

MIMO Broadcast Channels with Finite Rate Feedback

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Abstract—Multiple transmit antennas in a downlink channel can provide tremendous capacity (i.e. multiplexing) gains, even when receivers have only single antennas. However, receiver and transmitter channel state information is generally required. In this paper, a system where the receiver has perfect channel knowledge, but the transmitter only receives quantized information regarding the channel instantiation is analyzed. Simple expressions for the capacity degradation due to finite rate feedback as well as the required increases in feedback load per mobile as a function of the number of access point antennas and the system SNR are provided.

I. INTRODUCTION

In multiple antenna broadcast (downlink) channels, capacity can be tremendously increased by adding antennas at only the access point [1] [2]. In essence, an access point (AP) equipped with M antennas can support downlink rates up to a factor of M times larger than a single antenna access point, even when the mobile devices have only single antennas¹. In order to realize these benefits, however, the access point must:

- Simultaneously transmit to multiple users over the same bandwidth (orthogonal schemes such as TDMA or CDMA are generally highly sub-optimal).
- Obtain accurate channel state information (CSI).

Practical transmission structures that allow for simultaneous transmission to multiple mobiles (such as downlink beamforming) do exist. The requirement that the AP have accurate CSI, however, is far more difficult to meet, particularly in FDD systems. Training can be used to obtain channel knowledge at each of the mobile devices, but obtaining CSI at the AP generally requires feedback from each mobile. Such feedback channels do exist in current systems (e.g., for power control), but the required rate of feedback is clearly an important quantity for system designers.

In this paper, we consider the practically motivated *finite rate feedback* model, in which each mobile feeds back a finite number of bits regarding its channel instantiation at the beginning of each block or frame². This model was first considered for point-to-point MIMO channels in [4] [5] [6].

¹In fact, this is true on the uplink as well, by the multiple-access/broadcast channel duality [2]

²Note that the same model was considered independently by Ding, Love, and Zoltowski [3]

In point-to-point channels, the transmitter uses such feedback to more accurately direct its transmitted energy towards the receiver's antenna array, and even a small number of bits per antenna can be quite beneficial [7]. It is important to note, however, that the level of CSI available at the transmitter only affects the SNR-offset of point-to-point MIMO links; it does not affect the slope of the capacity vs. SNR curve, i.e., the *multiplexing gain*.

The capacity benefits of MIMO downlink channels, however, crucially depend on the CSI available to the AP. If the AP has perfect CSI, a multiplexing gain of M is generally achievable. If the AP has no CSI, however, a multiplexing gain of only one is achievable, and capacity can be reduced by up to a factor of M . Therefore, channel feedback is considerably more important for MIMO downlink channels than for point-to-point channels.

In this work we propose a simple downlink transmission scheme that uses downlink beamforming in conjunction with finite rate feedback. We quantify the degradation in capacity as a function of the feedback load, and show that the feedback load per mobile must increase approximately linearly with the number of AP antennas as well as the system SNR (in dB) in order to guarantee performance close to that with perfect CSI.

The remainder of this paper is organized as follows: In Section II we describe the channel model and in Section III we summarize capacity results for the MIMO broadcast channel with different degrees of CSI. In Section IV we introduce the finite rate feedback model and provide analytical results on systems with a fixed number of feedback bits as well as systems in which the feedback load increases with the SNR.

II. SYSTEM MODEL

We consider a K receiver multiple antenna broadcast channel in which the transmitter (access point) has M antennas, and each of the receivers has a single antenna. The broadcast channel is mathematically described as:

$$\mathbf{y}_i = \mathbf{h}_i^T \mathbf{x} + \mathbf{n}_i, \quad i = 1, \dots, K \quad (1)$$

where $\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K$ are the channel matrices (with $\mathbf{h}_i \in \mathbb{C}^{M \times 1}$) of users 1 through K , the vector $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the transmitted signal, and $\mathbf{n}_1, \dots, \mathbf{n}_K$ are independent complex

Gaussian noise terms with unit variance. There is a transmit power constraint of P , i.e. we must satisfy $E[||\mathbf{x}||^2] \leq P$.

