The “Monte Hall Problem” is as follows. Monte Hall was emcee of the game show “Let’s Make a Deal”. He would invite a member of the audience up to the stage, where there were three closed doors. Behind one of the doors was a prize, behind the other two doors nothing much. He would invite the contestant to pick one of the doors. When they picked a door, he would open one of the other doors, behind which there was nothing, and he would then invite the person to switch their pick to the remaining door or stick with the door they originally chose.

We want to analyze this problem using the method of conditional probabilities to determine the optimum strategy for the game. That is, there are a few strategies one could pick:

1. Just stick with the door originally chosen.
2. Always switch to the remaining door.
3. Flip a fair coin to decide whether to stick with the original door or switch.

Analyze these strategies to determine what the probability is, in each case, of winning the prize.

If there is sufficient time, here are two more problems which have what appear to be counter-intuitive answers.

**B)** Discuss (and simulate) the following problem, where a gambler has a certain amount \(K\) of money, which the gambler wagers (one unit per bet, and one bet per minute) with a probability of winning (or losing) a dollar .5 on each bet. The question is: do you expect the gambler to be able to play forever, or not?

Simulate the result using MATLAB to approximate the function \(P(t(n) = 0)\) (which is the probability that the gambler goes broke at time \(n\)).

**C)** Two people in this class are given (fair) coins and told to flip them 12 times and report to the class the results. One says: "I got HTTHHHTHHTHTT". The other says "I got "HHHHHHHHHHHH". Which one is lying (the answer could be "neither")? Demonstrate your conclusion using analysis as well as MATLAB simulation.