• We derived an approximate formula for the bandwidth of FM (not just for NBFM, like we did in the last class, but for FM in general). We did this by assuming that the message signal is constant for a certain interval of duration $\frac{1}{2B}$, which allowed us to simplify the FM waveform expression (specifically, we did not have to deal with the integral of $m(t)$ inside a cosine function, because the integral over a constant is just constant$^*t$). The Fourier Transform then gave us a sinc pulse. By taking the worst case positions of this sinc pulse, we were able to say that a large amount of the FM signal is concentrated in a bandwidth of $2(\Delta f + 2B)$. This is described on pages 261-263.

• We saw last class that in practice, we use Carson’s rule to estimate the bandwidth, which is $2(\Delta f + B)$, because the actual FM bandwidth is not actually as high as the worst case estimate we came up with above.

• When $m(t) = \alpha \cos(2\pi f_m t)$, which is a simple cosine signal of frequency $f_m$ and amplitude $\alpha$, we can compute the FM bandwidth exactly, without approximations. The Fourier Transform of the FM signal is then bunch of delta functions, centered at $f_c, f_c \pm f_m, f_c \pm 2f_m, f_c \pm 3f_m, \ldots$, and so on. Contrast this to the DSB-SC AM spectrum, which just had two delta functions (on the positive side): one centered at $f_c - f_m$ and another at $f_c + f_m$. FM thus needs infinite bandwidth, while the corresponding AM waveform only needed $2f_m$.

• Fortunately, the delta functions in the FM spectrum are not all the same magnitude. The delta function at $f_c + nf_m$, where $n = \ldots, -2, -1, 0, 1, 2, \ldots$, has a magnitude of $J_n(\beta)$, where $J_n(\beta)$ is called the $n^{th}$ order Bessel Function of the 1st kind. This is a standard function, and the values of this function for various $n$ and $\beta$ can be looked up in tables, or computed using software. Further, the value of $J_n(\beta)$ goes to zero as $n$ goes to either $\infty$ or $-\infty$, and in fact, $J_n(\beta)$ is very small if $|n| > \beta + 1$. Hence, we only consider a subset of these delta functions to constitute the FM bandwidth in practice, and don’t actually have to deal with an infinite bandwidth signal.

• At $n = \pm(\beta + 1)$, the corresponding delta function in the spectrum of the FM signal (for $m(t) = \alpha \cos(2\pi f_m t)$) is centered at $f_c \pm (\beta + 1)f_m$. If these are the highest/lowest frequency components we consider, and we ignore all frequencies beyond this because they have too small a magnitude, we will have an FM bandwidth of $2(\beta + 1)f_m = 2(\Delta f + B)$, where we have used the definition of $\beta$ from last class (and $B$ for $m(t) = \alpha \cos(2\pi f_m t)$ is just $f_m$). Notice that our exact analysis for a cosine message signal is the same as the Carson rule estimate. The above derivation and all relevant details are in pages 264-266. You will not be asked to repeat such a derivation in the exam/quiz, but be aware of the results and implications of all this.

• We already saw how a VCO can be used to generate an FM or PM signal. Before we had ways of building variable capacitors that can change quickly for a given input signal (which is how a VCO works), we had to rely on other methods of FM generation. One such method is to use the NBFM approximation we studied last class, which will yield an FM signal, as long as $\Delta f$ only needs to be small (that is, small $k_f$ values). This is described in Section 5.3, under “NBFM generation” (Figures 5.8, 5.9 depict the block diagram).
• To generate WBFM (commercial FM, for instance, is not NBFM), we can use *frequency multipliers*. If we feed an FM signal with center frequency $f_c$ and frequency deviation $\Delta f$ into a frequency multiplier, we can produce an FM waveform with center frequency $nf_c$, and frequency deviation $n\Delta f$, for some positive integer $n$. In other words, if we select $n$ to be large enough, we can convert NBFM to WBFM (because the frequency deviation is increased $n$-fold).

• A frequency multiplier can be built using a non-linear device followed by a band pass filter. The non-linear device produces harmonics of the input signal at $f_c, 2f_c, 3f_c$, and so on, and a band pass filter can be used to select the desired frequency component for a given value of $n$. The details, and the math behind this, is given on page 275. This method of generating NBFM and then using a frequency multiplier to get WBFM is called *Armstrong’s Indirect Method* of FM generation.

• FM can be demodulated using a PLL, as was demonstrated a few classes ago. Another method is to use a “slope detector” (also called a “frequency discriminator”). This method involves simply differentiating the FM waveform, which, if you work out the math (page 282), turns it into an AM waveform (with a DC offset). If the DC offset is large enough, we can use an envelope detector to demodulate this signal. Note that the derivative of an FM waveform still has a carrier of variable frequency, which means that a synchronous AM demodulator will not work, and we need to use an envelope detector. The block diagram is in Figure 5.12.

• A differentiator can be thought of as just a filter (a derivate in time domain is just multiplication by $j2\pi f$ in the frequency domain). We can therefore build a differentiator using any filter that has a linear response (that is, something proportional to $j2\pi f$) over the bandwidth of the input FM signal. The filter can have any arbitrary response outside the bandwidth of the input FM signal, since the output will not depend on that. Figure 5.12 depicts this concept, and Page 283 talks about how we might build a practical differentiator using a filter. A derivative is the “slope” of an input signal, hence the name “slope detector”.

• FM is preferred over AM because it has better noise rejection. We also saw the “Capture Effect” last class. Yet another advantage of FM is the immunity to non-linear distortion. As we saw above for the frequency multiplier, when we introduce non-linearities to an FM signal, we can use a BPF to reject the higher frequency harmonics. For AM, on the other hand, even if we use a BPF to reject higher frequency harmonics, we will be stuck with a signal that has distorted amplitude. For FM, we don’t care about amplitude distortions, but for AM, amplitude distortions are very important, and non-linearities might corrupt the message signal carried using AM. The math that explains this is on page 284 (be comfortable with this). In practice, this means we can use efficient Class C amplifiers for FM signals, which are known to produce non-linearities.

• FM is generally used to transmit audio signals (such as in analog TV broadcast or FM radio), and unfortunately, audio signals usually have a lot of power in the lower frequencies and lesser power in the higher frequencies. This means that the higher power low frequency components will be less affected by noise, when compared to the lower powered high frequency components. Hence, we often “preemphasize” an audio message signal before we use it to generate an FM waveform, using a preemphasis filter which has a higher gain at higher frequencies, which boosts the power of those frequencies. At the receiver, after demodulation, we would need to apply the reverse of the preemphasis filter (this is called deemphasis), to remove the effects of preemphasis and recover the original audio signal. Figures 5.15, 5.16 depict all this well.