For simplicity of exposition we assume that $K = M$. Straightforward generalizations for systems with $K < M$ are provided in [8]. If $K > M$, M of the mobiles can randomly be selected to reduce to the $K = M$ scenario.

We denote the concatenation of the channels by $\mathbf{H}^T = [\mathbf{h}_1 \ \mathbf{h}_2 \ \dots \ \mathbf{h}_K]$, i.e. \mathbf{H} is $K \times M$ with the i -th row equal to the channel of the i -th receiver (\mathbf{h}_i^T).

We consider the scenario where the entries of the channel matrices are distributed as iid complex Gaussian (Rayleigh fading). Each of the receivers is assumed to have perfect and instantaneous knowledge of its own channel vector, i.e. \mathbf{h}_i . Notice it is not required for mobiles to know the channel of other mobiles. The channel is assumed to be block fading, with independent fading from block to block.

Notation: We use boldface to denote vectors and matrices, \mathbf{A}^T denotes the transpose of \mathbf{A} , and \mathbf{A}^\dagger refers to the conjugate transpose, or Hermitian, of \mathbf{A} . The notation $||\mathbf{x}||$ refers to the Euclidean norm of the vector \mathbf{x} .

III. CAPACITY RESULTS

In this section we provide some background on known capacity results for the multiple-antenna broadcast channel. When perfect CSI is available at transmitter and receivers, the capacity region of the channel is achieved by dirty-paper coding [9], which is a technique that can be used to pre-cancel multi-user interference at the transmitter [10]. In this paper we study the total system throughput, or the sum rate, which we denote as $C_{sum}(\mathbf{H}, P)$. At high SNR, the sum rate capacity of the MIMO BC can be approximated as [11]:

$$C_{TX/RX-CSI}^{sum}(\mathbf{H}, P) \approx M \log(P) + c \quad (2)$$

where c is a constant depending on the channel matrix \mathbf{H} . The key feature to notice is that capacity grows *linearly* as a function of M . Though the K (total) receive antennas are distributed amongst K receivers, the linear growth is the same as in a M -transmit, K -receive antenna point-to-point MIMO system, i.e. both systems have the same *multiplexing gain*. Though downlink beamforming (without DPC) is not capacity-achieving, it does achieve the same multiplexing gain because it utilizes all M spatial dimensions. In downlink beamforming, the AP chooses a different beamforming vector (i.e., signalling direction) for each mobile.

If the transmitter does not have CSI, the situation is very different. In this scenario, the channels of all receivers are statistically identical, and thus any codeword receiver 1 can decode can also be decoded by any other receiver [12, Section VI] [13]. This implies that a TDMA strategy is optimal, and the sum capacity of this channel is equal to the capacity of the point-to-point channel from the transmitter to any one of the mobiles:

$$C_{RX-CSI}^{sum} = E_{\mathbf{h}_1} \left[\log \left(1 + \frac{P}{M} ||\mathbf{h}_1||^2 \right) \right] \quad (3)$$

The multiple AP antennas provide an SNR boost, but a multiplexing gain of only unity.

Clearly, there is a large gap between the performance with transmitter CSI (multiplexing gain of M) and without transmitter CSI (multiplexing gain of 1). Thus, it is of interest to investigate the more practical assumption of partial CSI at the AP. Though progress on the capacity region of such a channel has recently been made [14], the capacity region is still unknown. Our work shows that the full multiplexing gain can be achieved with partial CSI if the quality of the feedback (i.e. the quality of the CSI) is increased as a function of SNR.

