Multi-User Diversity vs. Accurate Channel State Information in MIMO Downlink Channels

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Abstract—In a multiple transmit antenna, single antenna per receiver downlink channel with limited channel state feedback, we consider the following question: given a constraint on the total system-wide feedback load, is it preferable to get low-rate/coarse channel feedback from a large number of receivers or high-rate/high-quality feedback from a smaller number of receivers? Acquiring feedback from many receivers allows multi-user diversity to be exploited, while high-rate feedback allows for very precise selection of beamforming directions. We show that there is a strong preference for obtaining high-quality feedback, and that obtaining near-perfect channel information from as many receivers as possible provides a significantly larger sum rate than collecting a few feedback bits from a large number of users. In terms of system design, this corresponds to a preference for acquiring high-quality feedback from a few users on each time-frequency resource block, as opposed to coarse feedback from many users on each block.

I. INTRODUCTION

Multi-user multiple-input, multiple-output (MU-MIMO) communication is very powerful and has recently been the subject of intense research. A transmitter equipped with $N_t$ antennas can serve up to $N_t$ users simultaneously over the same time-frequency resource, even if each receiver has only a single antenna. Such a model is relevant to many applications, such as the cellular downlink from base station (BS) to mobiles (users). However, knowledge of the channel is required at the BS in order to fully exploit the gains offered by MU-MIMO.

In systems without channel reciprocity (such as frequency-division duplexed systems), the BS obtains Channel State Information (CSI) via channel feedback from mobiles. In the single antenna per mobile setting, feedback strategies involve each mobile quantizing its $N_t$-dimensional complex channel vector and feeding back the corresponding bits approximately every channel coherence time.

Although there has been considerable prior work on the MIMO downlink channel feedback [1]–[9], there is a dichotomy between results on systems with a small number of receivers/users (on the order of $N_t$) and systems with many users. The throughput of systems with the number of users on the order of $N_t$ has been shown to be extremely sensitive to the accuracy of the CSI, and thus many feedback bits are needed from each user in order to achieve a large sum rate.

This has been shown from a fundamental information theoretic perspective [5] [10], as well as in terms of particular transmit strategies. For example, Zero-Forcing (ZF) beamforming has been shown to require CSI quality that scales proportional to the SNR in dB [2] [3]. At the other extreme, it has been shown that systems can achieve a very large sum rate even with very few feedback bits per user, in the asymptotic limit as the number of users is taken to infinity. For example, Random Beamforming (RBF) requires only $\log_2 N_t$ bits and one real number per user [1].

The presence of these two extremes leads to a natural question: how should a MIMO downlink system be designed such that feedback resources are most efficiently utilized to maximize sum rate? In order to answer this question, the aggregate feedback load in the system must be considered, instead of the conventional focus on the per-user feedback load.

Fig. 1. Two possible design strategies for a MIMO downlink system

An example helps illustrate the importance of this design question. Consider a MIMO downlink system with 25 users and 5 independent coherence bands (in frequency). One system design is to have all 25 users feed back 4 bits of quantized CSI for each of the 5 coherence bands, and then select, for each band, a subset of users for beamforming, as depicted by Design 1 in Figure 1. An alternative is Design 2, where the 25 users are divided into 5 arbitrary (or randomly assigned) groups of 5 users each, where each group is assigned a single band\(^1\). Each user then feeds back 20 bits of quantized CSI for its assigned band, and for each band a subset of the 5 eligible users is chosen for beamforming, in that band. Although both of the above designs use 100 bits of feedback per coherence band, it is not clear which design provides a larger sum rate.

\(^1\)This grouping is random or arbitrary, and the number of actual users is immaterial. This model also applies when users are grouped based on scheduling priorities, and hence the grouping process should not incur significant additional overhead. Alternatively, users can also be grouped based on their instantaneous channel, but this would introduce a random-access component to the feedback link and is not considered in the present work - see Section II for a short discussion.

Manuscript received July 27, 2010; revised May 2, 2011 and October 21, 2011. The material in this paper was presented in part at the IEEE International Conference on Communications, Beijing, China, May 2008.

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Thereby motivated, in this paper we ask the following fundamental design question, which we do not believe has been sufficiently addressed before: *Given a constraint on the aggregate number of feedback bits, is a larger system sum rate achieved by collecting a few feedback bits from a large number of users, or by collecting more bits from fewer users?*

Assuming an aggregate feedback load of $T_{fb}$ bits (for a block of symbols over which the channel is constant), we consider a system where $T_{fb}/B$ users quantize their channel direction to $B$ bits each and feed back these bits along with one real number (per user) representing the channel quality. The BS then selects, based upon the feedback received from the $T_{fb}/B$ users, up to $N_t$ users for transmission using multi-user beamforming. The parameter $B$, which is an offline design parameter, allows the system designer to choose the desired mix of multi-user diversity and CSI accuracy: a small value of $B$ emphasizes multi-user diversity over CSI accuracy, whereas CSI is emphasized if a large value of $B$ is chosen.

In a real system, this optimization can be interpreted as determining the appropriate grouping of mobiles in frequency and/or time, as illustrated in the earlier example. It is important to realize that the problem considered here is an *offline* optimization, meaning that the per-user feedback rate is decided beforehand for a given $T_{fb}$, $\text{sNIR}$ and $N_t$, and does not vary dynamically from one coherence block to another (i.e., $B$ is a deterministic function of $T_{fb}$, $\text{sNIR}$ and $N_t$). Considering dynamic optimization would only further improve the performance and importance of this approach, but we only consider offline optimization in this work.

