Problem 1

Consider a QAM system (Figure 1), where two message signals $m_1(t)$ and $m_2(t)$ (each of bandwidth $B$ Hz) are modulated by the carriers $\cos(2\pi f_c t)$ and $\cos(2\pi f_c t + \phi)$, respectively, where $f_c >> B$. If $\phi = \pi/2$, then the second carrier becomes a sine signal, and this is a standard QAM system where $v_1(t) = \text{constant} \ast m_1(t), v_2(t) = \text{constant} \ast m_2(t)$, as we discussed in class. What will $v_1(t)$ and $v_2(t)$ be for a general value of $\phi$? Hence conclude why $\phi = \pi/2$ (which means that the two carriers are sine and cosine signals) was chosen for the standard QAM system.

Problem 2

(a) You are given signals $m_1(t), m_2(t), m_3(t)$ and $m_4(t)$, each of bandwidth 5 KHz, where $|m_i(t)| \leq 2$ for $i = 1, 2, 3, 4$. You are licensed to transmit no more than a total of 32 W in the 500 - 522 KHz frequency band. No power may be transmitted outside this band. The 4 signals are live audio broadcasts, and all 4 need to be transmitted at the same time (you cannot transmit them
one-by-one). Explain, with a block diagram, how you would transmit and receive these signals, under this license. Your transmitter should generate a signal \( s(t) \) that will be transmitted, and the receiver should operate on \( s(t) \) to recover \( m_1(t), m_2(t), m_3(t) \) and \( m_4(t) \) (you can design a single receiver that demodulates all 4 at the same time, or 4 separate receivers that demodulate each one).

Specify any oscillator signal frequencies and amplitudes, as well as filter cutoffs (assuming ideal filters) you use. For whatever carrier signals you use at the transmitter, assume you have perfectly synchronized carriers with the same frequency and phase available at the receiver (so you don’t have to worry about frequency and phase errors during demodulation). There might be more than one solution, specify any design that will work.

(b) The signal at the receiver input in (a) might be too weak as a result of having traveled over a long distance. You have an amplifier (of sufficiently large gain) that operates only in the 100-110 KHz band. Further, all the filters you are using are of poor design and generate a lot of noise (even though they have an ideal frequency response). Hence, you should amplify signals before sending them through a filter, rather than after, to avoid amplifying any additional noise introduced by the filter. Design the receiver which will recover \( m_i(t) \), while including this amplifier (you can pick any one convenient value for \( i \), i.e., 1, 2, 3 or 4).

**Problem 3**

**Note:** This problem was originally proposed with 4 parts, but the first 2 parts were incorrect. The problem was been modified to include just the last 2 parts. Quiz 2 will NOT include a question based on Problem 3.

(a) The periodic signal in Figure 2 switches between 0 Hz and 500 Hz every 5 seconds. Explain mathematically why this signal has infinite bandwidth.

(b) When a signal has many frequency components extending to infinity, such as in this case, we usually define an “effective bandwidth” beyond which the frequency components are not very large. For this signal, we will consider all frequency components that are larger than 20% of the largest frequency component (i.e. the “peak” value in the spectrum), and ignore all other frequency components. Under this definition, what is the bandwidth of the signal (the difference between the largest and smallest positive frequencies)?

Hint: The math we developed while discussing the switching modulator for AM (using the diode bridge) will come in handy.