IV. FINITE RATE FEEDBACK MODEL

In the finite rate feedback model (shown in Fig. 1), the i -th receiver quantizes \mathbf{h}_i to N_{FB} bits and feeds back the bits perfectly (i.e. error-free) and instantaneously to the access point. The quantization is performed using a quantization codebook that is known at the transmitter and the receivers. A quantization codebook \mathcal{C} consists of $2^{N_{FB}}$ M -dimensional unit norm vectors $\mathcal{W} \triangleq \{\mathbf{w}_1, \dots, \mathbf{w}_{2^{N_{FB}}}\}$, where N_{FB} is the number of feedback bits per mobile. Similar to point-to-point MIMO systems, each receiver quantizes his channel to the beamforming vector that is closest to its channel vector, where closeness is measured in terms of the angle between two vectors or equivalently the inner product [5] [6]. Thus, the feedback from user i is chosen as:

$$F_i = \arg \max_{j=1, \dots, 2^{N_{FB}}} |\mathbf{h}_i^\dagger \mathbf{w}_j|. \quad (4)$$

For analytical simplicity, we use *random vector quantization* (RVQ), in which each of the $2^{N_{FB}}$ quantization vectors is independently chosen from the isotropic distribution on the M -dimensional unit sphere. We analyze performance averaged over all such choices of random codebooks, in addition to averaging over the fading distribution. Note that RVQ has a number of strong optimality properties for point-to-point MIMO channels [15].

After the transmitter has received feedback bits from each of the K receivers, an appropriate multi-user transmission strategy is chosen. In this work we use *zero-forcing beamforming*, both for its effectiveness and analytical simplicity³. When the transmitter has perfect CSI, zero-forcing can be used to completely eliminate multi-user interference by precoding transmission by the inverse of the channel matrix \mathbf{H} . This creates a parallel, non-interfering channel to each of the M receivers, and thus leads to a multiplexing gain of M .

In the finite rate feedback setting, the transmitter performs zero-forcing assuming that the quantized versions of the channels are correct. Since the quantizations are incorrect with probability one, zero-forcing does not eliminate all multi-user interference. Let $\hat{\mathbf{h}}_i$ refer to the quantized version of the

³Note that DPC cannot be directly applied to eliminate multi-user interference in this scenario because perfect transmitter CSI is required in order to do so. This is because DPC requires knowledge of the additive interference at the receiver, not the transmitter. In the finite rate feedback model, DPC can be used in conjunction with the transmitter's estimate of the channel vectors, but cancellation is not perfect and any error in the channel estimate leads to an additional source of noise.

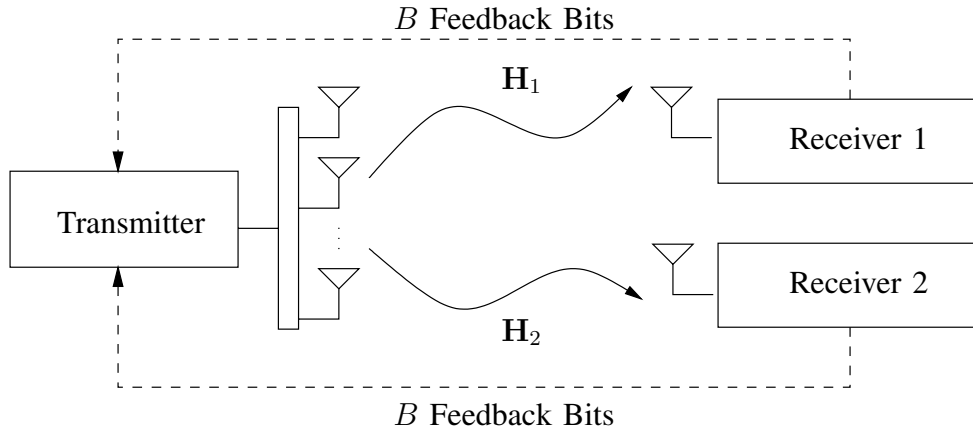


Fig. 1. Finite Rate Feedback System Model

mobile i 's channel. These quantized vectors are compiled into a matrix: $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1 \ \hat{\mathbf{h}}_2 \ \dots \ \hat{\mathbf{h}}_M]^T$.