In the example introduced in Figure 1, $T_{fb} = 100$ bits and Design 1 has 25 users feeding back $B = 4$ bits of information each for each band, thereby offering the BS many users to select from (more multi-user diversity), albeit with limited CSI accuracy. On the other hand, Design 2 has 5 users feeding back information per band which reduces the multi-user diversity available, but each of the 5 users feed back $B = 20$ bits of information, which corresponds to much more accurate CSI. Assuming $N_t = 4$, $\text{sNIR} = 10$ dB, ZF beamforming and greedy user selection, Design 1 achieves a sum rate of 4.67 bps/Hz (per band), while Design 2 achieves a much higher sum rate of 9.97 bps/Hz. Clearly, choosing the wrong value of $B$ can lead to a substantial sum rate loss.

By comparing the sum rates of different beamforming and user selection schemes at different values of $B$, we reach the following simple but striking conclusion: for almost any number of antennas $N_t$, average SNR, and feedback budget $T_{fb}$, *sum rate is maximized by choosing $B$ (feedback bits per user) such that near-perfect CSI is obtained for each of the $T_{fb}/B$ users that do feedback.* In other words, we find that accurate CSI is more valuable than multi-user diversity.

For the above comparison, we consider commonly proposed RBF-based strategies and ZF with greedy user selection, although we find that our basic result holds even when other beamforming and user selection schemes are included. Further, we find that ZF generally outperforms RBF, which is rather surprising in the context of prior work on schemes with a very small per-user feedback load. For example, RBF achieves a sum rate that scales with the number of users in the same manner as the perfect-CSI sum rate [1]. Conventional wisdom has been that RBF could compensate for coarse feedback through multi-user diversity and outperform ZF if there were enough users. But on the contrary, we find that RBF achieves a significantly smaller sum rate than a ZF-based system with highly accurate CSI, when both systems have the same aggregate feedback load, $T_{fb}$ bits. This is true even when $T_{fb}$ is extremely large, in which case the number of users feeding back under RBF is also very large, and thus multi-user diversity is plentiful. Because much of our work is supported by numerical results, the associated MATLAB code has been made available online [11].

**II. RELATED WORK**

While there already exists a large body of work focusing on ZF, RBF and Per unitary basis stream user and rate control (PU2RC) [12] [6] for the MIMO downlink, we believe that the questions considered in our work have not been addressed before, and are critical with respect to maximizing the sum rate. Prior work has compared ZF and PU2RC assuming a fixed number of per-user feedback bits and users [6], and it is shown that for certain combinations of bits and users, PU2RC outperforms ZF. As we shall see, our findings are quite different because we allow additional flexibility in terms of choosing the per-user feedback rate subject to a constraint on the aggregate feedback load. Comparisons with a fixed per-user feedback rate make sense for wireless standards that are currently being finalized and lack the flexibility to vary per-user feedback, however, we argue that this results in a significant loss in sum rate, and that future standards should certainly consider feedback from an aggregate rather than per-user perspective, with per-user feedback chosen optimally.

A closely related work is [13], where the tradeoff between multi-user diversity and accurate CSI is studied in the context of two-stage feedback. In the first stage all users feed back coarse estimates of their channel, based on which the transmitter runs a selection algorithm to select $N_t$ users who feed back more accurate channel quantization during the second feedback stage, and the split of the feedback budget between the two stages is optimized. Our work differs in that we consider only a *single stage* approach, and more importantly in that we optimize the number of users (randomly selected) who feed back accurate information rather than limiting this number to $N_t$. Indeed, this optimization is precisely why our approach shows such large gains over simple RBF or unoptimized ZF.

There has also been related work on systems with *channel-dependent* feedback, in which each user determines whether or not to feed back on the basis of its current channel condition (i.e., channel norm and quantization error) [6] [7] [14] [15] [16]. As a result, the BS does not *a priori* know who feeds back for each coherence block, and thus there is a random-access component to the feedback. Channel-dependent feedback intuitively appears to provide an advantage because only users with good channels feed back. Although some of this prior work has considered aggregate feedback load (c.f., [17]), that work has not considered optimization of
To the transmitter, where constraint is the vector of channel input symbols transmitted by the user $x$ (AWGN),

$$h_k \sim \mathcal{CN}(0, 1)$$

with elements constant within a block but vary independently from block to block according to a block-fading model, where the channels are assumed to be known perfectly. Here (meaning the users who do feed back are arbitrary in terms of their channel conditions).

III. SYSTEM MODEL & BACKGROUND

We consider a multi-input multi-output (MIMO) Gaussian downlink channel in which the Base Station (abbreviated BS, also called transmitter) has $N_t$ antennas and each of the users or User Terminals (abbreviated UT, also called mobile or stationary) have 1 antenna each (Figure 2). The channel output $y_k$ at user $k$ is given by:

$$y_k = h_k^T x + z_k, \quad k = 1, \ldots, K$$

where $z_k \sim \mathcal{CN}(0, 1)$ models Additive White Gaussian Noise (AWGN), $h_k \in \mathbb{C}^{N_t \times 1}$ is the vector of channel coefficients from the $k^{th}$ user antenna to the transmitter antenna array and $x$ is the vector of channel input symbols transmitted by the base station. The channel input is subject to an average power constraint $E[|x|^2] \leq \text{SNR}$. We assume that the channel state, given by the collection of all channel vectors, varies in time according to a block-fading model, where the channels are constant within a block but vary independently from block to block. The entries of each channel vector are i.i.d. Gaussian with elements $\sim \mathcal{CN}(0, 1)$. Each user is assumed to know its own channel perfectly.

