**Problem 9.1** A random variable $r$ is exponential with mean 1. Given that $r$ has been sampled and you obtained the value $r$, we have another random variable $x$ that is exponential with mean $\frac{1}{r+1}$.

Find:
(a) the MMSE estimate for $x$ given $r$.

(b) the MMSE estimate for $r$ given $x$.

(c) the LMMSE estimate for $x$ given $r$.

(d) the LMMSE estimate for $r$ given $x$.

Using MATLAB, simulate each of the above and calculate the Mean Square Error for each.

**Problem 9.2** A receiver at a radial distance $R$ from a radio beacon measures the beacon power to $x = y - 40 - 40\log_{10}R$ (in dB) where $y$ is a Gaussian $N(0, 8)$ random variable independent of $R$, which is a uniform random variable in $[0, 1000]$. Find the MAP and ML estimate of $R$ given $x = x$. Are they the same in this case?