The matrix $\hat{\mathbf{H}}$ is the estimate of the channels, upon which zero-forcing is performed. Thus, the zero-forcing precoding matrix is given by: $\mathbf{T} \triangleq \hat{\mathbf{H}}^\dagger (\hat{\mathbf{H}} \hat{\mathbf{H}}^\dagger)^{-1}$. Let \mathbf{v}_i denote the i -th column of \mathbf{T} normalized to have unit norm. Then, the transmitted signal is given by

$$\mathbf{x} = \sum_{j=1}^K \mathbf{v}_j s_j, \quad (5)$$

where $\mathbf{s} = [s_1 \ s_2 \ \dots \ s_K]^T$ is the vector of symbols intended for the K receivers, which are chosen iid zero-mean Gaussian with power P/M . The received signal at user i is therefore given by:

$$y_i = \mathbf{h}_i^T \mathbf{x} + n_i = \sum_{j=1}^K \mathbf{h}_i^T \mathbf{v}_j s_j + n_i, \quad (6)$$

and the SINR at mobile i is:

$$SINR_i = \frac{\frac{P}{M} |\mathbf{h}_i^T \mathbf{v}_i|^2}{1 + \sum_{j \neq i} \frac{P}{M} |\mathbf{h}_i^T \mathbf{v}_j|^2}. \quad (7)$$

We study the ergodic capacity, or the average capacity averaged over the fading distribution. Therefore, the rate of transmission to User i is equal to:

$$R_i = E_{\mathbf{H}} [\log_2(1 + SINR_i)]. \quad (8)$$

For a system that achieves a sum rate of $R(P)$ (where P is the SNR, or power constraint), the multiplexing gain is defined as:

$$r = \lim_{P \rightarrow \infty} \frac{R(P)}{\log_2(P)}. \quad (9)$$

A. Fixed Feedback Quality

We first study the capacity of a system where the number of feedback bits per mobile N_{FB} is fixed. We provide an expression for the approximate SNR-loss relative to perfect CSI as a function of the number of feedback bits, and

then show that a fixed feedback system becomes interference limited at high SNR.

In order to quantify the SNR degradation due to finite rate feedback, consider the expression for the SINR at the i -th mobile:

$$SINR_i = \frac{\frac{P}{M} |\mathbf{h}_i^T \mathbf{v}_i|^2}{1 + \sum_{j \neq i} \frac{P}{M} |\mathbf{h}_i^T \mathbf{v}_j|^2}. \quad (10)$$

We are interested in studying the SNR degradation relative to zero-forcing beamforming with perfect CSI. If zero-forcing is used, the beamforming vector for user j (\mathbf{v}_j) is chosen orthogonal to all other users' channels, resulting in $|\mathbf{h}_i^T \mathbf{v}_j| = 0$ for all $i \neq j$. When finite-rate feedback is used, this condition is not satisfied because \mathbf{v}_j is chosen orthogonal to the quantized versions of other users' channels. Thus, finite rate feedback leads to an SNR degradation by increasing the denominator of the SINR expression in (10)⁴. By computing the expected value of the denominator (i.e. the average interference plus noise power), we arrive at an approximate upper bound to the power loss due to finite rate feedback:

Theorem 1: Finite rate feedback with random vector quantization incurs an SNR degradation of approximately

$$\Delta SINR_{dB} \approx 10 \log_{10} \left(1 + P \cdot 2^{-\frac{N_{FB}}{M-1}} \right)$$

where N_{FB} is the number of feedback bits and P is the power constraint.