At the beginning of each block, each user quantizes its channel to $B$ bits, feeds back the bits, in an error- and delay-free manner, to the BS (see Figure 2). Vector quantization is performed using a codebook $\mathcal{C}$ that consists of $2^B \times N_t$ dimensional complex unit norm vectors $C \doteq \{w_1, \ldots, w_{2^B}\}$. Each user quantizes its channel vector to the quantization vector that forms the minimum angle to it. Thus, user $k$ quantizes its channel to $\hat{h}_k$ and feeds the $B$-bit index back to the transmitter, where $\hat{h}_k$ is chosen according to:

$$\hat{h}_k = \arg \min_{w \in \mathcal{C}} \sin^2 (\angle(h_k, w)) \cdot (1 - \angle(h_k, w))$$

where $\cos^2(\angle(h_k, w)) = \frac{|h_k^T w|^2}{|h_k|^2 |w|^2} = 1 - \sin^2(\angle(h_k, w))$. The specifics of the quantization codebook are discussed later.

Each user also feeds back a single real number, which can be the channel norm or some other Channel Quality Indicator (CQI). We assume that this CQI is known perfectly to the BS, i.e., it is not quantized, and thus CQI feedback is not included in the feedback budget; this simplification is investigated in Section VII-B.

For a total aggregate feedback load of $T_{fb}$ bits, we are interested in the sum rate (of the different feedback/baemforming strategies described later in this section) when $T_{fb} / B$ users feed back $B$ bits each. The $T_{fb} / B$ users who feed back are arbitrarily selected from a larger user set. Furthermore, in our block fading setting, only those users who feed back in a particular block/coherence time are considered for transmission in that block; in other words, we are limited to transmitting to a subset of only the $T_{fb} / B$ users.

There are many schemes proposed for the MIMO downlink, and investigating all of these would be beyond the scope of this work. However, we believe that ZF and PU$^2$RC represent and capture the two basic ideas associated with most MIMO downlink strategies: that of exploiting accurate per-user CSI, and that of exploiting multi-user diversity, respectively. Further, ZF and PU$^2$RC have received a lot of attention in the research community, and we have therefore chosen to analyze these.

A. Zero Forcing Beamforming

We assume Zero-Forcing (ZF) beamforming with the channel norm $|h_k|$ being fed back perfectly as the CQI. The BS then uses the greedy user selection algorithm described in [18], adopted to imperfect CSI by treating the vector $|h_k| : \hat{h}_k$ as if it were user $k$’s true channel, and allocated power equally among the selected users.

We denote the indices of selected users by $\Pi(1), \ldots, \Pi(n)$, where $n \leq N_t$ is the number of users selected. By the ZF criterion, the unit-norm beamforming vector $\hat{v}_{\Pi(k)}$ for user $\Pi(k)$ is chosen in the direction of the projection of $\hat{h}_{\Pi(k)}$ on the nullspace of $\{\hat{h}_{\Pi(j)}\}_{j \neq k}$. Although ZF beamforming is used, there is residual interference because the beamformers are based on imperfect CSI. The (post-selection) SINR for selected user $\Pi(k)$ is

$$\text{SINR}_{\Pi(k)} = \frac{\text{SNR}}{1 + \sum_{j \neq k} \text{SNR} \frac{|\hat{h}_{\Pi(k)}|^2 \cos^2(\angle(\hat{h}_{\Pi(k)}, \hat{v}_{\Pi(k)}))}{|\hat{h}_{\Pi(k)}|^2 + \sum_{j \neq k} \cos^2(\angle(\hat{h}_{\Pi(k)}, \hat{v}_{\Pi(k)}))}}$$

and the corresponding sum rate is $\sum_{k=1}^n \log_2 (1 + \text{SINR}_{\Pi(k)})$.

For the sake of analysis and ease of simulation, each user utilizes an independently generated quantization codebook $C$ consisting of unit-vectors independently chosen from the isotropic distribution on the $2N_t$-dimensional unit sphere [19] (Random Vector Quantization or RVQ). The sum rate is averaged over this ensemble of quantization codebooks.

The RVQ process can be easily simulated for arbitrary values of $B$; see [20, Appendix B] for details.
\[ R_{ZF} \left( \frac{\text{SNR}, N_t, T_{fb}}{B}, B \right) = \mathbb{E} \left[ \sum_{k=1}^{n} \log_2 \left( 1 + \frac{\text{SNR}}{n} ||\mathbf{h}_k||^2 \sum_{j \neq k} \cos^2 \left( \angle \left( \mathbf{h}_k, \hat{\mathbf{v}}_{ij} \right) \right) \right) \right] \]  

\[ B = 10 \] corresponds to a sum rate loss of 4 bps/Hz (relative to perfect CSI) and 17 bits are required to reduce this loss to 1 bps/Hz.

