We saw that multipliers can be used both to modulate and demodulate a signal (Figure 4.1). Section 4.2 further talks about 4 implementations of multipliers: variable gain amplifiers, nonlinear modulators, switching modulators and ring modulators. In class, we discussed the first three in detail (Pages 184-187).

Demodulators built with multipliers are very good in that when we match the demodulation oscillator to the carrier frequency, we can demodulate a particular signal very well. Noise and interference from other channels present in other frequency bands are all rejected by the filter after the multiplier. However, we require a good match of the carrier signal to be generated at the demodulator (Problem 4 in HW2 deals with the effects of frequency and phase mismatch during demodulation).

Generating a matched carrier locally can be a problem because of time delays (which cause a phase shift in the transmitted carrier which might be difficult to estimate), and Doppler shifts (which causes the frequency of the transmitted carrier to change slightly).

Demodulators that need the carrier signal to be locally generated while demodulating a signal are called “synchronous” or “coherent” demodulators. However, instead of generating the carrier locally at the demodulator, we can transmit the carrier along with the modulated signal.

For instance, instead of using \( m(t) \) as the message signal, we can use \( A + m(t) \) (Page 190, 191), and transmit \( (A + m(t))c(t) \), where \( c(t) \) is the carrier signal. Hence, we transmit not only the usual modulated signal \( m(t)c(t) \), but also, the component \( Ac(t) \), which is the carrier amplified by the factor \( A \). The \( A \) is a constant voltage added to the message, and is also called the DC factor.

In the frequency domain, \( m(t)c(t) \) has a spike at the carrier frequency \( f_c \), with the message spectrum appearing on either side of the carrier (Figure 4.1). If we include the \( Ac(t) \) component, we will see a spike at \( f_c \) (Equation 4.8c). Transmitting only \( m(t)c(t) \) is referred to as DSB-SC AM, which you might recall from last class. You should be comfortable with what happens to signals when they are multiplied by cosine signals, and what happens to them when passed through filters (the beginning of Section 4.2 is a good refresher).

When we transmit the carrier along with the DSB-SC signal, we can “recover” this carrier and use it for demodulation. We will discuss this later on, but this class, we saw that transmitting the carrier allows us to use another kind of demodulator: the envelope detector. Recall from class that the envelope of the modulated waveform closely follows the message signal \( m(t) \), when \( A \) is large enough. The received signal is \( (A + m(t))c(t) \), and the envelope of this signal is simply \( |A + m(t)| \). Since we want the envelope to be equal to the message we transmitted, we want \( |A + m(t)| = A + m(t) \), or in other words, \( A + m(t) \geq 0 \) (this condition is in addition to having the condition \( f_c \gg \) (bandwidth of \( m(t) \)), which we discussed last class). These conditions are explained in Section 4.3. Related, is the quantity called the “modulation index” \( \mu \) (Page 192, 193). If \( \mu > 1 \), we call the modulated signal an “over-modulated” signal, which cannot be demodulated by envelope detection (however, it can be demodulated with a synchronous demodulator). These conditions on \( \mu \) arise from the fact that, generally, if we
are transmitting $x(t)c(t)$, where $x(t) = A + m(t)$, the envelope detector will not work unless $x(t)$ is always positive. If $x(t)$ takes negative values, then those negative values will be seen as positive values instead by the envelope detector (which is modeled as an absolute value operator), and so will appear as distortion (which we “heard” in class).

- We discussed a simple envelope detector, based on charging and discharging a capacitor using a diode (Figure 4.11). Such demodulators were once called “crystal radios” (because diodes were called crystals at one point) and they don’t need a power supply to operate. In effect, they can demodulate a signal using simply the power in the transmitted signal itself. Recall that the DC term, which we called $A$ in the class, has to be large enough for the envelope demodulator to work. Larger the value of $A$, the more the power we are transmitting with, so it is no surprise that we can use this power to demodulate.

- The capacitor in the diode demodulator needs to be of just the right value so that it charges and discharges at the correct rate to capture the message signal on the envelope (Page 197). An alternative envelope detector can be built using a rectifier (Figure 4.10).