Proof: This approximation is arrived at by computing the expectation of the interference and noise level over the

⁴The statistical distribution of the numerator of (10) is not affected by the use of finite rate feedback, as it is easy to show that the quantity $|\mathbf{h}_i^T \mathbf{v}_i|^2$ is chi-square with two degrees of freedom with either perfect CSI or finite rate feedback.

distribution of the channels and the random codebooks:

$$\begin{aligned}
 \Delta P &= E_{\mathbf{H}, \mathcal{W}} \left[1 + \sum_{j \neq i} \frac{P}{M} |\mathbf{h}_i^T \mathbf{v}_j|^2 \right] \\
 &= 1 + \frac{P}{M} (M-1) E \left[\|\mathbf{h}_i\|^2 \cos^2(\angle(\mathbf{h}_i, \mathbf{v}_j)) \right] \\
 &\stackrel{(a)}{=} 1 + \frac{P}{M} (M-1) M E \left[\cos^2(\angle(\mathbf{h}_i, \mathbf{v}_j)) \right] \\
 &\stackrel{(b)}{=} 1 + P \cdot E \left[\sin^2(\angle(\mathbf{h}_i, \hat{\mathbf{h}}_i)) \right] \\
 &\stackrel{(c)}{=} 1 + P \cdot 2^{N_{FB}} \cdot B \left(2^{N_{FB}}, \frac{M}{M-1} \right) \\
 &\stackrel{(d)}{\leq} 1 + P \cdot 2^{-\frac{N_{FB}}{M-1}},
 \end{aligned}$$

where $B(\cdot, \cdot)$ is the beta function. Step (a) is arrived at by noting that $\|\mathbf{h}_i\|^2$ is chi-square with $2M$ degrees of freedom, independent of the direction of \mathbf{h}_i . For the remainder of the proof, WLOG assume that $\|\mathbf{h}_i\| = 1$. The key step is (b), which follows from the fact that $E[\cos^2(\angle(\mathbf{h}_i, \mathbf{v}_j))] = \frac{1}{M-1} E[\sin^2(\angle(\mathbf{h}_i, \hat{\mathbf{h}}_i))]$. To see this, without loss of generality assume $\hat{\mathbf{h}}_i = [1 \ 0 \ \dots \ 0]^T$, and $\mathbf{v}_j = [0 \ 1 \ 0 \ \dots \ 0]^T$ for some $j \neq i$. This is always possible by a change of basis because \mathbf{v}_j is chosen orthogonal to $\hat{\mathbf{h}}_i$. Then we have $\cos^2(\angle(\mathbf{h}_i, \mathbf{v}_j)) = |\mathbf{h}_{i,2}|^2$, by definition of \mathbf{v}_j . Since RVQ is used, the vector \mathbf{h}_i is related to $\hat{\mathbf{h}}_i$ according to a distribution that only depends on the angle between \mathbf{h}_i and $\hat{\mathbf{h}}_i$. By symmetry we have $E[|\mathbf{h}_{i,2}|^2] = \frac{1}{M-1} E[\sum_{j=2}^M |\mathbf{h}_{i,j}|^2] = \frac{1}{M-1} E[\sin^2(\angle(\mathbf{h}_i, \hat{\mathbf{h}}_i))]$. The distribution and expectation of $\sin^2(\angle(\mathbf{h}_i, \hat{\mathbf{h}}_i))$ is given in [16], yielding (c). Finally, the inequality in (d) is reached using inequalities provided in [16]. ■

The most important feature to notice is that the SNR loss is an increasing function of the system SNR as well as the number of AP antennas. Note that this is only an approximation because the numerator and denominator in (10) are not independent.

Theorem 2: When the number of feedback bits N_{FB} is fixed, the system becomes interference limited at high SNR.