**B. Random Beamforming**

Random beamforming (RBF) was proposed in [1] [21], wherein each user feeds back \( \log_2 N_t \) bits along with one real number. In this case, there is a common quantization codebook \( \mathcal{C} \) consisting of \( N_t \) orthogonal unit vectors and quantization is performed according to (2). In addition to the quantization index, each user feeds back a real number representing its SINR. If \( w_m (1 \leq m \leq N_t) \) is the selected quantization vector for user \( k \), then \( \mathbf{w}_m \) is given by

\[ \frac{\text{SNR}}{N_t} ||\mathbf{h}_k||^2 w_{m,k}^2}{1 + \frac{\text{SNR}}{N_t} \sum_{n \neq m} ||\mathbf{h}_n||^2 w_{n,m}^2} = ||\mathbf{h}_k||^2 \cos^2 \angle \mathbf{h}_k, \mathbf{w}_m \]  

\[ \frac{\text{SNR}}{N_t} \sum_{n \neq m} ||\mathbf{h}_n||^2 w_{n,m}^2} = ||\mathbf{h}_k||^2 \sin^2 \angle \mathbf{h}_k, \mathbf{w}_m \]  

After receiving the feedback, the BS selects the user with the largest SINR on each of the \( N_t \) beams \( (\mathbf{w}_1, \ldots, \mathbf{w}_{N_t}) \), and beamforming is performed along these same vectors.

**C. PU²RC**

Per unitary basis stream user and rate control (PU²RC), proposed in [12] (a more widely available description can be found in [6]), is a generalization of RBF in which there is a common quantization codebook \( \mathcal{C} \) consisting of \( 2^{B - \log_2 N_t} \) ‘sets’ of orthogonal codebooks, where each orthogonal codebook consists of \( N_t \) orthogonal unit vectors, and thus a total of \( 2^B \) vectors. Quantization is again performed according to (2), and each user feeds back the same SINR statistic as in RBF. User selection is performed as follows: for each of the orthogonal sets the BS repeats the RBF user selection procedure and computes the sum rate (where the per-user rate is \( \log_2 (1 + \text{SNR}) \)), after which it selects the orthogonal set with the highest sum rate. If \( B = \log_2 N_t \), there is only a single orthogonal set and the scheme reduces to ordinary RBF.

For each of the two strategies ZF and PU²RC, we determine an optimal \( B \) that maximizes the respective sum rate and compare the resulting rates. PU²RC and ZF differ in that the latter strategy involves a dedicated downlink training phase, while the former does not. Also, PU²RC is restricted to selecting users within one of the orthogonal sets and thus has very low complexity, whereas the described ZF technique has no such restriction. We show, however, in Section VII-A, that our results hold even when ZF with selection strategies of comparable complexity is used.

**IV. OPTIMIZATION OF ZERO-FORCING BEAMFORMING**

Let \( R_{ZF} (\text{SNR}, N_t, T_{fb}, B) \) be the sum rate for a system using ZF with \( N_t \) antennas at the transmitter, signal-to-noise ratio \( \text{SNR} \), and \( T_{fb} \) users each feeding back \( B \) bits. From Section III-A, we have Equation (5), where the expectation is carried out over channels, as well as random codebooks. No closed form for this expression is known to exist, even in the case of perfect CSI, but this quantity can be easily computed via Monte Carlo simulation. We are interested in the number of feedback bits per user \( B_{ZF} (\text{SNR}, N_t, T_{fb}) \) that maximizes this sum rate for a total feedback budget of \( T_{fb} \):

\[ R_{ZF}^{OPT} (\text{SNR}, N_t, T_{fb}) = \arg \max_{B \leq \log_2 N_t} R_{ZF} (\text{SNR}, N_t, T_{fb}, B). \]

Although this optimization is not analytically tractable, it is well behaved and can be meaningfully understood (further, Monte Carlo simulations combined with a line-search can be used to numerically compute the optimal point).

\[ \text{Fig. 3. Sum rate Vs. Feedback load for Zero-forcing} \]

Consider first Figure 3, where the sum rate \( R_{ZF} (\text{SNR}, N_t, T_{fb}, B) \) is plotted versus \( B \) for 2 and 4-antenna systems for various values of \( \text{SNR} \) and \( T_{fb} \). Based on this plot it is immediately evident that the sum rate increases very rapidly with \( B \), and that the rate-maximizing \( B_{ZF}^{OPT} \) is very large, e.g., in the range 15 – 20 and 20 – 25 for \( N_t = 4 \) at 5 and 10 dB, respectively. Both of these observations indicate a strong preference for accurate CSI over multi-user diversity.

The achievable sum rate with ZF and user selection is considered in [4], where the behavior of the sum rate is

\[ \text{Because the SINR in (4) is monotonically decreasing in the sine of the angle between the channel and the quantized vector, quantizing according to (2) is equivalent to quantizing to the beam that maximizes SINR.} \]

