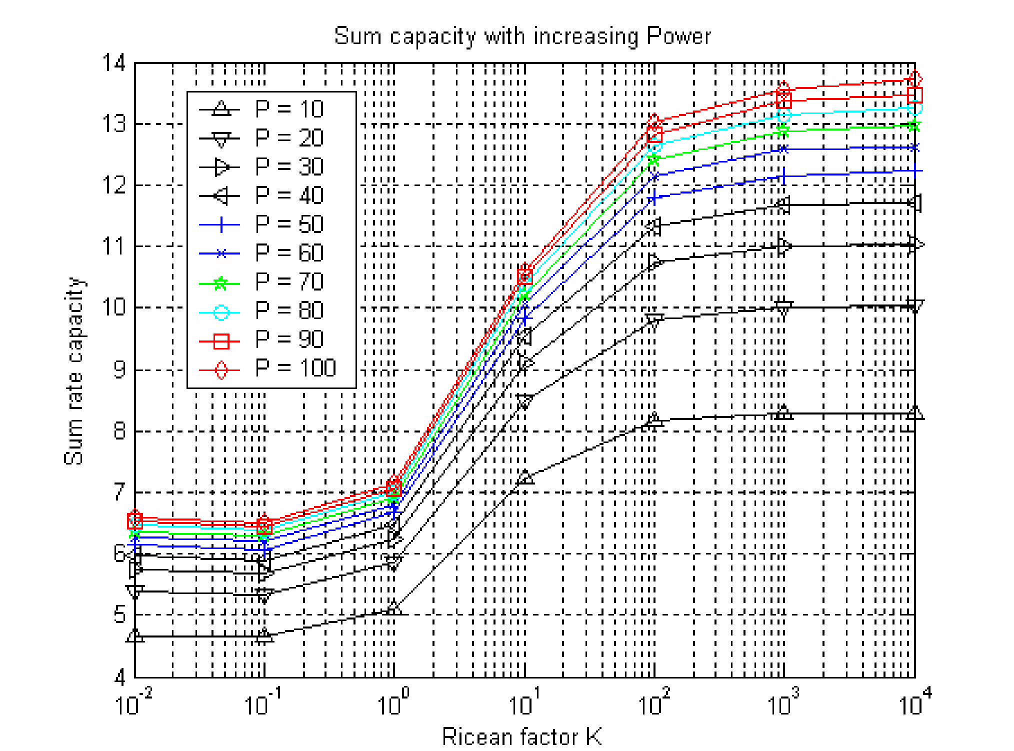

Fig. 4 Sum rate capacity versus the Ricean factor K for different Power When random beamforming has maximum SINR

Since $\mathrm{SINR}_{i,m}$ for $i = 1,\ldots,\mathrm{N}$, are i.i.d. random variables, the CDF of $\max_{1\le i\le \mathrm{N}} \mathrm{SINR}_{i,m}$ for $m = 1,\ldots,\mathrm{M}$ is $\mathrm{F}^n(x)$. Using the obtained CDF we can now evaluate the sum rate capacity of our proposed randomly chosen beam-forming technique:

$$\mathrm{M}\int_1^\infty \log(1+x)\mathrm{N}f(x)F^{\mathrm{N\text{-}1}}(x)dx \le R \le \mathrm{M}\int_0^\infty \log(1+x)\mathrm{N}f(x)F^{\mathrm{N\text{-}1}}(x)dx \qquad (10)$$

where $f(x)$ and $F(x)$ are as defined in (8) and (9), respectively.

From (8) through (10) are derived from [4]

IV. CONCLUSION

This project deals with multiple antenna broadcast channels where due to rapid time variations of the channel, limited resources, imperfect feedback, full channel state information for all the users cannot be provided at the transmitter. Since having no channel state information does not lead to gains, it is important to study MIMO broadcast channels with partial CSI. In this project, the influence of line of sight components on the overall performance of MIMO broadcast channels has been considered. We propose using random beams and choosing the users with the highest signal to interference plus noise ratio. The transmitter is sending M random beamforms to different users instead of sending them to only one user with best SINR to attain linear growth in the channel capacity to the number of transmit antennas. The method proposed is optimal in that it uses M beamforms efficiently than the method where all the M beamforms are concentrated to one user with the best overall channel.

References

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