Proof: The sum rate of the system is given by

$$R(P) = \sum_{i=1}^K E_{\mathbf{H}} [\log_2(1 + \text{SINR}_i)]. \quad (11)$$

By Jensen's inequality, $E_{\mathbf{H}} [\log_2(1 + \text{SINR}_i)] \leq \log_2(1 + E_{\mathbf{H}} [\text{SINR}_i])$. The expected SINR can be bounded as:

$$E_{\mathbf{H}} [\text{SINR}_i] = E_{\mathbf{H}} \left[\frac{\frac{P}{M} |\mathbf{h}_i^\dagger \mathbf{v}_i|^2}{1 + \sum_{j \neq i} \frac{P}{M} |\mathbf{h}_i^\dagger \mathbf{v}_j|^2} \right] \quad (12)$$

$$\leq E_{\mathbf{H}} \left[\frac{|\mathbf{h}_i^\dagger \mathbf{v}_i|^2}{\sum_{j \neq i} |\mathbf{h}_i^\dagger \mathbf{v}_j|^2} \right]. \quad (13)$$

This quantity is clearly finite and is independent of P . Thus, the average SINR of each user is bounded, and therefore $R(P)$ is also bounded. Evaluating the limit in (9), we see

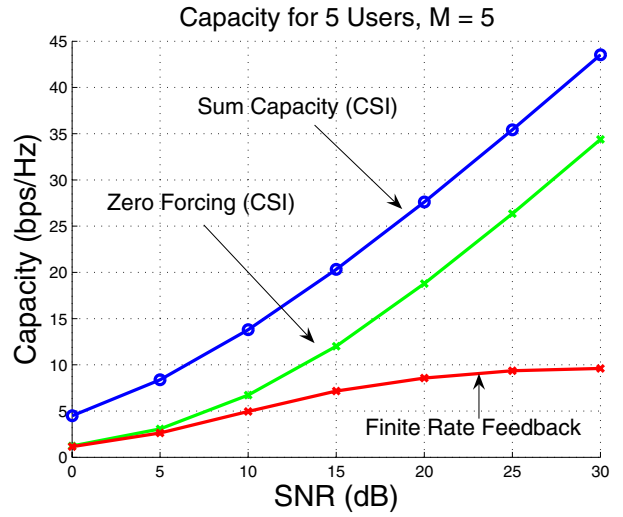


Fig. 2. Performance with Fixed # Feedback Bits

that the multiplexing gain is zero, implying that the system is interference limited. ■

Regardless of how many feedback bits (N_{FB}) are used, the system eventually becomes interference limited because interference and signal power both scale linearly with P . In Fig. 2 the performance of a 5 antenna, 5 user system with 15 feedback bits per mobile is shown. When the SNR is small, limited feedback performs nearly as well as zero-forcing. However, as the SNR is increased, the limited feedback system becomes interference limited and the rates converge to an upper limit, as expected.

B. Increasing Feedback Quality

In this section we study systems in which the number of feedback bits is increased as a function of the SNR in order to achieve the full multiplexing gain of the downlink channel. We show that the full multiplexing gain is achieved if N_{FB} is increased at the appropriate rate as a function of the SNR, and provide a simple characterization for the number of the number of feedback bits required to maintain a constant power offset relative to perfect CSI zero-forcing.

We are interested in scaling the number of feedback bits such that a constant power offset, or alternatively rate offset, is maintained between zero-forcing with perfect CSI (ZF) and the rates achievable using limited feedback (LF). Some simple algebra and Jensen's inequality yields the following upper bound on the rate difference per mobile:

$$\begin{aligned}
 \Delta R &= \lim_{P \rightarrow \infty} E_{\mathbf{H}, \mathcal{W}} [R_{ZF}(P) - R_{LF}(P)] \\
 &\leq \lim_{P \rightarrow \infty} E_{\mathbf{H}, \mathcal{W}} \left[\log \left(1 + \frac{P}{M} \sum_{j \neq i} |\mathbf{h}_i^\dagger \mathbf{v}_j|^2 \right) \right] \\
 &\leq \lim_{P \rightarrow \infty} \log \left(E_{\mathbf{H}, \mathcal{W}} \left[1 + \frac{P}{M} \sum_{j \neq i} |\mathbf{h}_i^\dagger \mathbf{v}_j|^2 \right] \right) \quad (14)
 \end{aligned}$$

Thus, if the the expected value of the interference and noise term can be kept constant as $P \rightarrow \infty$, the rates achievable with limited feedback will be within ΔR of those achieved with zero-forcing.