\[ \text{The optimization can alternatively be posed in terms of the numbers of users who feedback, i.e., } K \text{ users feedback } T_{fb}/K \text{ bits each. However, it turns out to be much more insightful to consider this in terms of } B, \text{ the feedback bits per user.} \]
characterized by identifying three regimes: the large user regime, the interference limited regime, and the high resolution regime. When the number of users feeding back is large, i.e., $T_h/B$ is large, the system operates in the large user regime, and the rate achievable with SINR feedback is detailed in [4, Eq. (41)]. In that regime, the achievable rate increases with the quantity $2^B(T_h/B)$, and thus increasing the per-user feedback by one bit is equivalent to maintaining the per-user feedback, but doubling the number of users. This effect is seen in Figure 4, where increasing the user pool from 10 to 100 results in a large increase in sum rate, but the rate grows very slowly with the number of users thereafter. For instance, the sum rate with 2000 users feeding back 15 bits each is the same as with 4000 users feeding back 14 bits each. Because $2^B(T_h/B)$ is increasing in $B$ (for $B > 2$), the sum rate is increasing in $B$ within the large user regime when the aggregate feedback load is fixed, and thus it is sub-optimal to operate in the large user regime. As $B$ increases, $T_h/B$ decreases, thus eventually driving the system away from the large user regime and into the high resolution regime. We will see shortly that it is similarly sub-optimal to remain in the interference limited regime as well, and hence it is important to understand the behavior of sum rate in the high resolution regime to truly understand the optimization problem.

Unfortunately, the characterization of the sum rate for the high resolution regime presented in [4, Eq. (47)] is not accurate enough to capture the behavior for the purposes of our optimization (and results in a trivial optimal point). Thus, in order to more precisely understand this behavior, we introduce the following sum rate approximation, denoted by $R_{ZF}$ ($SNR, N_t, T_h/B, B$), given by:

$$N_t \log_2 \left[ 1 + \frac{\left( SNR \over N_t \right) \log \left( T_h N_t \over B \right)}{1 + \left( SNR \over N_t \right) 2^{-B \over \eta + 1} \log \left( T_h N_t \over B \right)} \right],$$

where $R_{ZF} \approx \tilde{R}_{ZF}$. This approximation is obtained from the expression for $R_{ZF}$ in (5) by (a) assuming that the maximum number of users are selected (i.e., $n = N_t$), (b) replacing each $cos^2 \left( \angle (h_{HI(k)}, \tilde{v}_{HI(j)}) \right)$ in the SINR denominator with its expected value $2^{-B \over \eta + 1} / (N_t - 1)$ [2, Lemma 2], and (c) approximating the $cos^2 \left( \angle (h_{HI(k)}, \tilde{v}_{HI(j)}) \right)$ term in the SINR numerator with unity. Further, $||h_{HI(k)}||^2$ is treated as the largest channel norm among $T_h/B$ users, which stochastically dominates the maximum of $T_h N_t / B$ random variables with $\Gamma(1, 1)$ distribution, the latter having an expected value which behaves as $log \left( T_h N_t / B \right)$, for large $T_h$.

The approximation (6) is not meant to accurately capture the sum rate, but rather to capture the effect of $B$, and we will see that this captured with reasonable accuracy. Further, when the number of users feeding back is large, $||h_{HI(k)}||^2$ indeed behaves like the logarithm of the number of users available for selection (see [4]) and (c) becomes valid with high probability. The value of $n$ is difficult to characterize, but when the number of users feeding back is large, we generally expect to select $N_t$ users (except, when the $SNR$ is very small). Hence, (b) is merely Jensen’s inequality, and (6) becomes a lower bound to the sum rate when the number of users is large, and $SNR$ is large enough to warrant selection of $N_t$ users.

We first use the approximation to explain the rapid sum rate increase with $B$. From (6) we see that increasing $B$ by $N_t - 1$ bits reduces the interference power by a factor of 2. As long as the interference power is significantly larger than the noise power, this leads to (approximately) a 3 dB SINR increase and thus a $N_t$ bps/Hz sum rate increase. When the system is interference limited, the two instances of 1 in (6) can be dropped, which gives $R_{ZF} (SNR, N_t, T_h/B, B) \approx B N_t / (N_t - 1)$, i.e., sum rate increases almost linearly with $B$ for smaller values of $B$, consistent with Fig. 3, and the BS should take advantage of the maximum possible CSI accuracy that it can gather.
$\text{SNR} = 10$ dB. Motivated by [2, Theorem 1] (see Section III-A for discussion), we approximate the sum rate by the perfect CSI sum rate minus a multi-user interference penalty term:

$$R_{ZF} \left( \text{SNR}, N_t, \frac{T_{fb}}{B} B \right) \approx R_{ZF} \left( \text{SNR}, N_t, \frac{T_{fb}}{B}, \infty \right) - N_t \log_2 \left( 1 + \frac{\text{SNR}}{N_t} 2^{-\frac{T_{fb} N_t}{B}} \log \frac{T_{fb} N_t}{B} \right)$$

(7)

This penalty term reasonably approximates the loss due to imperfect CSI which is indicated in Figure 5. In the figure we see that for $B > 25$ the sum rate curves for perfect and imperfect CSI essentially match and thus the penalty term in (7) is nearly zero. As a result, it clearly does not make sense to increase $B$ beyond 25 because doing so reduces the number of users but does not provide a measurable CSI benefit. Keeping this in mind, the most interesting observation gleaned from Figure 5 is that $B_{ZF}^{\text{OPT}}$ corresponds to a point where the loss due to imperfect CSI is very small. In other words, it is optimal to operate at the point where effectively the maximum benefit of accurate CSI has been reapplied.

Although $B_{ZF}^{\text{OPT}}$ is quite large for many parameter choices, it appears to not be particularly dependent on the total feedback budget $T_{fb}$, but is very sensitive to (and increasing with) $\text{SNR}$ and $N_t$. We explain this by computing the optimal $B$ corresponding to (6), that is, $B_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right) \approx \tilde{B}_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right)$, where:

$$\tilde{B}_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right) \triangleq \arg \max_{\log_2 N_t \leq B \leq \frac{T_{fb}}{2}} \tilde{R}_{ZF} \left( \text{SNR}, N_t, \frac{T_{fb}}{B}, B \right).$$

The approximation is concave in $B$, and thus the following fixed point characterization of $\tilde{B}_{ZF}^{\text{OPT}}$ is obtained by setting the derivative of $\tilde{R}_{ZF} \left( \text{SNR}, N_t, \frac{T_{fb}}{B}, B \right)$ to zero:

$$\frac{\text{SNR}}{N_t} 2^{-\frac{\tilde{B}_{ZF}^{\text{OPT}}}{N_t}} \frac{\tilde{R}_{ZF}^{\text{OPT}} \log 2}{N_t - 1} \left( \log \frac{T_{fb} N_t}{B_{ZF}^{\text{OPT}}} \right)^2 = 1. \quad (8)$$

This quantity is easily computed numerically, but a more analytically convenient form is found by rewriting (8) as:

$$\tilde{R}_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right) = - \frac{N_t - 1}{\log 2} \left( 1 - \frac{\text{SNR}}{N_t} L \right) \quad (9)$$

where $L = \left( \log T_{fb} N_t / (\tilde{B}_{ZF}^{\text{OPT}}) \right)^2$ and $W_{-1} \left( \cdot \right)$ is branch -1 of the LambertW function [22]. Using the asymptotic expansion [22, Equation 4.19] for $W_{-1} \left( -x \right)$ and small $x > 0$ in (9), we have Equation 10.

By repeatedly applying the asymptotic expansion of $W_{-1} \left( \cdot \right)$ to the occurrences of $L$ in (10), we have that $\tilde{B}_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right)$ varies as $O(\log \log T_{fb})$, $(N_t - 1) \log_2 \text{SNR} + O(\log \log N_t)$ and $(N_t - 1) \log_2 (\text{SNR}/N_t)$.

In Figures 6 and 7, $B_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right)$ and the approximation $\tilde{B}_{ZF}^{\text{OPT}} \left( \text{SNR}, N_t, T_{fb} \right)$ are plotted versus $T_{fb}$ and $\text{SNR}$, respectively (we restrict ourselves to values of feedback bits such that the number of users feeding back is an integer). In both figures we see that the approximation is quite accurate, and that the behavior agrees with the scaling relationships described previously.

V. OPTIMIZATION OF PU$^2$RC

Per unitary basis stream user and rate control (PU$^2$RC) generalizes RBF to more than $\log_2 N_t$ feedback bits per user. A common quantization codebook, consisting of $2^{B_t} / N_t$ ‘sets’ of $N_t$ orthogonal vectors each, is utilized by each user. A user finds the best of the $2^B$ quantization vectors, according to (2), and feeds back the index of the set $(B - \log_2 N_t)$ bits and the index of the vector/beam in that set $(\log_2 N_t)$ bits. Although the quantization codebooks for ZF and PU$^2$RC are slightly different, the key difference is in user selection. While $O(\log \log \log \text{SNR})$ for large $T_{fb}$, $N_t$ and $\text{SNR}$, respectively. These scaling results are consistent with the scaling results without user selection or optimization of $B$, presented in [2].
ZF allows for selection of any subset of (up to) $N_t$ users, the low-complexity PU$^2$RC procedure described in Section III-C constrains the BS to select a set of up to $N_t$ users from one of the $2^B/N_t$ sets.

As a result of this difference, a very different conclusion is reached when we optimize the per-user feedback load $B$ for PU$^2$RC: we find that $B = \log_2 N_t$ (i.e., RBF) is near-optimal and thus the optimization provides little advantage. Sum rate is plotted versus $B$ (for PU$^2$RC) in Figure 8. Very different from ZF, the sum rate does not increase rapidly with $B$ for small $B$, and it begins to decrease for even moderate values of $B$.

If $B$ is too large, the number of orthogonal sets $2^B/N_t$ becomes comparable to the number of users $T_{fb}/B$ and thus it is likely that there are fewer than $N_t$ users on every set (there are on average $T_{fb}/N_t$ users per set). For example, if $T_{fb} = 500$ and $B = 8$, there are $2^8$ orthogonal sets and 40 users and thus less than a user per set on average. Hence, the BS likely schedules much fewer than $N_t$ users, thereby leading to a reduced sum rate. For moderate values of $B > \log_2 N_t$, there are a sufficient number of users per set but nonetheless this ‘thinning’ of users is the limiting factor. As $B$ increases the quantization quality increases, but because there are only $T_{fb}/N_t$ users per set (on average) the multi-user diversity (in each set) decreases sharply, so much so that the rate per set in fact decreases with $B$. (For ZF there is also a loss in multi-user diversity as $B$ is increased, but the number of users is inversely proportional to $B$, whereas here it is inversely proportional to $2^B$.) The BS does choose the best set (amongst the $2^B/N_t$ sets), but this is not enough to compensate for the decreasing per-set rate. Hence, we find that PU$^2$RC strongly prefers multi-user diversity.