In the previous section, the following upper bound is provided:

$$E_{H,W} \left[1 + \frac{P}{M} \sum_{j \neq i} |\mathbf{h}_i^\dagger \mathbf{v}_j|^2 \right] \leq 1 + P \cdot 2^{-\frac{N_{FB}}{M-1}}$$

If this upper bound can be kept constant by increasing N_{FB} as a function of P , then the rate difference between zero-forcing and limited feedback will also be kept constant. If we set $1 + P \cdot 2^{-\frac{N_{FB}}{M-1}} = b$, where $b \geq 1$ is some constant, we get:

$$\begin{aligned} N_{FB} &= (M-1) \log_2 P - (M-1) \log_2(b-1) \\ &= \frac{(M-1) \log_2 10}{10} P_{dB} - (M-1) \log_2(b-1) \\ &\approx \frac{M-1}{3} P_{dB} - (M-1) \log_2(b-1). \end{aligned}$$

In order to maintain a maximum power offset of 3 dB, it is sufficient to keep the interference and power term to be equal to 2, i.e. $b = 2$. Thus, the resulting scaling of bits takes on a particularly simple form when a 3 dB offset is desired:

$$N_{FB} = \frac{M-1}{3} P_{dB} \text{ bits/mobile} \quad (15)$$

In order to achieve a smaller power offset, b needs to be made appropriately smaller. For example, a 1-dB offset corresponds to $b = 10^{1/10} = 1.259$ and thus an additional $1.95(M-1)$ feedback bits are required at all SNR's.

Thus, in order to maintain a constant power gap, the feedback load must be scaled approximately linearly with the number of access point antennas (M) as well as with the SNR. Note that closed-form expressions for the gap between sum rate capacity and perfect CSI zero-forcing are provided in [11].

In Fig. 3, achievable rates vs. SNR are shown for a 5 antenna, 5 user system. The feedback load is assumed to scale by the relationship given in (15), and limited feedback is seen to perform within 2.6 dB of perfect CSI zero-forcing. Notice that actual power offset is smaller than 3 dB primarily due to the use of Jensen's inequality in deriving ΔR in (14).

V. CONCLUSIONS

We have proposed the use of finite rate feedback in a multiple antenna downlink channel as a method to exploit the enormous capacity benefits of multi-user MIMO. We showed that if the number of feedback bits scales with the SNR, then the full multiplexing benefit of the multiple transmit antennas can be realized. Furthermore, in order to maintain performance within a few dB of perfect CSI zero-forcing, the feedback load per mobile scales approximately linearly with the number of access point antennas.

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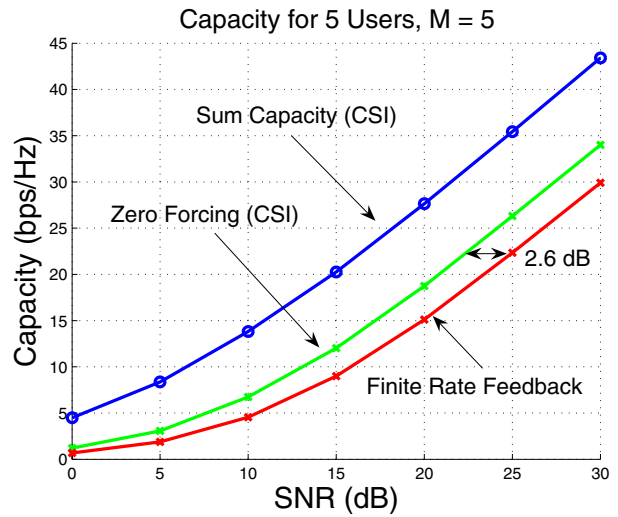


Fig. 3. Performance with Increasing # Feedback Bits

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