VI. COMPARISON OF MULTI-USER BEAMFORMING SCHEMES

In Figure 9, the sum rates of ZF and PU$^2$RC (with randomly generated codebooks) are compared for various values of SNR, $T_{fb}$ and $N_t$; for each strategy, $B$ has been optimized separately as discussed in Sections IV and V, respectively. It is seen that ZF maintains a significant advantage over PU$^2$RC for $N_t = 4$. At small $N_t$, both schemes perform similarly, but ZF maintains an advantage. Also included, is the performance of RBF/PU$^2$RC with the following enhancements: (a) the BS may choose to schedule fewer than $N_t$ beams (as proposed in [9]), (b) power is near-optimally (rather than uniformly) allocated across users using the iterative procedure proposed in [9], and (c) an optimal DFT-based codebook is designed, as suggested in [23]. Enhancements (a) and (b) require finer (if not perfect) knowledge of the channels and is hence an upper bound on what can be achieved with RBF/PU$^2$RC. In spite of these enhancements, we see that the performance of ZF is superior, and the advantage of ZF increases extremely rapidly with $N_t$ and $\text{SNR}$. Note the contrast with [6], where an unoptimized ZF system is found inferior to PU$^2$RC.

As optimized PU$^2$RC performs essentially the same as RBF (Section V), this large gap in sum rate can be explained by contrasting RBF and optimized ZF. From [1], we have that the SINR of the $k$th user (on a particular beam) under RBF has CDF $1 - e^{-\frac{\text{SNR}}{N_t} \log_2 \left(\frac{B}{N_t} \text{SNR} + 1\right)}$. By basic results in order statistics, the expectation of the maximum amongst $K$ i.i.d. random variables is accurately approximated by the point at which the CDF equals $(K - 1)/K$ [24]. Hence, in order to achieve a target SINR $S$, RBF requires about $K = \exp\left(S \frac{N_t}{\text{SNR}} (1 + S)^{-1}\right)$ users. Setting $S = \log_2 \left(\frac{\text{SNR} K}{N_t} \log_2 \left(\frac{B}{N_t} \text{SNR} + 1\right)\right)$, which approximately describes the ZF SINR with a total feedback budget of $T_{ZF}$ bits (while operating at near-perfect CSI, as described in Section IV), we see that RBF requires $K = \frac{T_{fb} N_t}{B T_{ZF}} \left(1 + \frac{\text{SNR}}{N_t} \log_2 \left(\frac{B T_{fb}}{T_{ZF} N_t} \text{SNR} + 1\right)\right) N_t^{-1}$ users to match the rate achieved by optimized ZF. Equivalently, RBF requires $(\log_2 N_t) \frac{T_{fb} N_t}{B T_{ZF}} \left(1 + \frac{\text{SNR}}{N_t} \log_2 \left(\frac{B T_{fb}}{T_{ZF} N_t} \text{SNR} + 1\right)\right) N_t^{-1}$ total bits to match the sum rate of optimized ZF with $T_{ZF}$ bits. When $N_t = 4$, $\text{SNR} = 5$ dB, RBF requires 5000 users (10000 total bits) in order to match the sum rate of ZF with only 300 bits. Clearly, it is impractical to consider RBF in such a setting, and the RBF bit requirement increases rapidly with $T_{ZF}$, $N_t$ as well as $\text{SNR}$, making RBF increasingly impractical.

Although RBF uses a very small codebook, it may appear that this is compensated by the large number of users, as has been suggested in prior work [1], as it might be possible to select users that are well-aligned to one of the $N_t$ quantization vectors/beamformers. To understand this, consider the smallest
quantization error amongst the $T_b/\log_2 N_t$ users, which, due to independence of the channels, is effectively the same as the error of a single codebook of size $B = \log_2 (T_b N_t/\log_2 N_t)$.

For example, with $N_t = 4$ and $T_b = 300$ bits, the best quantization error is only as good as an 8-bit quantization. As we saw in Section IV, the sum rate is very sensitive to quantization error and multi-user diversity cannot compensate for this. Hence, the “large-user” advantage of RBF is never realized, even as $T_b$ grows.

VII. FURTHER CONSIDERATIONS

A. Complexity considerations

While comparing ZF and RBF, we have not explicitly compared the complexity requirements for implementing these two algorithms. Particularly, RBF is considered a popular choice for beamforming because (a) user selection can be performed with $O(N_t/\log N_t)$ complexity [1], compared to $O(N_t^2)$ complexity with ZF and greedy selection [18], and (b) each user can quantize the channel with $O(N_t^2)$ complexity using a codebook that is uniform across users in the RBF scheme, compared to a worst-case exhaustive search of $O(N_t N_t^2)$ for an arbitrary vector codebook (these complexity calculations are expressed in terms of $N_t$, assuming $T_b$ and $B$ grow linearly with $N_t$ as per (10)). However, we show in this section that ZF-based selection and quantization methods that have complexity similar to RBF can still maintain the large advantage in sum rate.

![Graph showing sum rate with optimized $B$ and various user selection schemes with $N_t = 4$, $T_b = 300$ bits and $SNR = 5$, $10$ dB](image)

We first consider the following “simplified” user selection algorithm for ZF while addressing (a): the BS computes the ZF rate while assuming that the users with the $j$ largest channel norms are selected, for $j = 1, \ldots, N_t$, and then picks $j$ that provides the largest sum rate. This simple algorithm involves just $N_t$ sum rate computations. Figure 10 compares the sum rate for ZF with greedy and simplified selection, as well as RBF/PU^2RC. The selected user set with simplified selection is likely to have less orthogonal users than greedy selection and thus performs worse, but nonetheless is seen to outperform RBF/PU^2RC at the optimal operating point.

We address (b) using the the scalar quantization scheme proposed in [25] as an example, which independently quantizes the relative magnitudes and phases of each component of the channel vector with $O(N_t)$ complexity ($O(1)$ complexity per entry in the vector). From Figure 10, we see that scalar quantization provides a sum rate only slightly smaller than RVQ at the optimal point. However, because CSI is strongly preferred to multi-user diversity, it is still worthwhile to operate at the “essentially perfect” CSI point, even with a suboptimal quantization codebook, and the performance with scalar quantization is still competitive. For large $B$, which is typically the preferred operating point, the same quantizer can be used by all users with small probability of the quantized vectors being the same, allowing for uniformity of codebooks across users. For small $B$, structured vector codebooks can be used (with low complexity due to the small size) to avoid quantizing different vectors to the same direction.

The simplified schemes presented here for ZF have slightly higher complexity at the BS and slightly lower complexity at the user terminal compared to RBF, and can be considered to be of similar complexity overall. We note that they may not be the best low-complexity algorithms, but they serve to illustrate that complexity considerations do not invalidate our main conclusion.

B. CQI Feedback

Prior work has shown that CQI quantized to 3-4 bits (per user) is virtually the same as unquantized CQI [17] [4], and we have hence thus far considered only quantization of the channel direction. However, if we account for CQI feedback in the budget, the actual per-user feedback is the $B$ directional bits in addition to the CQI bits, meaning that by ignoring CQI bits we have artificially inflated the number of users. If the CQI bits are accounted for, strategies that utilize few directional bits become even less attractive, i.e., CQI bits make multi-user diversity more expensive, and thus our basic conclusion is unaffected. For example, with $N_t = 4$, $T_b = 300$ bits and $SNR = 10$ dB, ZF with unquantized CQI (i.e., not accounting for CQI feedback) is optimized with 13 users and $B = 23$, while ZF with CQI quantized to 4 bits per user is optimized with 10 users and $B = 26$, i.e., multi-user diversity is even less important in the latter case.

An alternative CQI is the expected SINR, as discussed in [4]:

$$\text{SNR} = \frac{1}{|h_k|^2 \cos^2 \theta (\angle h_k, \theta_k)},$$

which is similar to the CQI in RBF/PU^2RC. This allows the BS to select users that have not only large channels, but also small quantization errors. However, as presented earlier, the system preferably operates in the high resolution regime, as described in [4], and the advantage of SINR feedback over channel norm feedback is minimal in this regime as the quantization error is small, and there is no real difference between the two CQI feedback schemes at the optimal point. This is seen in Figure 10.

\footnote{The expected distortion of an idealized codebook that achieves the quantization upper bound in [26] is only a factor of $N_t^{-1}$ smaller than with RVQ, and thus it is also not possible to greatly improve upon RVQ either.}
C. Effect of Receiver Training and Feedback Delay

Both imperfect CSI at the users and/or delay in the channel feedback loop results in inherent imperfection in the CSI provided to the BS, which results in additional multi-user interference, even if $B$ is extremely large. This can be accounted for by reducing the $\text{SNR}$ [8], which suggests that our results can be applied to this setting, but with a shift in $\text{SNR}$. In general, the case of imperfect CSI at the users is a complicated problem, and a more complete study would be required to fully characterize the effects, which is beyond the scope of this paper.

VIII. CONCLUSION

In this paper, we have considered the basic but apparently overlooked question of whether low-rate feedback/many user systems or high-rate feedback/limited user systems provide a larger sum rate in MIMO downlink channels. This question simplifies to a comparison between multi-user diversity (many users and accurate channel information (high-rate feedback)), and the surprising conclusion is that there is a very strong preference for accurate CSI. Multi-user diversity provides a throughput gain that is only double-logarithmic in the number of users who feed back, whereas the marginal benefit of increased per-user feedback is very large up to the point where the CSI is essentially perfect.

In terms of channel correlation across time and frequency, we note that a recent work has studied a closely related tradeoff in the context of a frequency-selective channel [27]: should each user quantize its entire frequency response or only a small portion of the frequency response (i.e., quantize only a single resource block)? The first option corresponds to coarse CSI (even though frequency-domain correlation is exploited) but a large user population, while the second corresponds to accurate CSI but fewer users per resource block. Consistent with our results, the second option is seen to provide a considerably larger sum rate than the first. We suspect the same holds true in the context of temporal correlation, where the comparison is between a user quantizing its channel across many continuous blocks (possibly exploiting the correlation of the channel by using a differential quantization scheme) and a user finely quantizing its current channel at only a few limited time instants.

In closing, it is worth emphasizing that our results do not imply that multi-user diversity is inconsequential. On the contrary, multi-user diversity actually provides a significant benefit. However, the basic design insight is that feedback resources should first be used to obtain accurate CSI and only afterwards be used to exploit multi-user diversity. Given the increasing importance of multi-user MIMO in single-cell and multi-cell (i.e., network MIMO) settings, it seems that this point should be fully exploited in the design of future cellular systems.

REFERENCES


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